Important websites:

Course website: Notes, written and reading assignments, etc.

zdaugherty.ccnysites.cuny.edu/teaching/m201f18/

My Math Lab (MML): Online assignments.

www.pearson.com/mylab

(See main course website for instructions.)

Upcoming deadlines:

Due Sunday 9/2

- From MML: Orientation Assignment

Due Tuesday 9/4

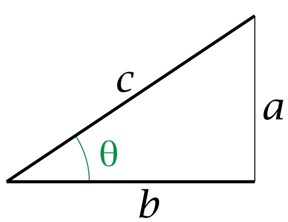
- From MML: Section 1.1, Section 1.2
- From course website: Homework 0 email

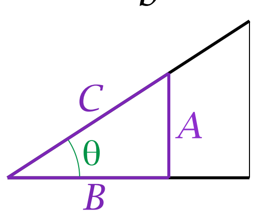
Due Thursday 9/6

- From MML: Section 1.3, Section 1.5
- From course website: summaries

First quiz: In class, Tuesday 9/4.

Trigonometric functions, step one: similar triangles





Two similar triangles have the same set of angles, and have the properties that

$$\frac{A}{B} = \frac{a}{b}, \quad \frac{B}{C} = \frac{b}{c}, \text{ and } \frac{A}{C} = \frac{a}{c}.$$

Define

$$\cos(\theta) = \frac{b}{c} \quad \text{and} \quad \sin(\theta) = \frac{a}{c}.$$

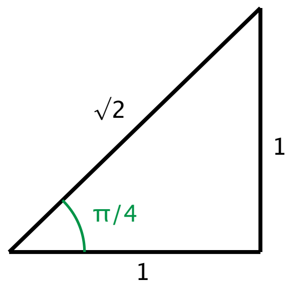
Then let

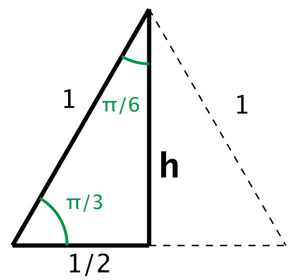
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{a}{b}, \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a},$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}, \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{b}{a}.$$

Easy angles:

isosceles right triangle: equilateral triangle cut in half:

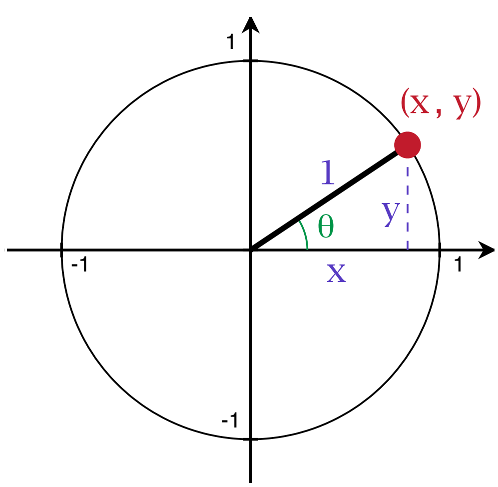




$h = \sqrt{1-(1/2)^2} = \sqrt{3}/2$

	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$						
$\pi/3$						
$\pi/6$						

Step two: the unit circle



For $0 < \theta < \frac{\pi}{2}$...

$$\cos(\theta) = \frac{x}{1} = x$$

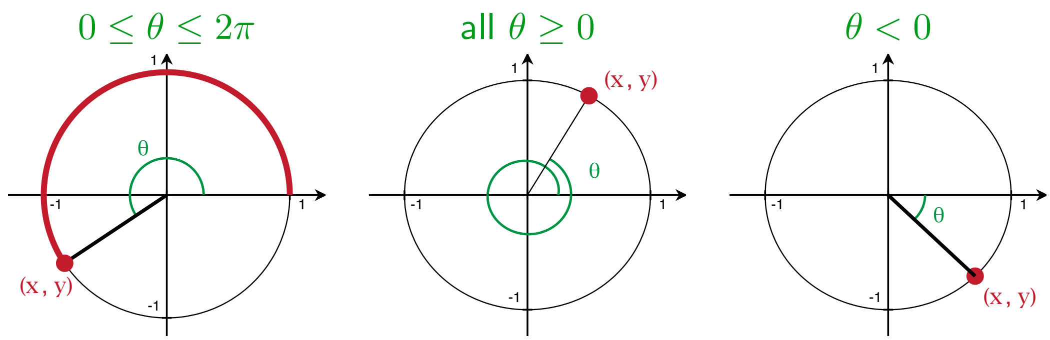
$$\sin(\theta) = \frac{y}{1} = y$$

Use this idea to extend trig functions to any θ...

