Inverse functions and logarithms

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Examples:

1. $f : \mathbb{R} \to \mathbb{R}$ defined by $x \mapsto x^2$; e.g.

$$\begin{array}{ccccc} 1 & -2 & -\pi & 1/3 & \text{etc.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ 1 & 4 & \pi^2 & 1/9 & \end{array}$$

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2. $f : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ defined by $x \mapsto |\sqrt{x}|$; e.g.

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Note that $\sqrt{x}$ is only a function when we go to extra effort to decide that we're always going to choose the positive answer.

3. Let bacteria grow, and measure population over time.
Consider $N : \mathbb{N} \to \mathbb{N}$ by $N(t) = \#$ bacteria at time t.

t (hours)	$N(t)$ = pop. at time t
0	100
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Answer: between 4 and 5 hours

Inverse functions

Given a function f, the inverse function f^{-1} is the machine that takes in f's output, and returns the corresponding input.

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Non-Example: If $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is given by

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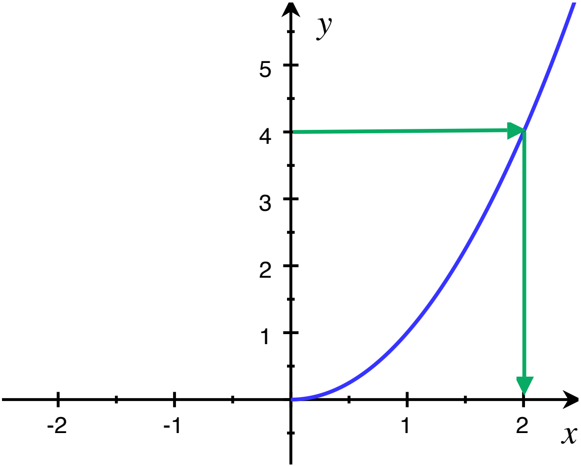

If $y = x^2$ and $x \geq 0$, then $x = |\sqrt{y}|$.

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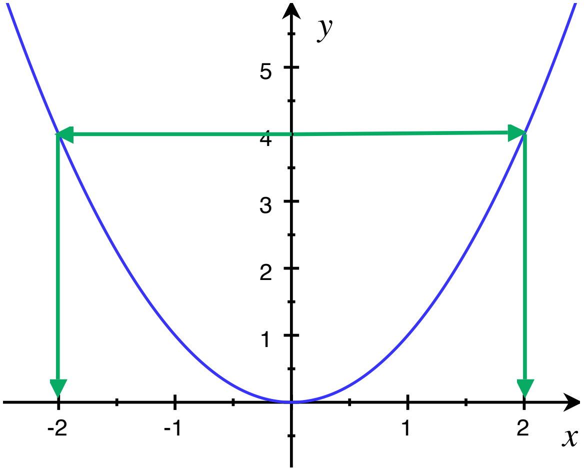
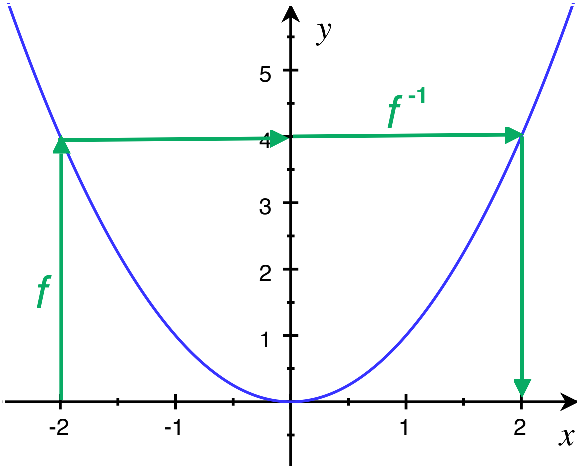


If $y = x^2$, then $x = |\sqrt{y}|$ or $-|\sqrt{y}|$. Which one???

If $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is given by

$f(x) = x^2$, then $|\sqrt{x}|$ is not the inverse.



If $y = x^2$ and $x < 0$, then $x \neq |\sqrt{y}|$!

When is a function invertible?

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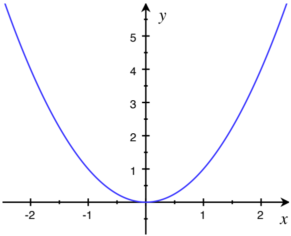

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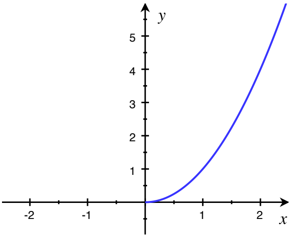

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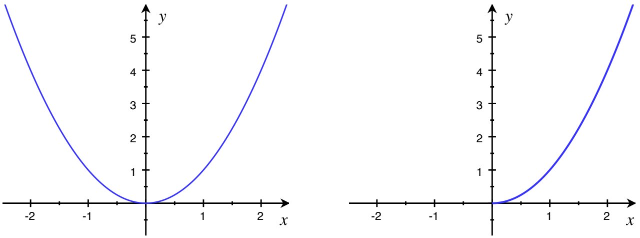

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Horizontal line test: A function is one-to-one if and only if no horizontal line intersects the function's graph more than once.

