Important websites:

Course website: Notes, written and reading assignments, etc.

zdaugherty.ccnysites.cuny.edu/teaching/m201f18/

My Math Lab (MML): Online assignments.

www.pearson.com/mylab

(See main course website for instructions.)

Upcoming deadlines:

Due Sunday 9/2

- From MML: Orientation Assignment

Due Tuesday 9/4

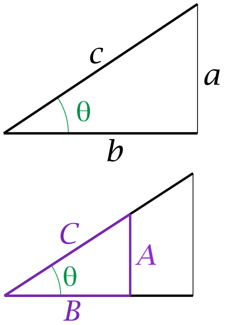
- From MML: Section 1.1, Section 1.2
- From course website: Homework 0 email

Due Thursday 9/6

- From MML: Section 1.3, Section 1.5
- From course website: summaries

First quiz: In class, Tuesday 9/4.

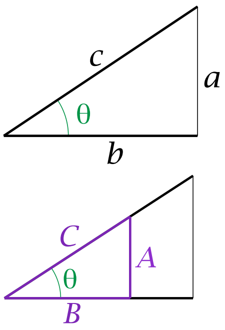
Trigonometric functions, step one: similar triangles



Two similar triangles have the same set of angles, and have the properties that

$$\frac{A}{B} = \frac{a}{b}, \quad \frac{B}{C} = \frac{b}{c}, \text{ and } \frac{A}{C} = \frac{a}{c}.$$

Trigonometric functions, step one: similar triangles



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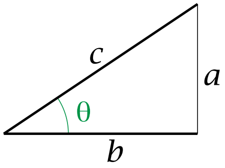
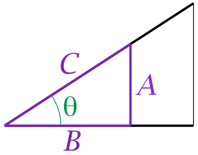
Define

$$\cos(\theta) = \frac{b}{c} \quad \text{and} \quad \sin(\theta) = \frac{a}{c}.$$

Trigonometric functions, step one: similar triangles





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Define

$$\cos(\theta) = \frac{b}{c} \quad \text{and} \quad \sin(\theta) = \frac{a}{c}.$$

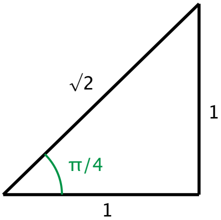
Then let

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{a}{b}, \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a},$$

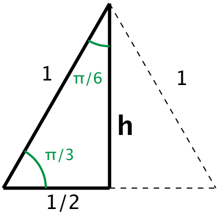
$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}, \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{b}{a}.$$

Easy angles:

isosceles right triangle:



equilateral triangle cut in half:



$$h = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$						
$\pi/3$						
$\pi/6$						

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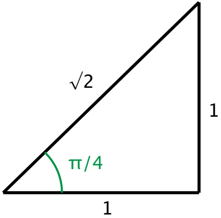

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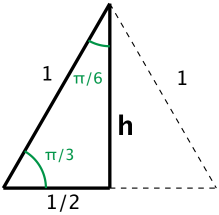
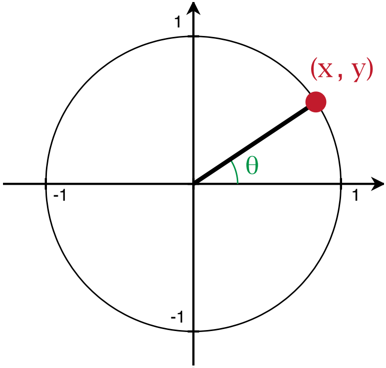


$$h = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

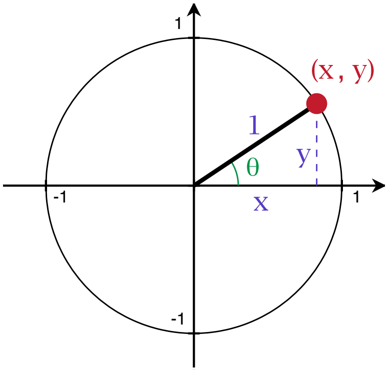
	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\pi/4$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\pi/3$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
$\pi/6$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	2	$\sqrt{3}$

Step two: the unit circle



For $0 < \theta < \frac{\pi}{2}$...

Step two: the unit circle



For $0 < \theta < \frac{\pi}{2}$...

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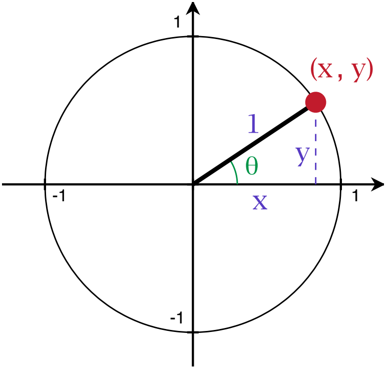


For $0 < \theta < \frac{\pi}{2}$...

$$\cos(\theta) = \frac{x}{1} = x$$

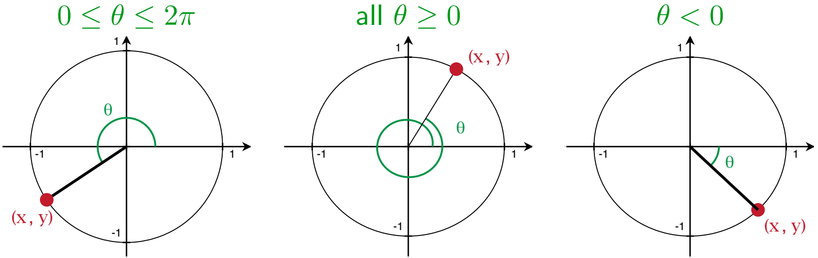
$$\sin(\theta) = \frac{y}{1} = y$$

Use this idea to extend trig functions to any θ...