Define

$$\cos(\theta) = x \qquad \sin(\theta) = y,$$

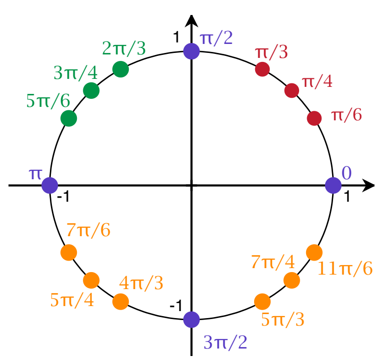
where θ is defined by...



Sidebar: In calculus, radians are king. Where do they come from?
Circumference of a unit circle: 2π
Arclength of a wedge with angle θ*:*

$\frac{\theta}{360°} * 2\pi$ *(if in degrees)* *or* $\frac{\theta}{2\pi} * 2\pi = \boxed{\theta}$ *(if in radians)*

Reading off of the unit circle



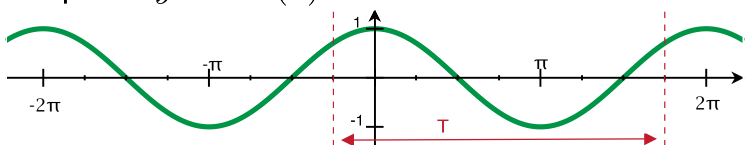
$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$
$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$
$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$
$$x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$							
$\sin(\theta)$							

	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos(\theta)$									
$\sin(\theta)$									

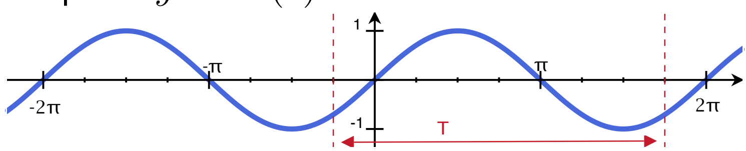
Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



A = Amplitude = $\frac{1}{2}$ length of the range = 1
T=Period = time to repeat = 2π

Graph of $y = \sin(\theta)$:



A=Amplitude = $\frac{1}{2}$ length of the range = 1
T=Period = time to repeat = 2π

You try: Transform the graph of $\sin(\theta)$ into the graph of $2\sin(\frac{1}{2}\theta + \pi/6) - 1$, one step at a time. (See notes)
What is the amplitude of $2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1$? What is the period?

Trig identities to know and love:

Even/odd:

$$\cos(-\theta) = \cos(\theta) \quad \text{(even)} \qquad \sin(-\theta) = -\sin(\theta) \quad \text{(odd)}$$

Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

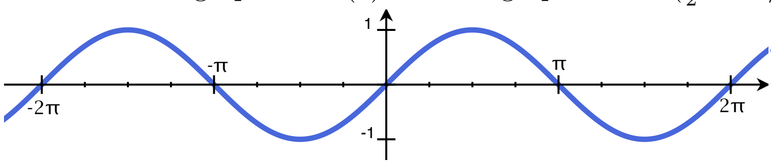
Angle addition:

$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

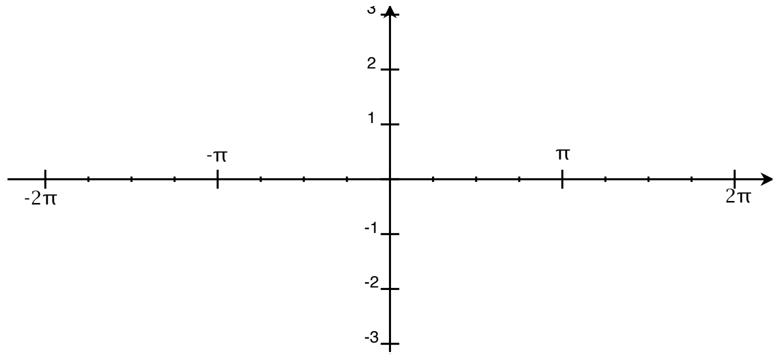
$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

(in particular $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ *and* $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$*)*

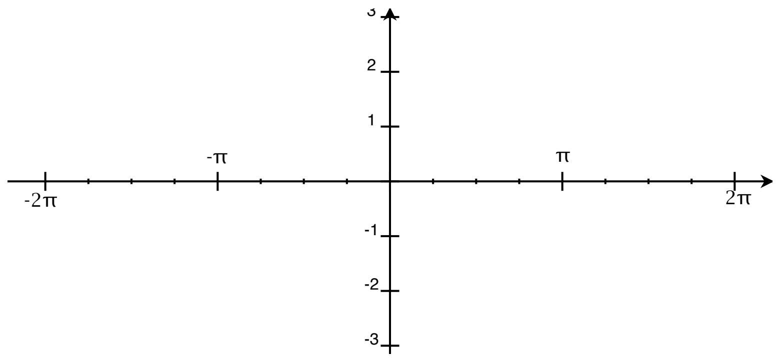
Transform the graph of $\sin(\theta)$ into the graph of $2\sin(\frac{1}{2}\theta + \pi/6) - 1$:



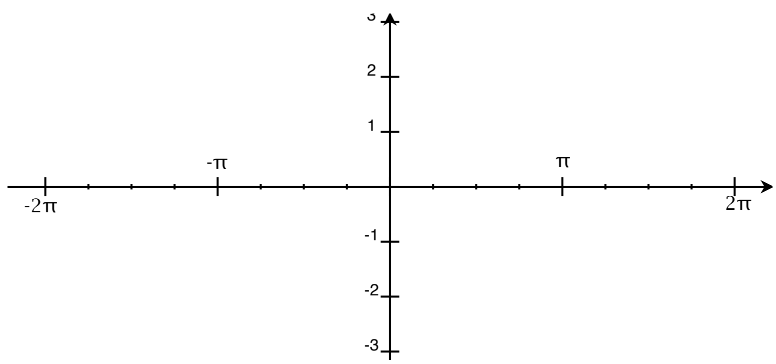
$\xrightarrow{\sin\left(\frac{1}{2}\theta\right)}$



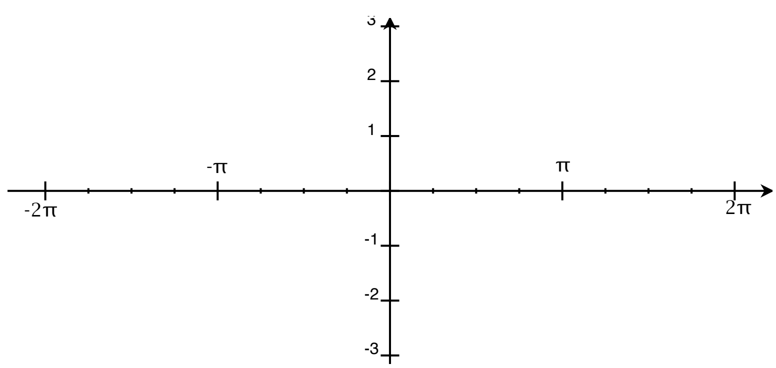
$\xrightarrow{\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)}$



$\xrightarrow{2\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)}$



$\xrightarrow{2\sin\left(\frac{1}{2}(\theta+\frac{\pi}{3})\right)-1}$

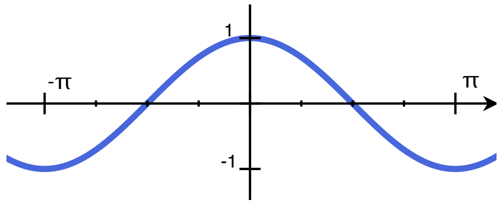


What is the amplitude of $2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1$?

What is the period?

Other trig functions

$y = \cos(\theta)$



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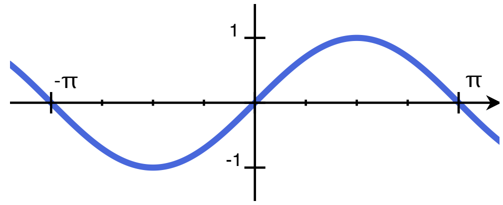
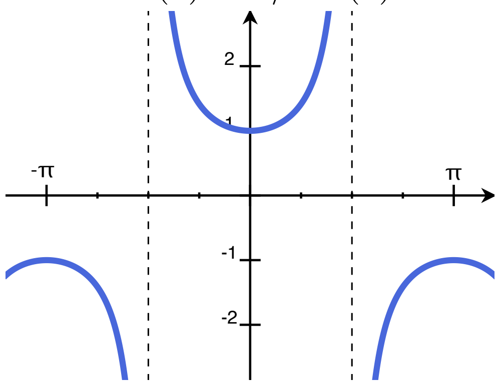
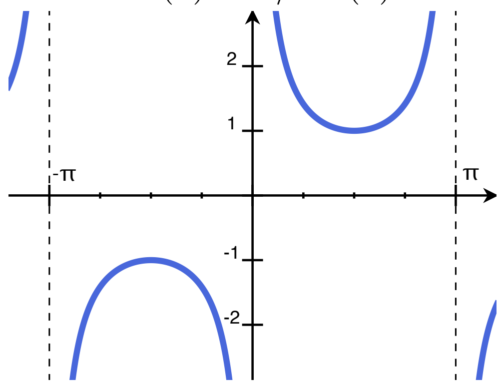