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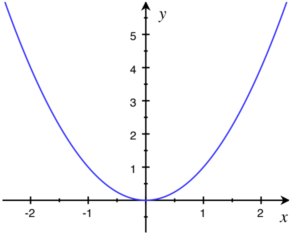

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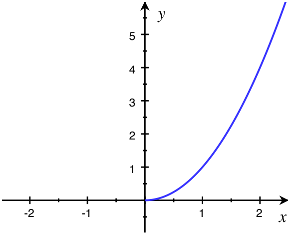

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Answer: A function is invertible if and only if it is one-to-one.

Graphing inverses

For a one-to-one function f, we have

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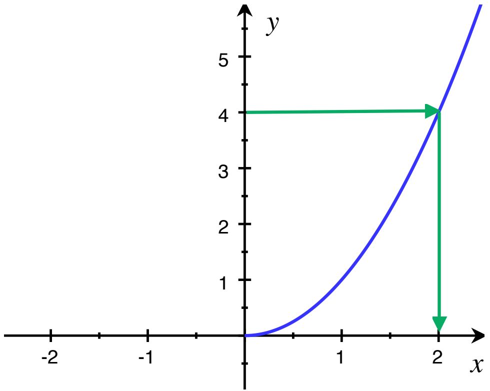

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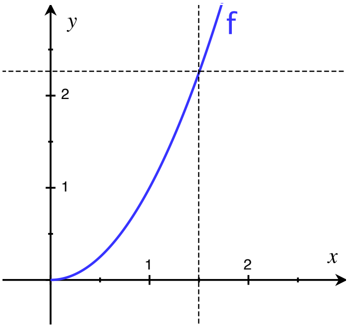

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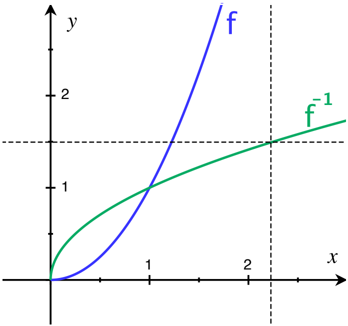

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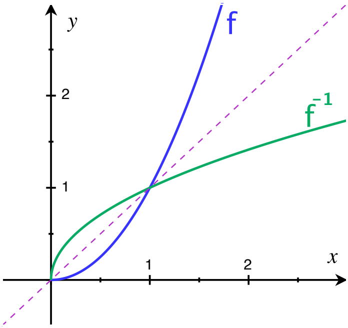

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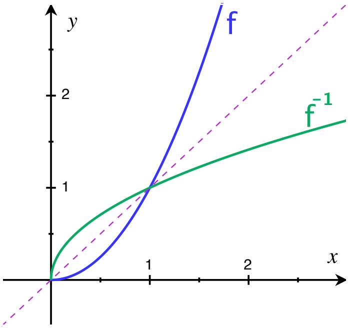

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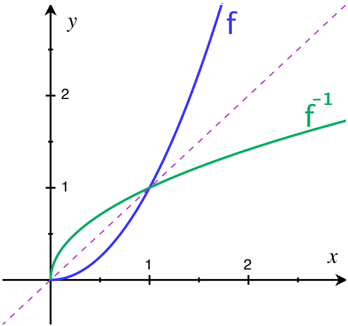


The graph of $y = f^{-1}(x)$ is the reflection of the graph of f over the line $y = x$ (i.e. swap the axes). Further,

the domain of f is the range of f^{-1},

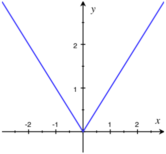
and

the range of f is the domain of f^{-1}.

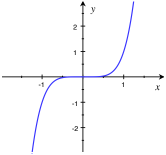
You try:

- For each of the following functions, (a) give the domain and range of f, and (b) decide if f is invertible.
- If f is invertible, then (c) sketch a graph of f^{-1}, (d) give the domain and range of f^{-1}, and (e) try to write a formula for f^{-1}.
- If f is not invertible over all of the real numbers, what is a restricted domain over which f is invertible? Over that restricted domain, do (c) and (d) from above.