Define

$$\cos(\theta) = x \qquad \sin(\theta) = y,$$

where θ is defined by...



Define

$$\cos(\theta) = x \qquad \sin(\theta) = y,$$

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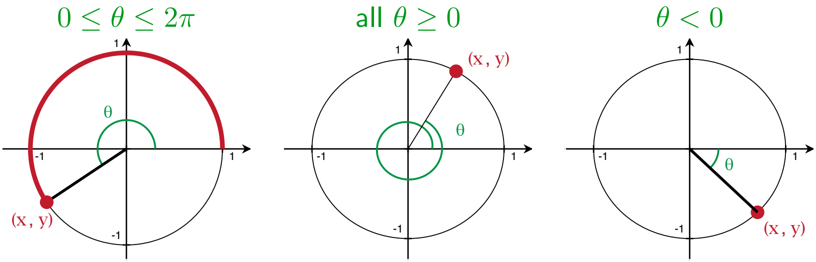


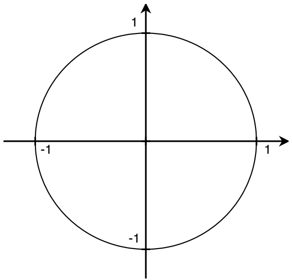
Sidebar: In calculus, radians are king. Where do they come from?

Circumference of a unit circle: 2π

Arclength of a wedge with angle θ*:*

$\frac{\theta}{360^\circ} * 2\pi$ *(if in degrees)* *or* $\frac{\theta}{2\pi} * 2\pi = \boxed{\theta}$ *(if in radians)*

Reading off of the unit circle



	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$							
$\sin(\theta)$							

	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos(\theta)$									
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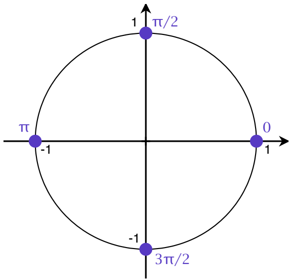

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$	1	0	-1	0			
$\sin(\theta)$	0	1	0	-1			

	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
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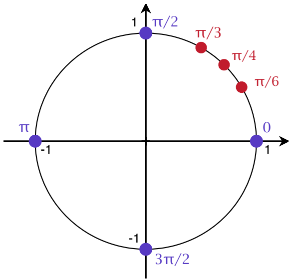

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$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
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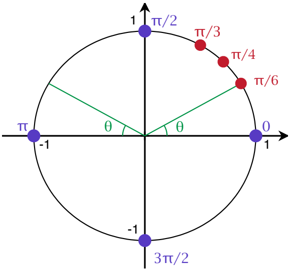

$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(\theta)$	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
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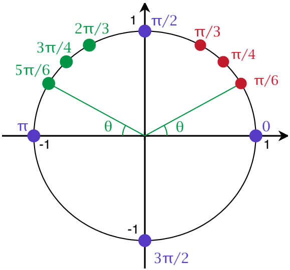

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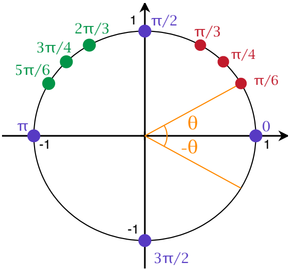

$$\cos(\pi - \theta) = -\cos(\theta) \quad \sin(\pi - \theta) = \sin(\theta)$$

$$\cos(-\theta) = \cos(\theta) \quad \sin(-\theta) = -\sin(\theta)$$

$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
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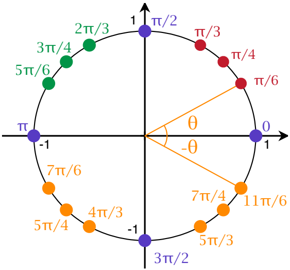

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	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$

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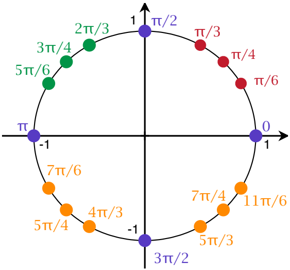