$\sec(\theta) = 1/\cos(\theta)$

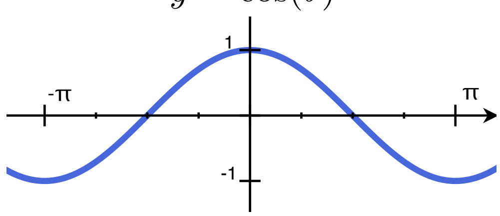


$\csc(\theta) = 1/\sin(\theta)$

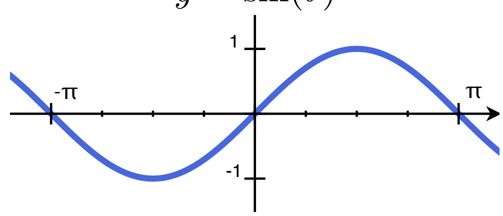


Other trig functions

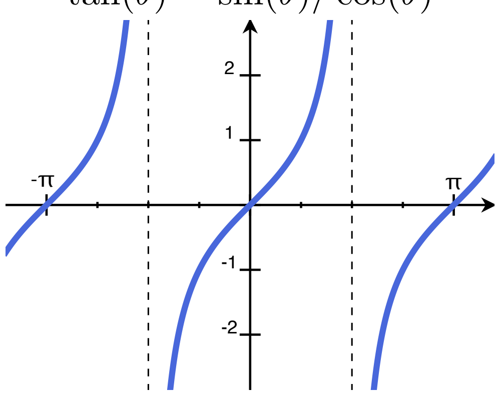
$y = \cos(\theta)$



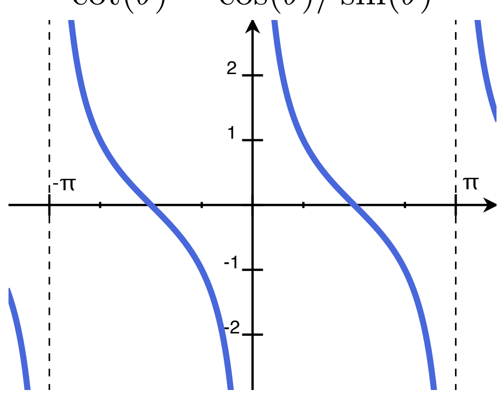
$y = \sin(\theta)$



$\tan(\theta) = \sin(\theta)/\cos(\theta)$



$\cot(\theta) = \cos(\theta)/\sin(\theta)$



Exponential functions

The basics: Let n and m be positive integers, and a be a real number.

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n} \qquad \text{(MML: } a\hat{\ }n\text{)}$$

Examples:

$$\begin{aligned}
2^5 &= 2*2*2*2*2 \\
2^5 * 2^3 &= (2*2*2*2*2)*(2*2*2) = 2^8 \\
(2^3)^5 &= (2*2*2)*(2*2*2)*(2*2*2)*(2*2*2)*(2*2*2) = 2^{15} \\
2^{3^5} &= 2^{243} >> \left(2^3\right)^5 = 2^{15} \\
2^3 * 5^3 &= (2*2*2)*(5*5*5) = (2*5)*(2*5)*(2*5) = (2*5)^3
\end{aligned}$$

Some identities:

$$a^n * a^m = a^{n+m} \qquad\qquad (a^n)^m = a^{n*m}$$

(Notice: a^{m^n} means $a^{(m^n)}$, since $\left(a^m\right)^n$ can be written another way)

$$a^n * b^n = (a*b)^n$$

Pushing it further...

Take for granted: If n and m are positive integers,

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n}, \qquad a^n * a^m = a^{n+m}, \qquad (a^n)^m = a^{n*m}.$$

Notice:

1. What is a^0?

$$a^n = a^{n+0} = a^n * a^0, \qquad \text{so } \boxed{a^0 = 1}.$$

2. What is a^x if x is negative?

$$a^n * a^{-n} = a^{n-n} = a^0 = 1, \qquad \text{so } \boxed{a^{-n} = 1/(a^n)}.$$

3. What is a^x if x is a fraction?

$$(a^n)^{1/n} = a^{n*\frac{1}{n}} = a^1 = a, \qquad \text{so } \boxed{a^{1/n} = \sqrt[n]{a}}$$

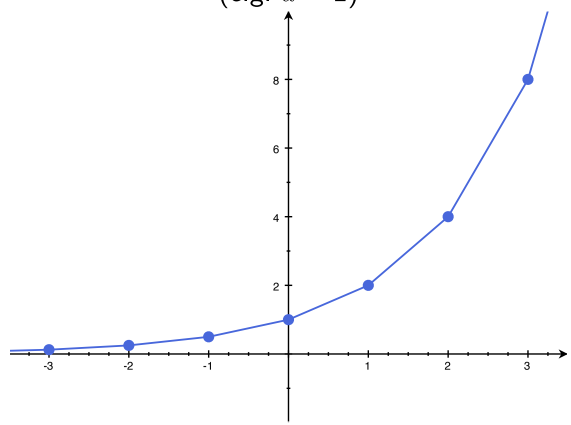
$$\text{and } \boxed{a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m}.$$

Example: $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$ or $8^{5/3} = \sqrt[3]{8^5} = \sqrt[3]{32{,}768} = 32$

What is a^x for all x?