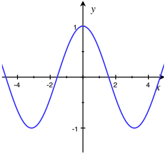
(1) $f(x) = |x|$



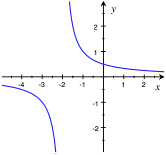
(2) $f(x) = x^5$



(3) $f(x) = \cos(x)$



(4) $f(x) = 1/(x+2)$



Calculating the inverse function algebraically

Given an invertible f, solve for f^{-1} by setting $f(y) = x$, and solving for $y = f^{-1}(x)$.

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You try:

1. Check that if $f(x) = x^3 + 2$ then $f^{-1}(x) = (x-2)^{1/3}$ by calculating $f(f^{-1}(x))$ and $f^{-1}(f(x))$.
2. For the following functions, calculate $f^{-1}(x)$ and verify your answer as above. (a) $f(x) = 3/(x-1)$ (b) $f(x) = 5\sqrt{x-2}$

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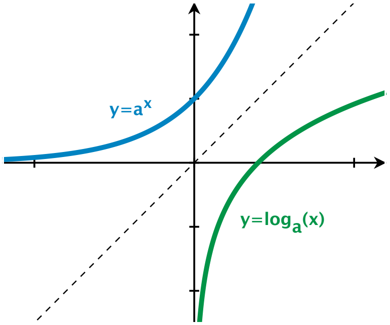
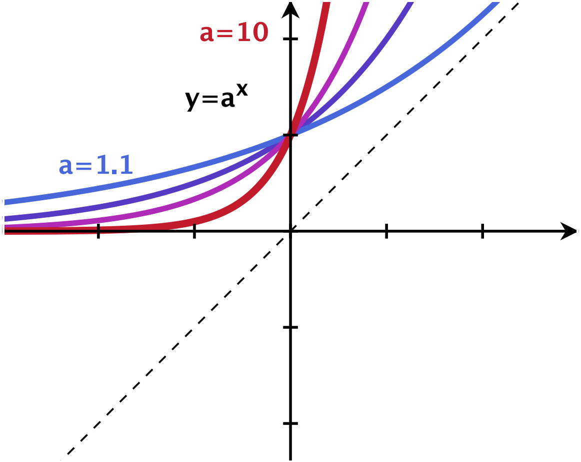


Properties of Logarithms



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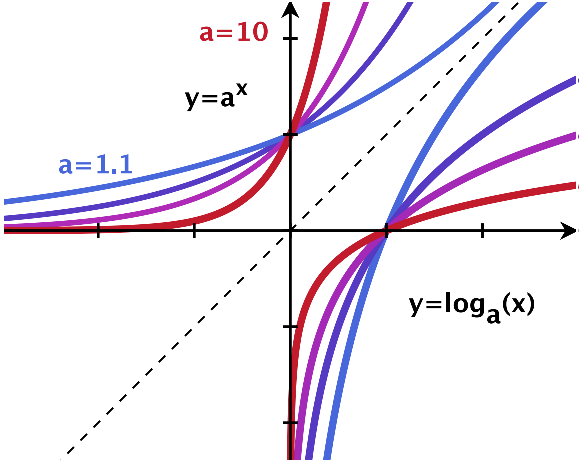

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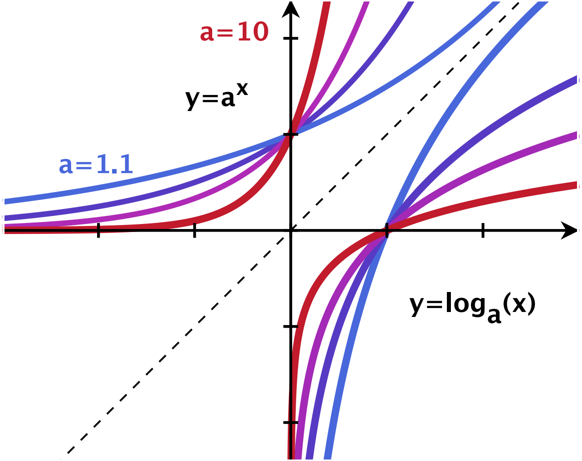
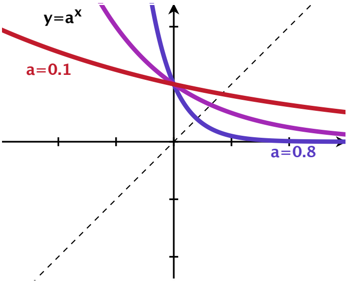
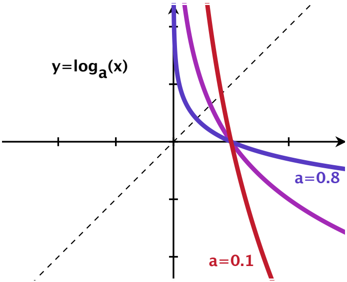


Domain: $(0, \infty)$ i.e. all $x > 0$ Range: $(-\infty, \infty)$ i.e. all x

Properties of Logarithms

$0 < a < 1$:





Domain: $(0, \infty)$ i.e. all $x > 0$

Range: $(-\infty, \infty)$ i.e. all x

Properties of Logarithms

Since. . . we know. . .