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$$\cos(2\pi n + \theta) = \cos(\theta) \quad \sin(2\pi n + \theta) = \sin(\theta)$$

$$x^2 + y^2 = 1 \implies \cos^2(\theta) + \sin^2(\theta) = 1$$

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
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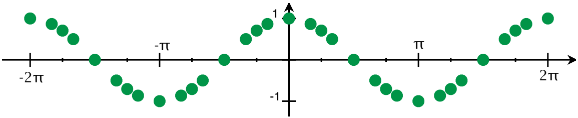
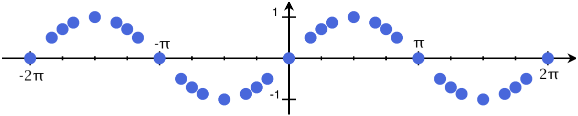
$\cos(\theta)$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\sin(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$

Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



Graph of $y = \sin(\theta)$:



Plotting on the θ-y axis

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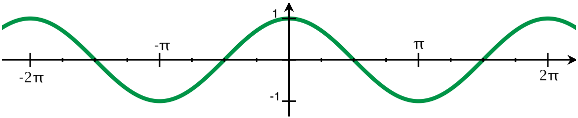
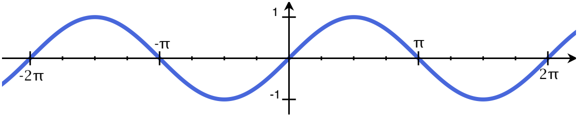
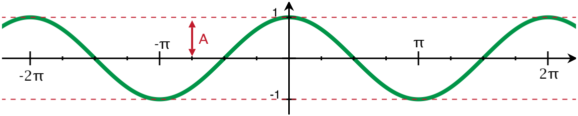


Graph of $y = \sin(\theta)$:



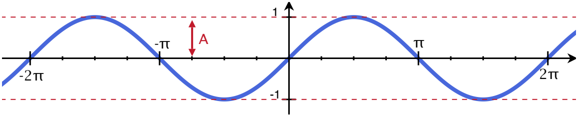
Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



A = Amplitude = $\frac{1}{2}$ length of the range = 1

Graph of $y = \sin(\theta)$:



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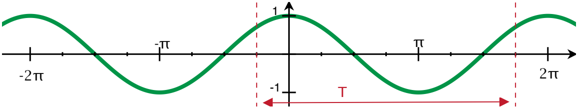
Plotting on the θ-y axis

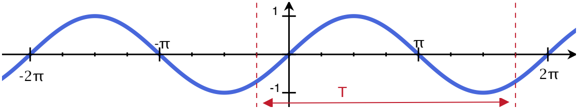
Graph of $y = \cos(\theta)$:



A = Amplitude = $\frac{1}{2}$ length of the range = 1

T=Period = time to repeat = 2π

Graph of $y = \sin(\theta)$:



A=Amplitude = $\frac{1}{2}$ length of the range = 1

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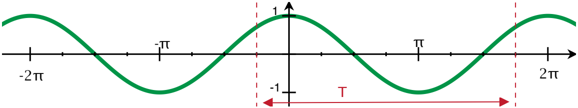
Plotting on the θ-y axis

Graph of $y = \cos(\theta)$:



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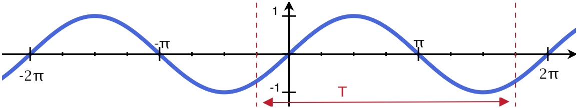
Graph of $y = \sin(\theta)$:



A=Amplitude = $\frac{1}{2}$ length of the range = 1
T=Period = time to repeat = 2π

You try: Transform the graph of $\sin(\theta)$ into the graph of $2\sin(\frac{1}{2}\theta + \pi/6) - 1$, one step at a time. (See notes)
What is the amplitude of $2\sin\left(\frac{1}{2}\theta + \frac{\pi}{6}\right) - 1$? What is the period?

Trig identities to know and love:

Even/odd:

$$\cos(-\theta) = \cos(\theta) \quad \text{(even)} \qquad \sin(-\theta) = -\sin(\theta) \quad \text{(odd)}$$

Pythagorean identity:

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

Angle addition:

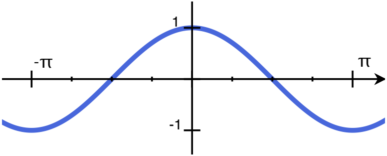
$$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)$$

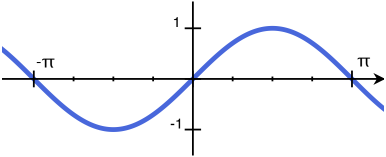
(in particular $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ *and* $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$*)*

Other trig functions

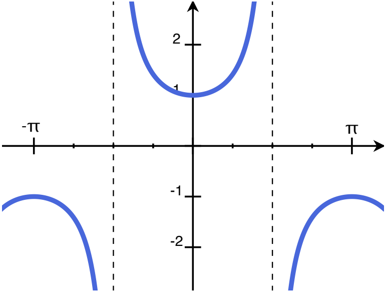
$$y = \cos(\theta)$$



$$y = \sin(\theta)$$



$$\sec(\theta) = 1/\cos(\theta)$$



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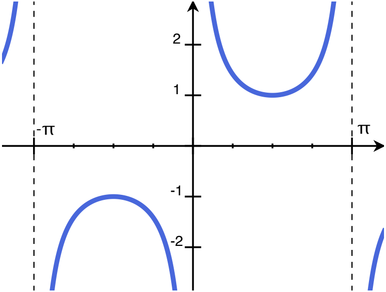
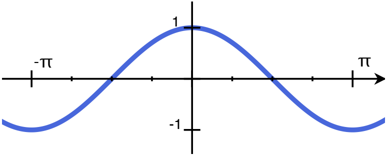