If $a > 1$:

(e.g. $a = 2$)



$x = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

What is a^x for all x?

If $a > 1$:

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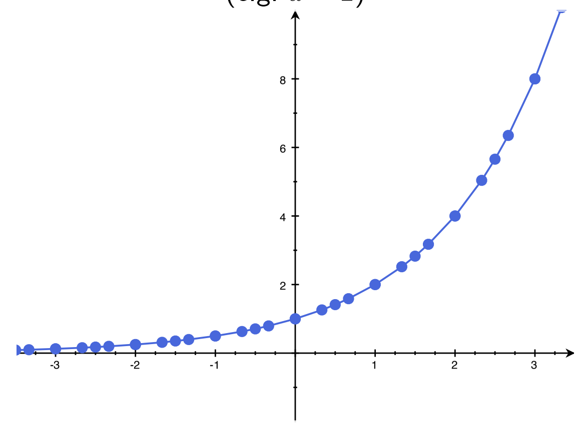

$x = n/2$ and $n/3$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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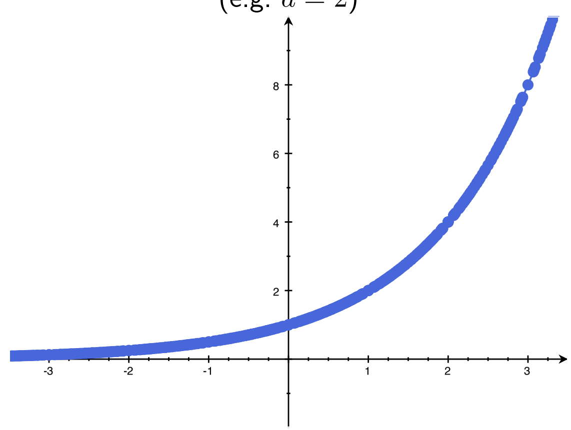
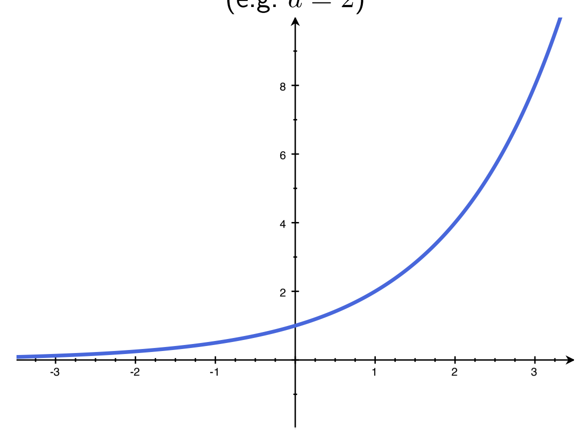


$x = n/2,\ n/3,\ \ldots,\ n/15$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

What is a^x for all x?

If $a > 1$:

(e.g. $a = 2$)

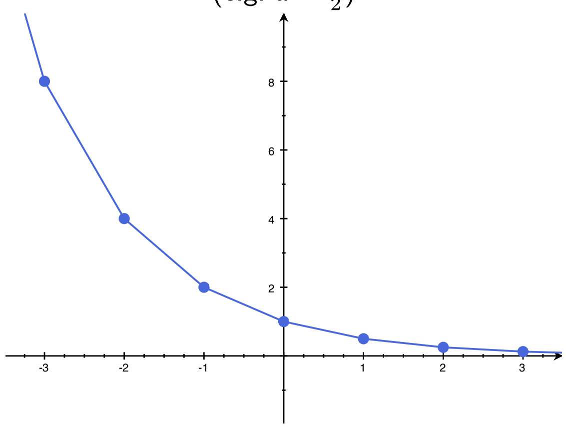


$y = a^x$

What is a^x for all x?

If $0 < a < 1$:

(e.g. $a = \frac{1}{2}$)