Properties of Logarithms

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1. $a^0 = 1$

we know. . .

1. $\log_a(1) = 0$

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2. $a^1 = a$

we know…

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Properties of Logarithms

Since...

1. $a^0 = 1$
2. $a^1 = a$
3. $a^b * a^c = a^{b+c}$

we know...

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3. $\log_a(b * c) = \log_a(b) + \log_a(c)$

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Suppose $y = \log_a(b) + \log_a(c)$.

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Properties of Logarithms

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3. $a^b * a^c = a^{b+c}$	3. $\log_a(b * c) = \log_a(b) + \log_a(c)$
4. $(a^b)^c = a^{b*c}$	4. $\log_a(b^c) = c \log_a(b)$

Example: why $\log_a(b * c) = \log_a(b) + \log_a(c)$:

Suppose $y = \log_a(b) + \log_a(c)$.

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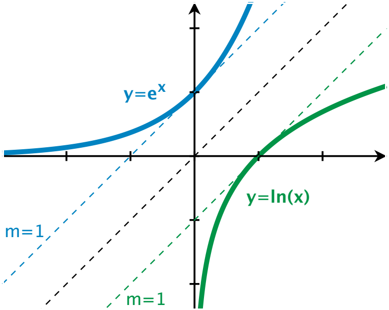
So $y = \log_a(b * c)$ as well!

Lastly: $\dfrac{\log_a(b)}{\log_a(c)} = \log_c(b)$

Favorite logarithmic function

Remember: $y = e^x$ is the function whose slope through the point (0,1) is 1.
The inverse to $y = e^x$ is the natural log:

$$\ln(x) = \log_e(x)$$



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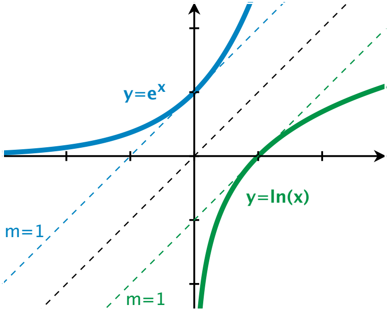

We will often use the facts that $e^{\ln(x)} = x$ (for $x > 0$) and $\ln(e^x) = x$ (for all x)

Two super useful facts:

Explain why:

(1) $\log_a(b) = \ln(b)/\ln(a)$

(2) $a^b = e^{b\ln(a)}$ [hint: start by rewriting $b\ln(a)$, and use the fact that $e^{\ln(x)} = x$]

Two super useful facts:

Explain why:

(1) $\log_a(b) = \ln(b)/\ln(a)$

Since $\ln(b) = \log_e(b)$ and $\ln(a) = \log_e(a)$, we have

$$\frac{\ln(b)}{\ln(a)} = \frac{\log_e(b)}{\log_e(a)} = \log_a(b)$$

(2) $a^b = e^{b\ln(a)}$ [hint: start by rewriting $b\ln(a)$, and use the fact that $e^{\ln(x)} = x$]

Since $b\ln(a) = \ln(a^b)$ and $e^{\ln(x)} = x$, we have

$$e^{b\ln(a)} = e^{\ln(a^b)} = a^b$$

Examples:

(1) Condense the logarithmic expressions

$$\frac{1}{2}\ln(x)+3\ln(x+1) \qquad 2\ln(x+5)-\ln(x) \qquad \frac{1}{3}(\log_3(x)-\log_3(x+1))$$

(2) Solve the following expressions for x:

$$e^{-x^2} = e^{-3x-4} \qquad 3(2^x) = 24$$

$$2(e^{3x-5}) - 5 = 11 \qquad \ln(3x+1) - \ln(5) = \ln(2x)$$

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$$\boxed{\ln(\sqrt{x}(x+1)^3)} \qquad \boxed{\ln\left(\frac{(x+5)^2}{x}\right)} \qquad \boxed{\log_3\left(\left(\frac{x}{x+1}\right)^{1/3}\right)}$$

(2) Solve the following expressions for x:

$$e^{-x^2} = e^{-3x-4} \qquad 3(2^x) = 24$$

$$\boxed{x = -1, 4} \qquad \boxed{x = 3}$$

$$2(e^{3x-5}) - 5 = 11 \qquad \ln(3x+1) - \ln(5) = \ln(2x)$$

$$\boxed{x = \frac{\ln(8)+5}{3}} \qquad \boxed{x = \frac{1}{7}}$$

Inverse trig functions

Two notations:

$f(x)$	$f^{-1}(x)$
$\sin(x)$	$\sin^{-1}(x) = \arcsin(x)$
$\cos(x)$	$\cos^{-1}(x) = \arccos(x)$
$\tan(x)$	$\tan^{-1}(x) = \arctan(x)$
$\sec(x)$	$\sec^{-1}(x) = \text{arcsec}(x)$
$\csc(x)$	$\csc^{-1}(x) = \text{arccsc}(x)$
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There are lots of points we know on these functions...