Other trig functions

$y = \cos(\theta)$



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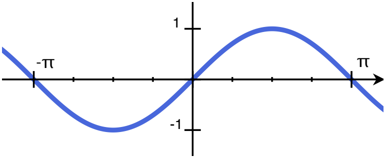
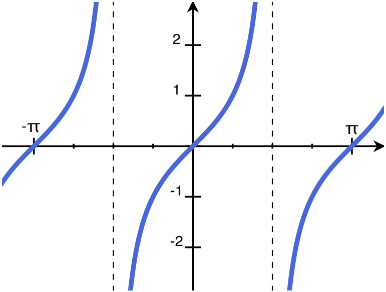


$\tan(\theta) = \sin(\theta)/\cos(\theta)$



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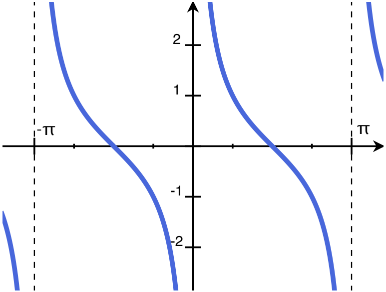

Exponential functions

The basics: Let n and m be positive integers, and a be a real number.

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n} \qquad \text{(MML: } a^\wedge n\text{)}$$

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Pushing it further...

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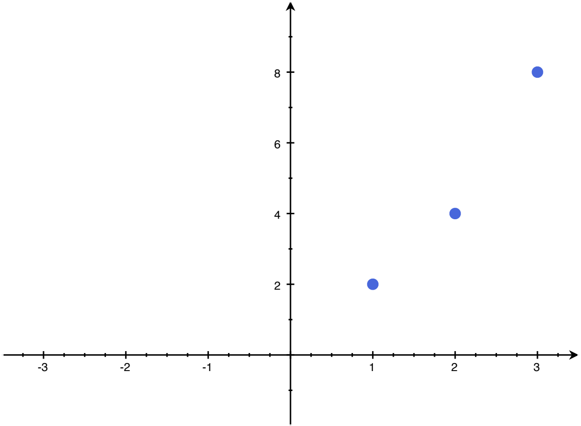
Example: $8^{5/3} = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$ or $8^{5/3} = \sqrt[3]{8^5} = \sqrt[3]{32{,}768} = 32$

What is a^x for all x?

If $a > 1$:

(e.g. $a = 2$)



$$x = 1, 2, 3, \ldots$$

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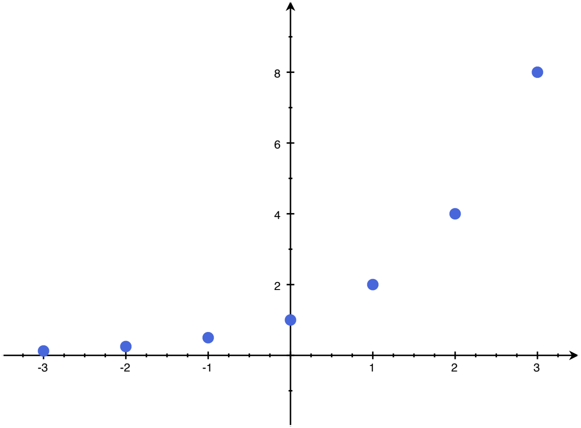

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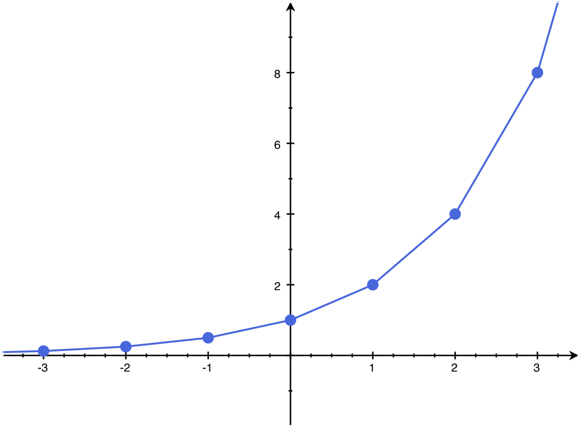

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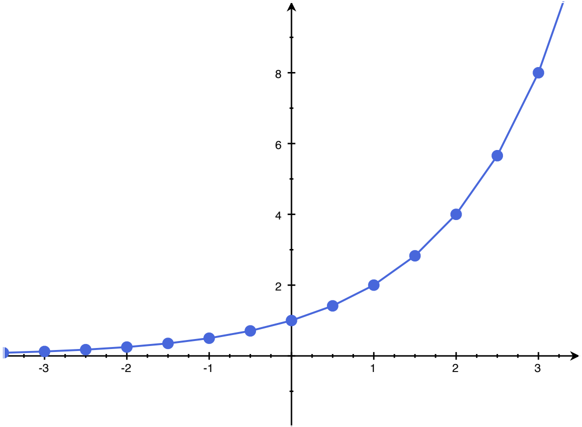