$x = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

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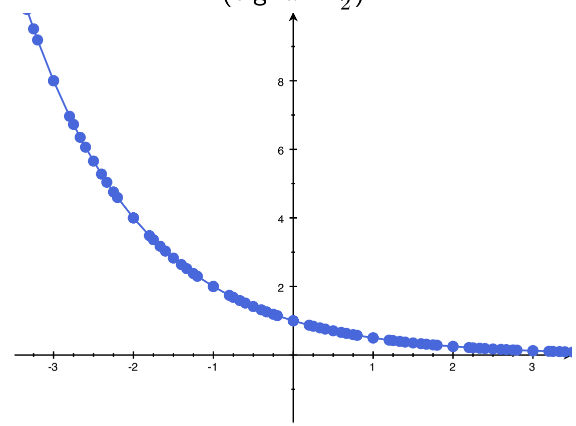
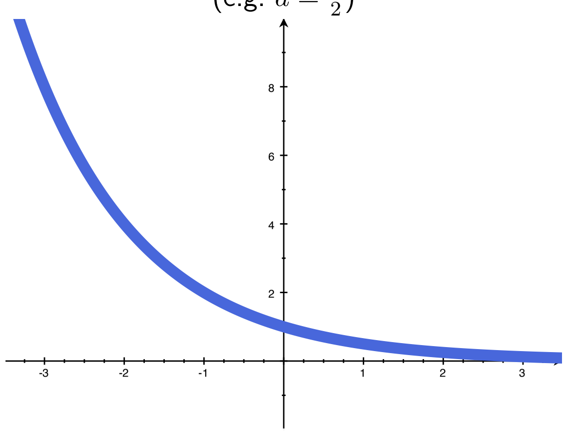


$x = n/2, n/3, n/4, n/5$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

What is a^x for all x?

If $0 < a < 1$:

(e.g. $a = \frac{1}{2}$)

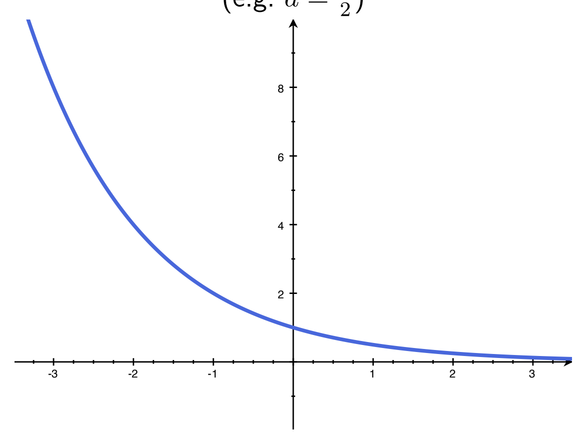


$x = n/2,\ n/3,\ \ldots,\ n/100$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

What is a^x for all x?

If $0 < a < 1$:

(e.g. $a = \frac{1}{2}$)

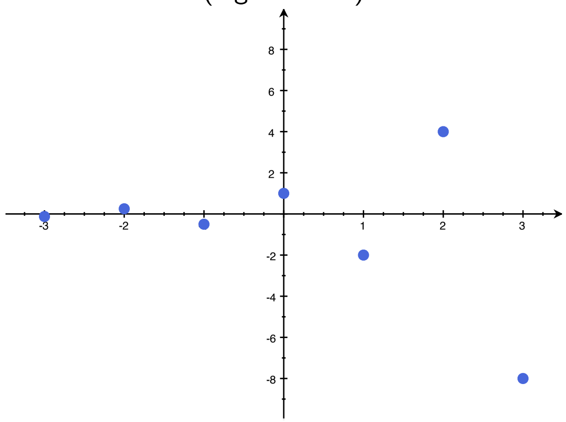


$y = a^x$

What is a^x for all x?

If $0 > a$:

(e.g. $a = -2$)



$x = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

What is a^x for all x?

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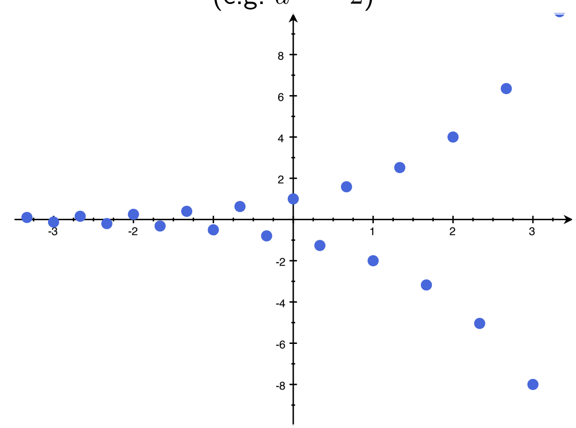

$x = n/3$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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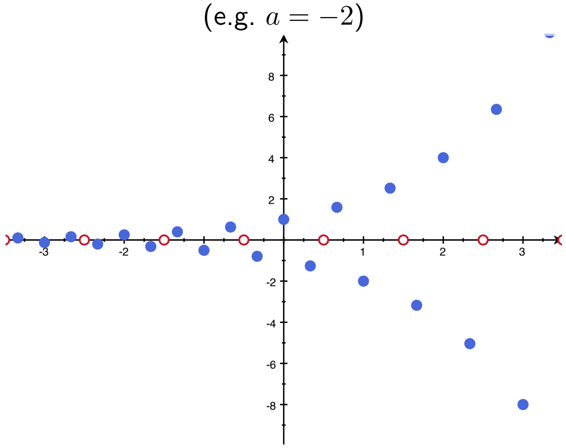

$x = n/3$ and $n/2$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

What is a^x for all x?