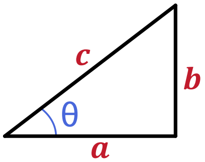
Examples:

1. Since $\sin(\pi/2) = 1$, we have $\arcsin(1) = \pi/2$
2. Since $\cos(\pi/2) = 0$, we have $\arccos(0) = \pi/2$

Etc...

In general:

arc___(-) takes in a ratio and spits out an angle:



$\cos(\theta) = a/c$ so $\arccos(a/c) = \theta$

$\sin(\theta) = b/c$ so $\arcsin(b/c) = \theta$

$\tan(\theta) = b/a$ so $\arctan(b/a) = \theta$

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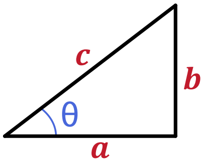

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$\tan(\theta) = b/a$ so $\arctan(b/a) = \theta$

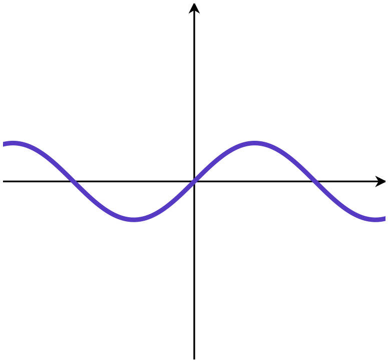
Domain problems:

$\sin(0) = 0, \quad \sin(\pi) = 0, \quad \sin(2\pi) = 0, \quad \sin(3\pi) = 0, \ldots$

So which is the right answer to arcsin(0), really?

Domain/range

$$y = \sin(x)$$



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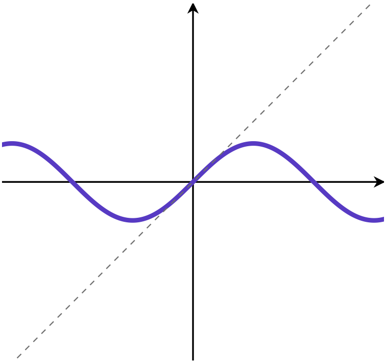

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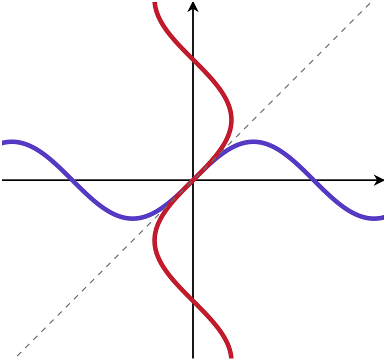

Domain: $-1 \leq x \leq 1$

Domain/range

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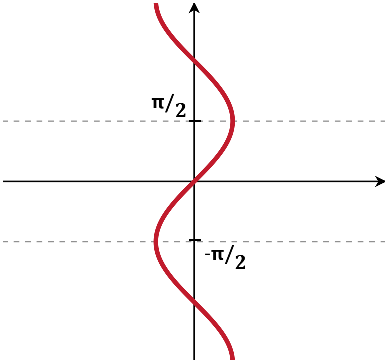

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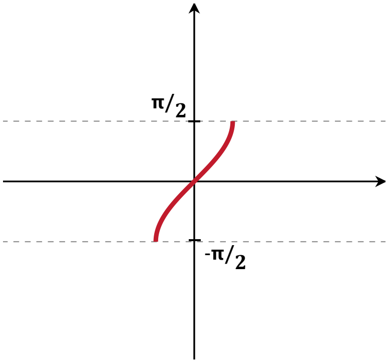
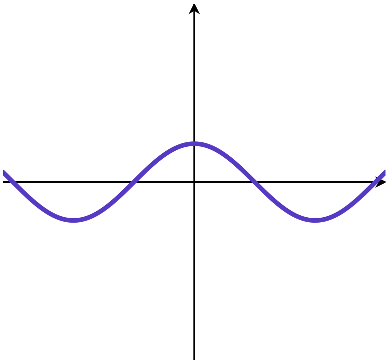