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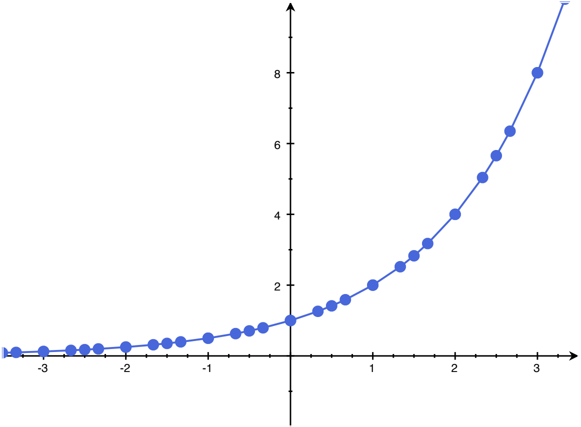

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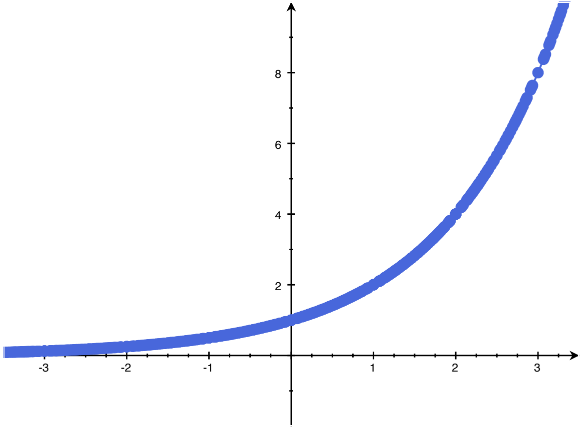

$x = n/2,\ n/3,\ \ldots,\ n/15$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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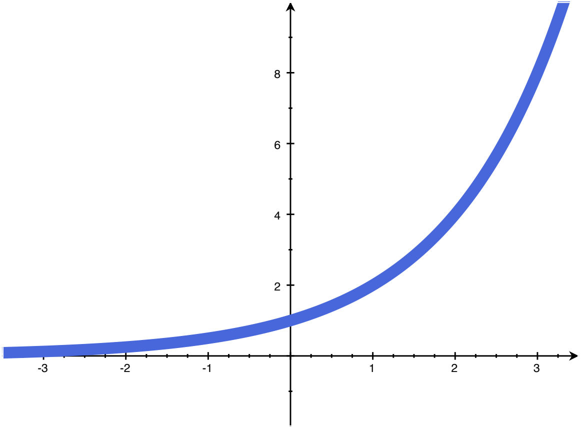

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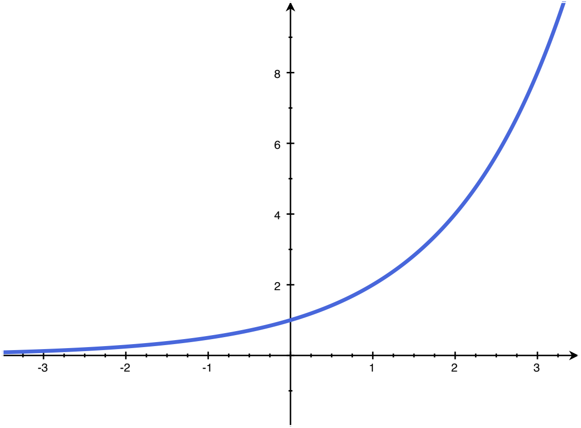

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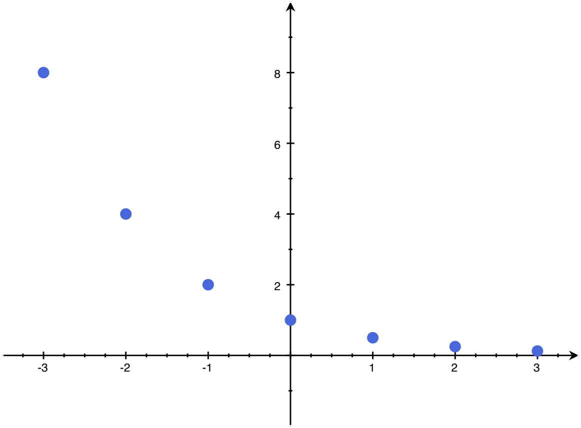