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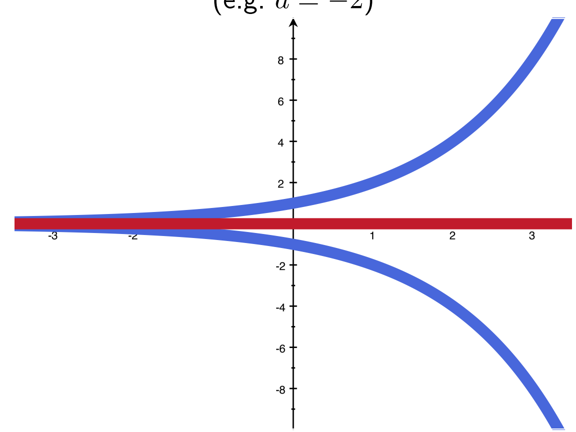
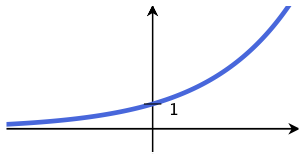


$x = n/2,\ n/3,\ \ldots,\ n/100$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

OH NO!

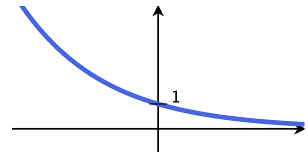
The function a^x:

$1 < a$:



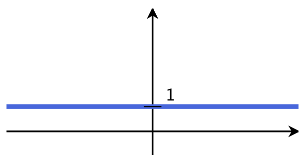
D: $(-\infty, \infty)$, R: $(0, \infty)$

$0 < a < 1$:



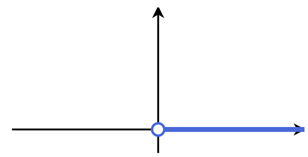
D: $(-\infty, \infty)$, R: $(0, \infty)$

$a = 1$:



D: $(-\infty, \infty)$, R: $\{1\}$

$a = 0$:



D: $(0, \infty)$, R: $\{0\}$

(If $a < 0$, then a^x is not defined as a function on the real numbers.)

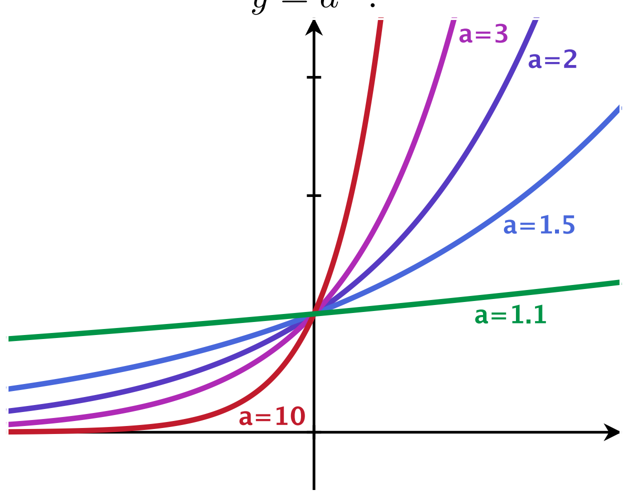
Properties:

$a^b * a^c = a^{b+c}$ $\quad (a^b)^c = a^{b*c}$ $\quad a^{-x} = 1/a^x$ $\quad a^c * b^c = (ab)^c$

Our favorite exponential function:

Look at how the function is increasing through the point $(0, 1)$:

$y = a^x$:



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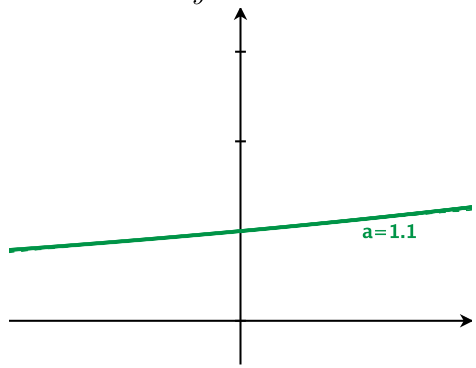

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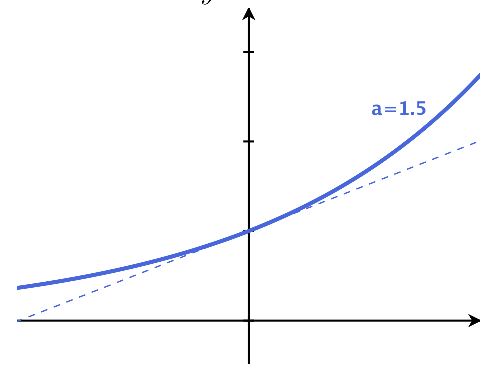