Domain: $-1 \leq x \leq 1$ Range: $-\pi/2 \leq y \leq \pi/2$

Domain/range

$$y = \cos(x)$$



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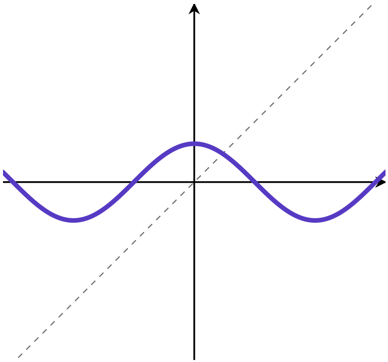

Domain/range

$y = \cos(x)$
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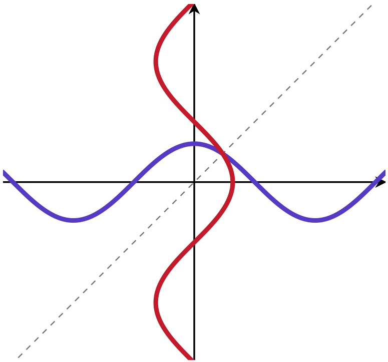

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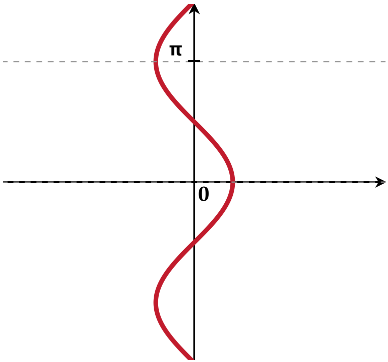

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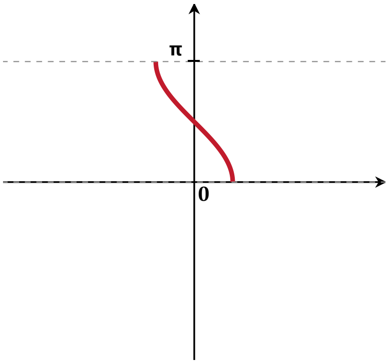
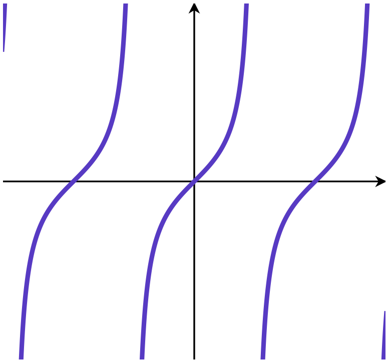


Domain: $-1 \le x \le 1$ Range: $0 \le y \le \pi$

Domain/range

$y = \tan(x)$



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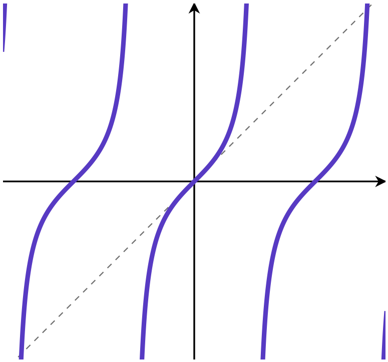

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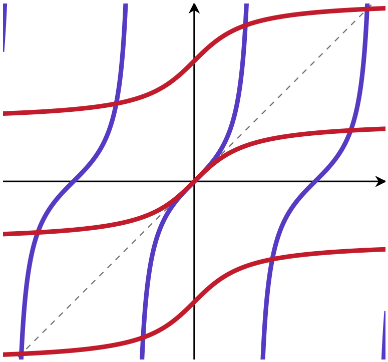

Domain: $-\infty \leq x \leq \infty$

Domain/range

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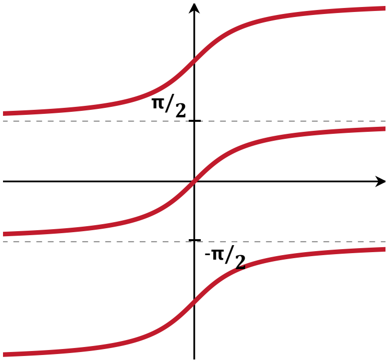

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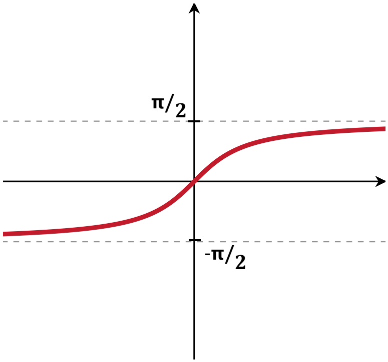
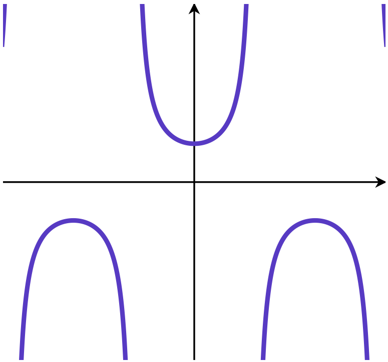