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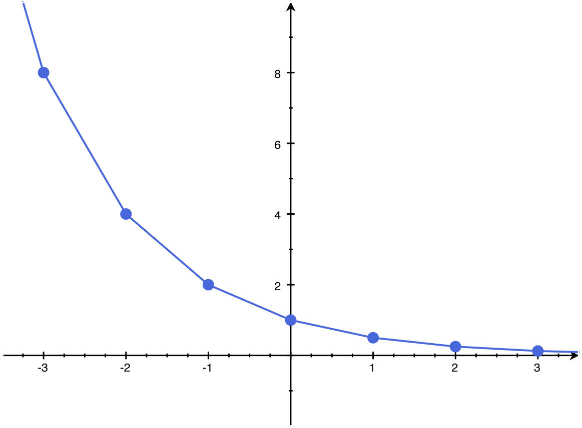

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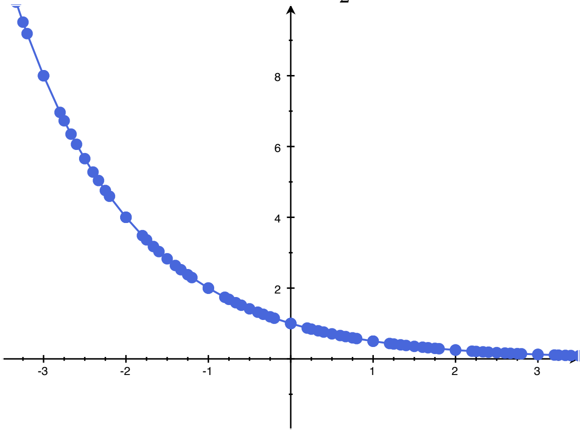

$x = n/2, n/3, n/4, n/5$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

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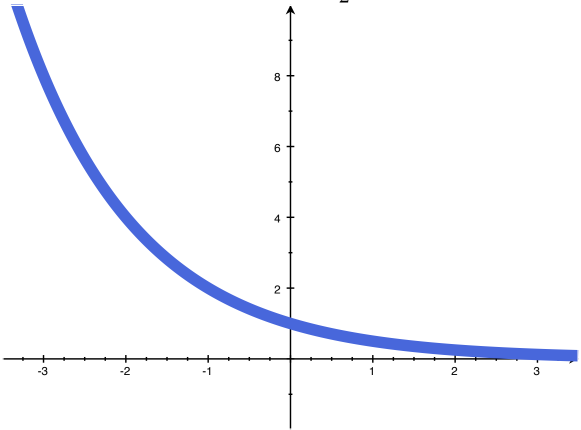

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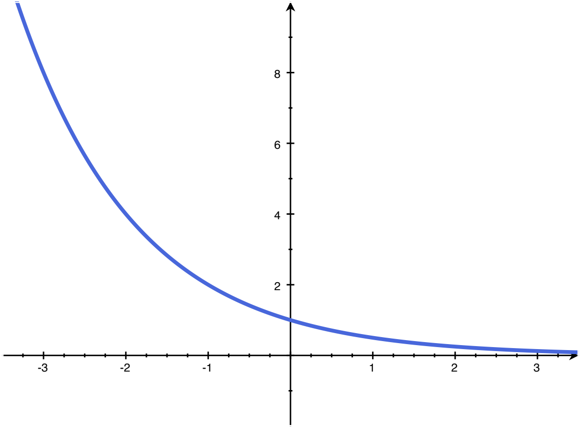

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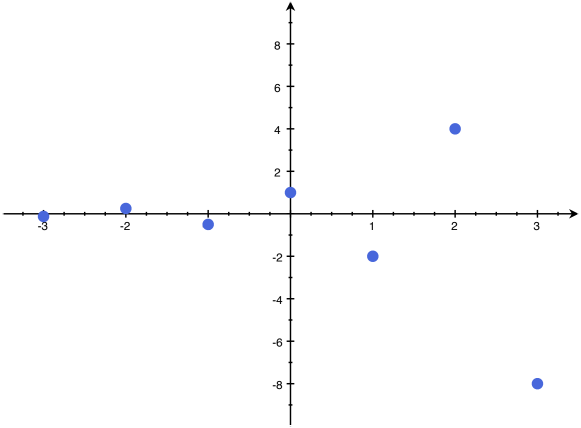

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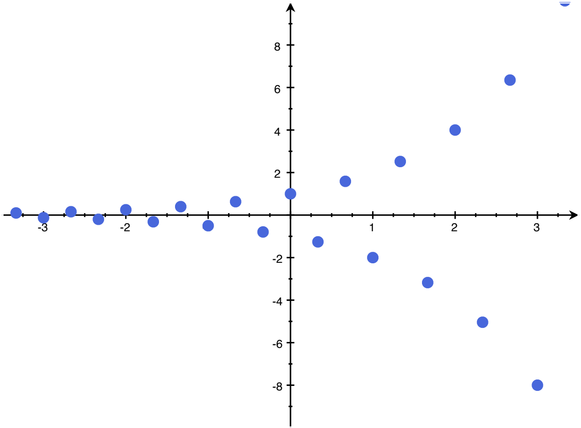