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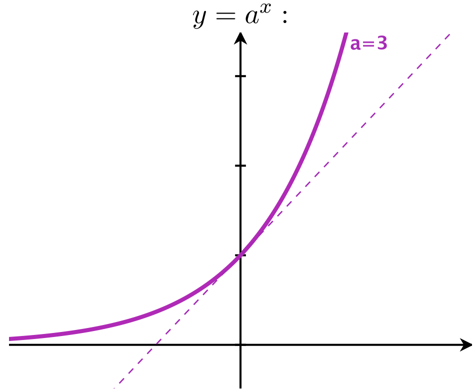

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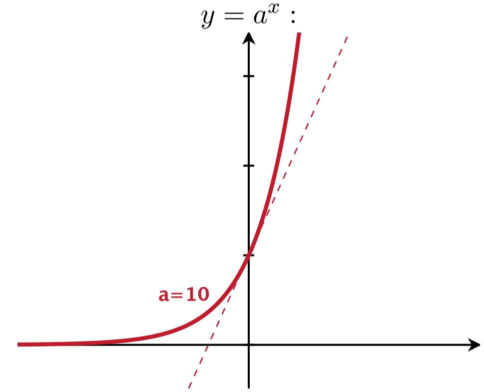

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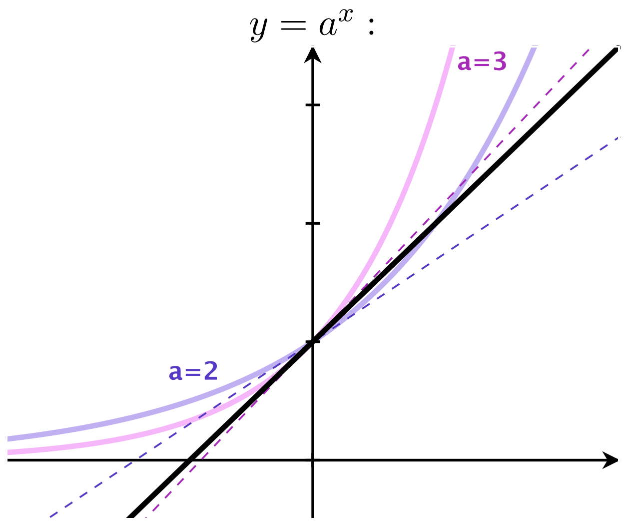

Q: Is there an exponential function whose slope at (0,1) is 1?

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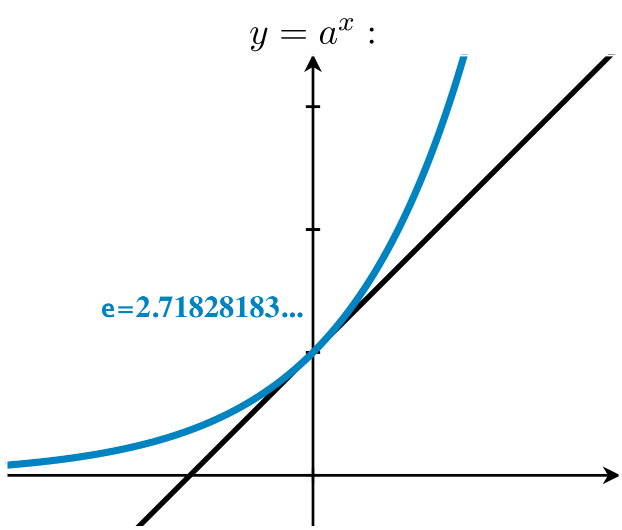

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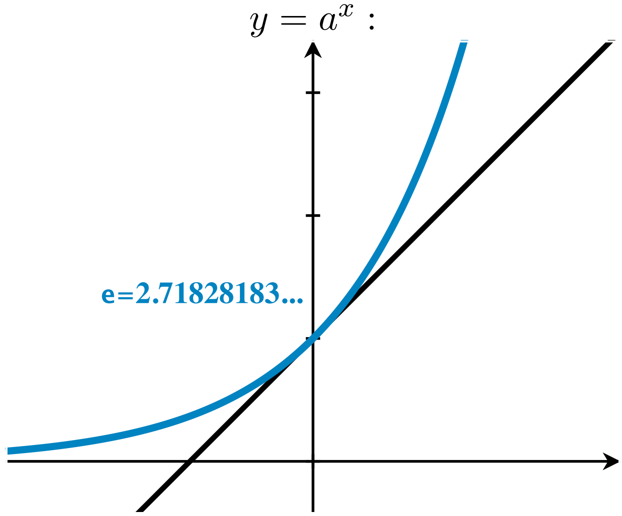

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Read: "Exponential growth and decay", examples 3 and 4.