Domain: $-\infty \leq x \leq \infty$ Range: $-\pi/2 < y < \pi/2$

Domain/range

$y = \sec(x)$



Domain/range

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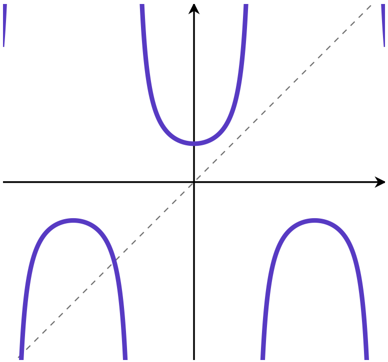
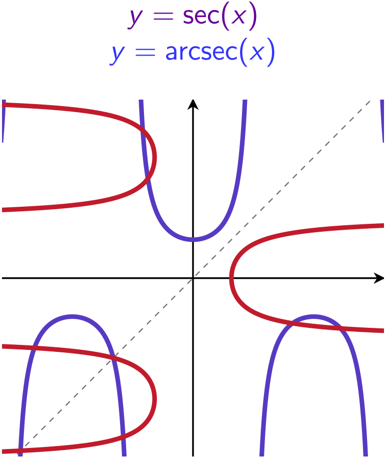


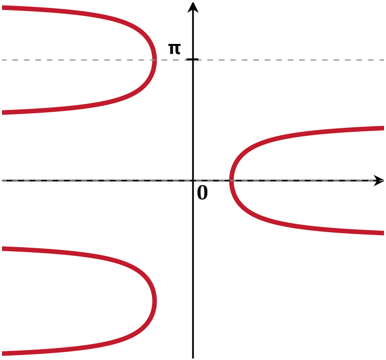
Domain/range



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

$y = \text{arcsec}(x)$



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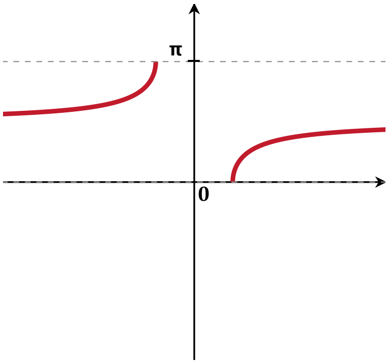
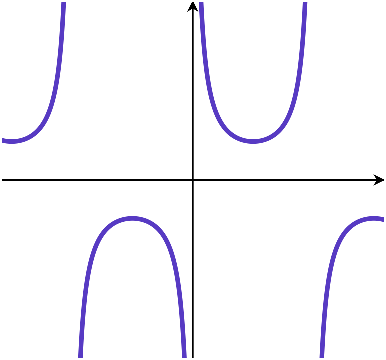


Domain: $x \leq -1$ and $1 \leq x$ Range: $0 \leq y \leq \pi$

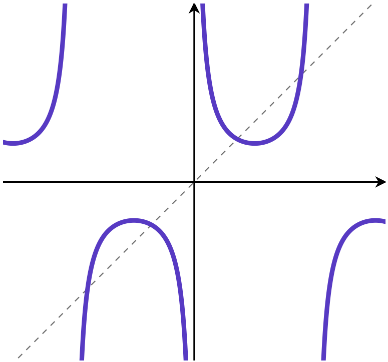
Domain/range

$$y = \csc(x)$$



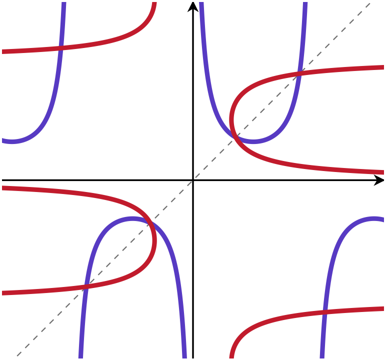
Domain/range

$$y = \csc(x)$$



Domain/range

$y = \csc(x)$
$y = \operatorname{arccsc}(x)$



Domain: $x \leq -1$ and $1 \leq x$

Domain/range

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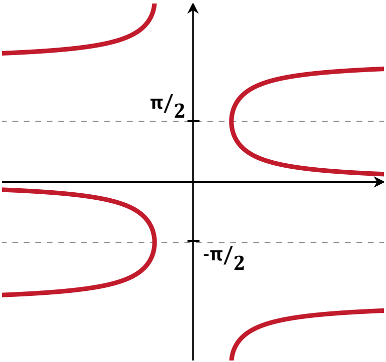