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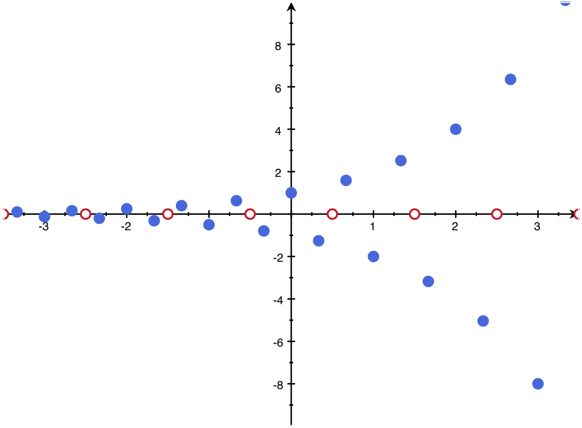

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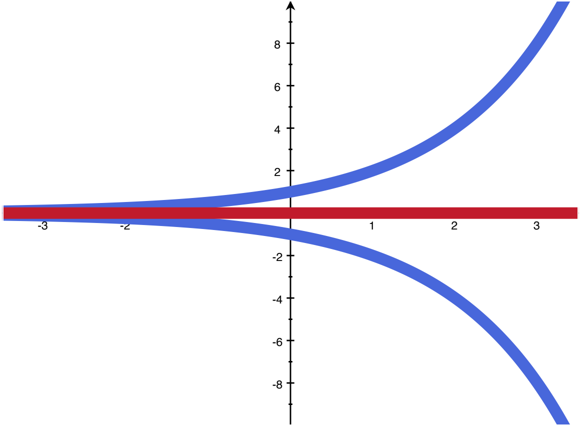
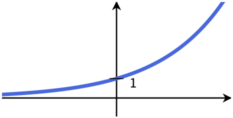


$x = n/2$, $n/3$, $\ldots$, $n/100$, for $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

OH NO!

The function a^x:

$1 < a$:



D: $(-\infty, \infty)$, R: $(0, \infty)$

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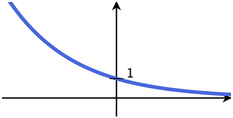

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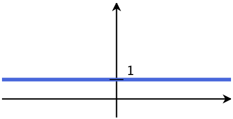
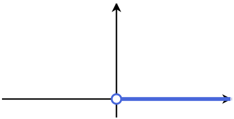


D: $(-\infty, \infty)$, R: $\{1\}$

$a = 0$:



D: $(0, \infty)$, R: $\{0\}$

(If $a < 0$, then a^x is not defined as a function on the real numbers.)

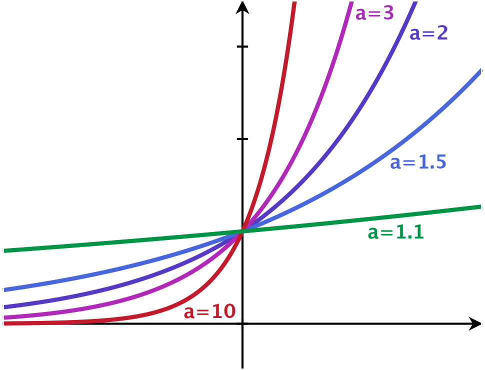
Properties:

$a^b * a^c = a^{b+c}$ $\quad (a^b)^c = a^{b*c}$ $\quad a^{-x} = 1/a^x$ $\quad a^c * b^c = (ab)^c$

Our favorite exponential function:

Look at how the function is increasing through the point $(0, 1)$:

$y = a^x$:



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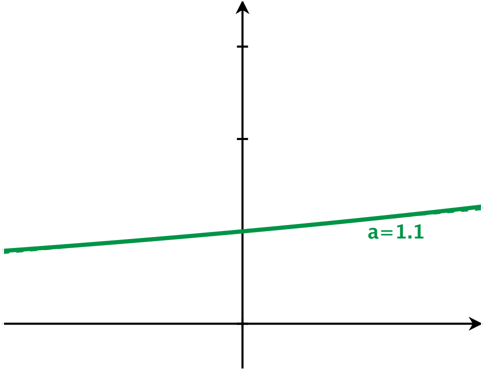

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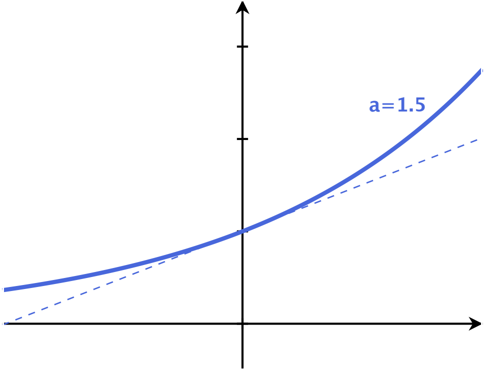

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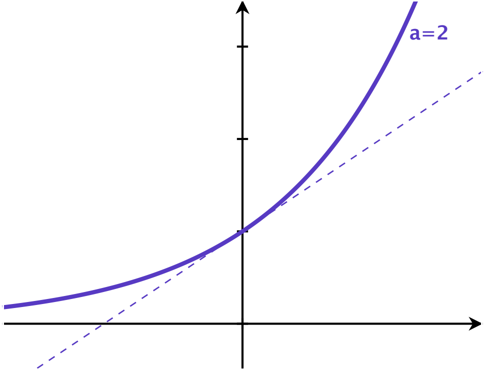

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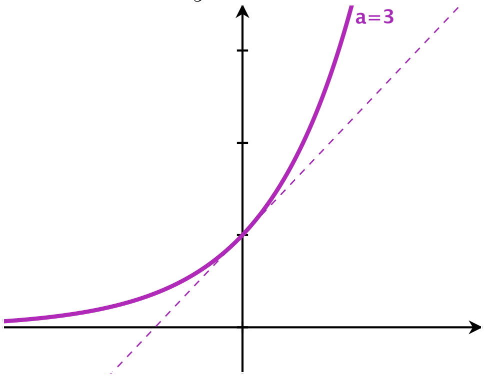

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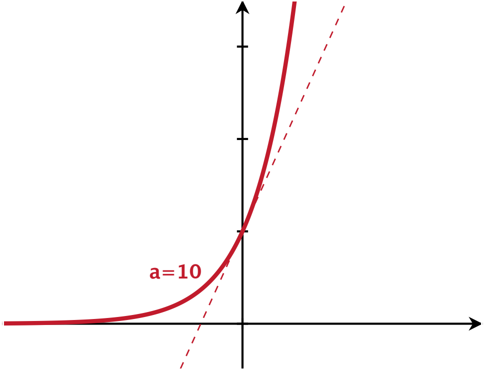
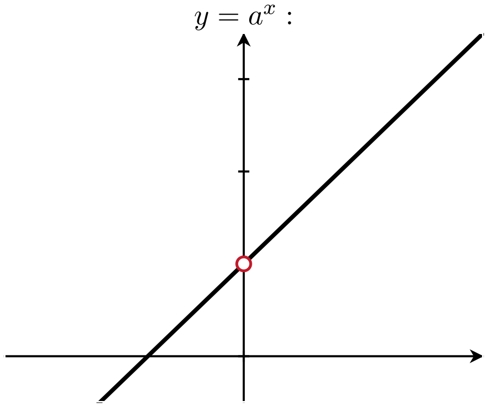


Our favorite exponential function:

Look at how the function is increasing through the point $(0,1)$:



Q: Is there an exponential function whose slope at (0,1) is 1?

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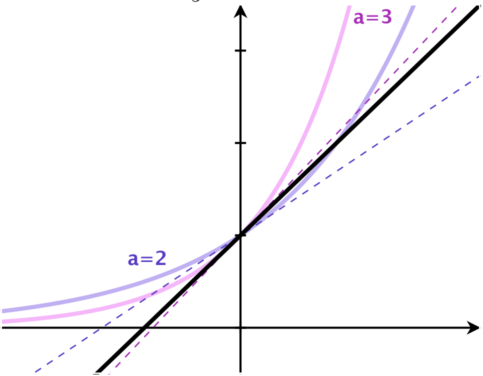

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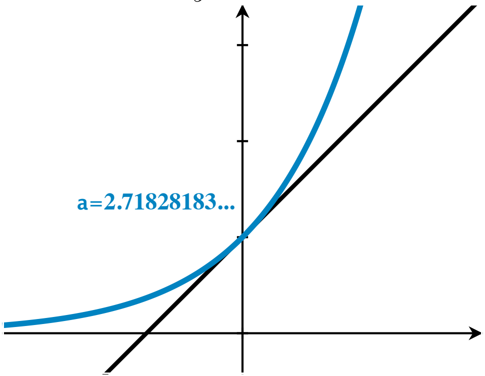

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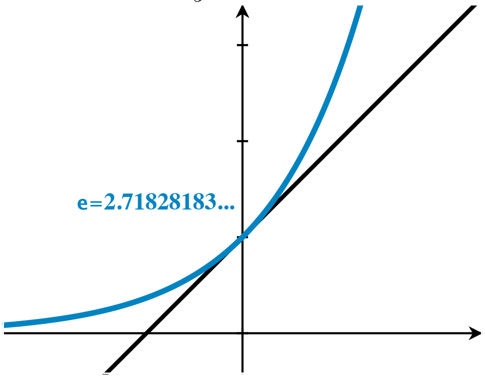

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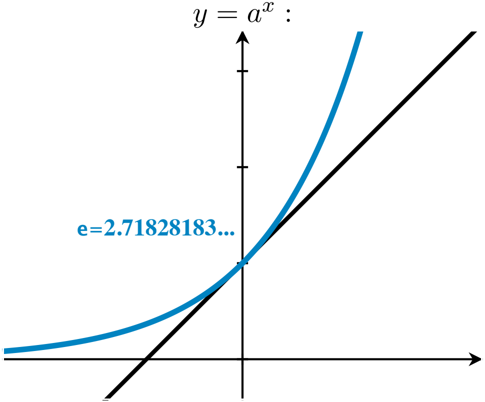

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Read: "Exponential growth and decay", examples 3 and 4.