Domain: $x \leq -1$ and $1 \leq x$

Domain/range

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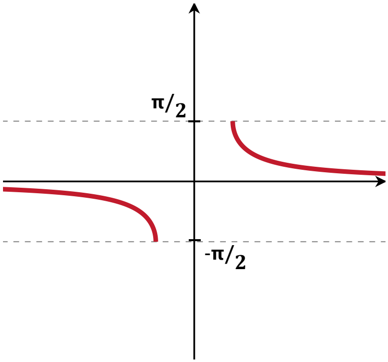
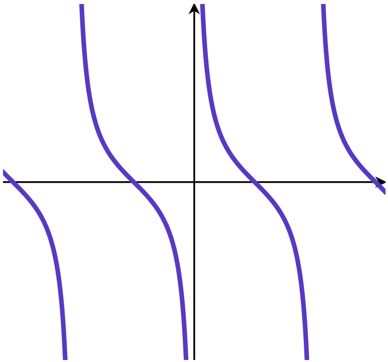


Domain: $x \leq -1$ and $1 \leq x$ Range: $-\pi/2 \leq y \leq \pi/2$

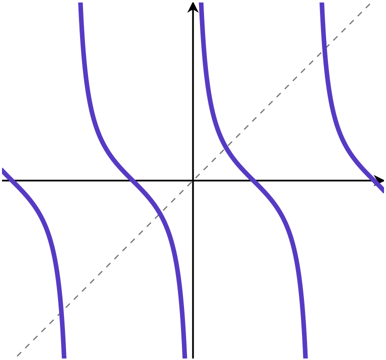
Domain/range

$$y = \cot(x)$$



Domain/range

$$y = \cot(x)$$



Domain/range

$y = \cot(x)$

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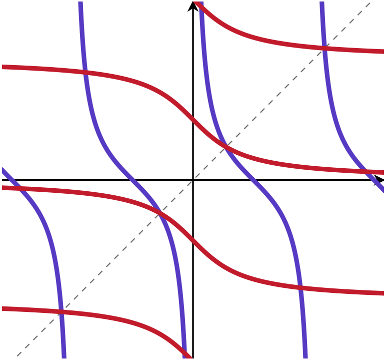
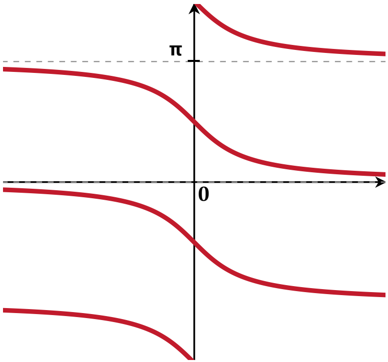


Domain: $-\infty \leq x \leq \infty$

Domain/range

$y = \text{arccot}(x)$



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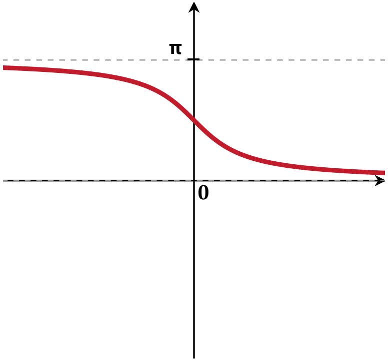
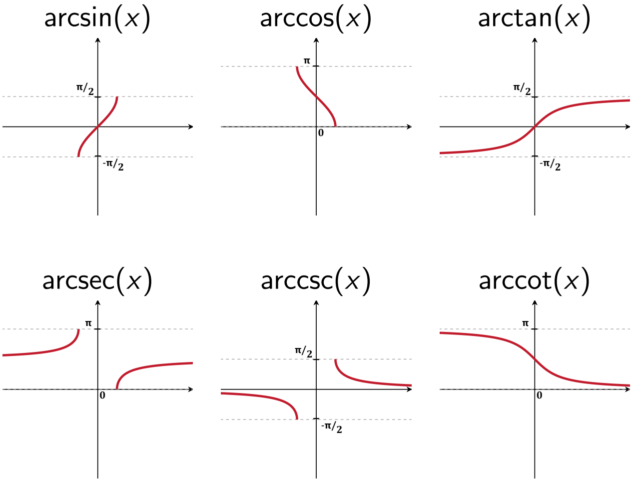


Domain: $-\infty \leq x \leq \infty$ Range: $0 < y < \pi$

Graphs



arcsin(x)
π/2
-π/2
arccos(x)
π
0
arctan(x)
π/2
-π/2
arcsec(x)
π
0
arccsc(x)
π/2
-π/2
arccot(x)
π
0