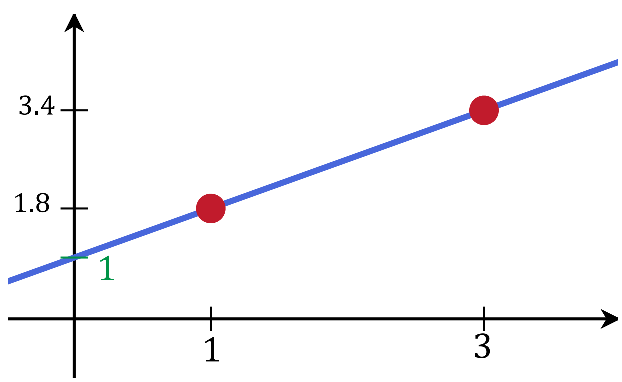
Functions and their graphs
(Sections 1.1 & 1.2)

Simplest functions: Lines!



Example:

Slope: $m = \dfrac{3.4 - 1.8}{3 - 1} = 0.8$ (rise/run)

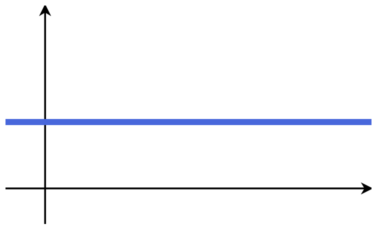
Point-slope form: $y - 1.8 = 0.8 * (x - 1)$ (good for writing down lines)

Slope-intercept form: $y = 0.8 * x + 1$ (good for graphing)

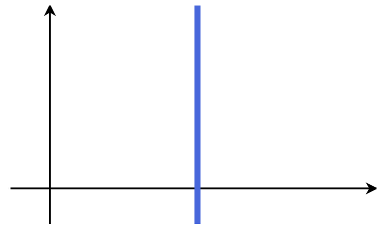
General form: $-0.8 * x + y - 1 = 0$ (accounts for ∞ slope)

Lines: Special cases

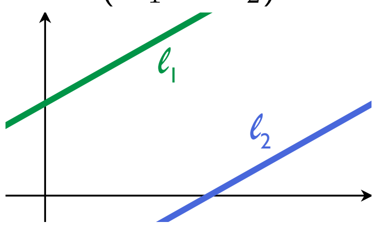
Constant functions
$(m = 0)$



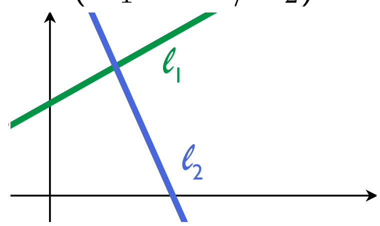
Vertical lines
$(m = \infty)$



Parallel lines
$(m_1 = m_2)$



Perpendicular lines
$(m_1 = -1/m_2)$



Other good functions to know: polynomials.

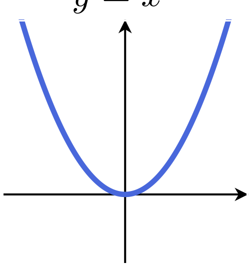
$$y = a_0 + a_1 x + \cdots + a_n x^n$$

(n is the degree)

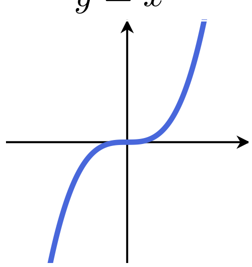
The basics (know these graphs!)

$n = 0$:	$n = 1$:	$n = 2$:
constants	lines	parabolas

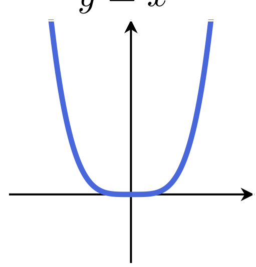
$y = x^2$



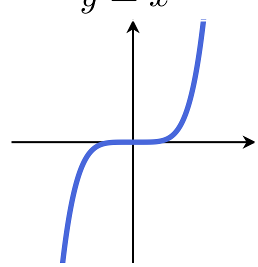
$y = x^3$



$y = x^4$



$y = x^5$



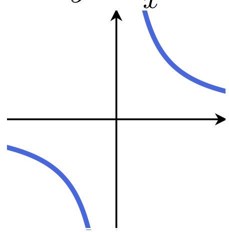
Question: How many points to you need to solve for a polynomial of degree n?

Other good functions to know: rationals.

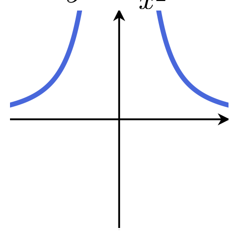
$$y = \frac{a_0 + a_1 x + \cdots + a_n x^n}{b_0 + b_1 x + \cdots + b_m x^m}$$

The basics (know these graphs!)

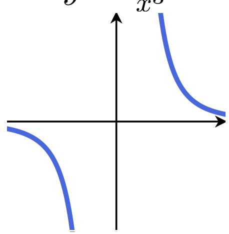
$y = \frac{1}{x}$



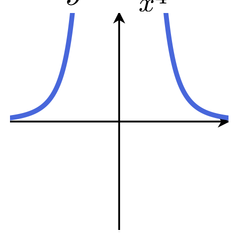
$y = \frac{1}{x^2}$



$y = \frac{1}{x^3}$



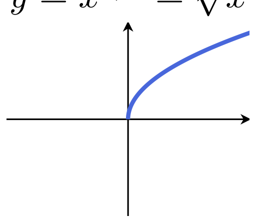
$y = \frac{1}{x^4}$



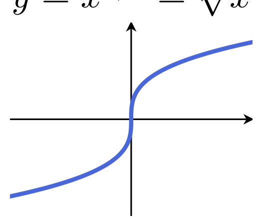
Other powers: $y = x^a$.

The basics (know these graphs!)

$y = x^{1/2} = \sqrt{x}$



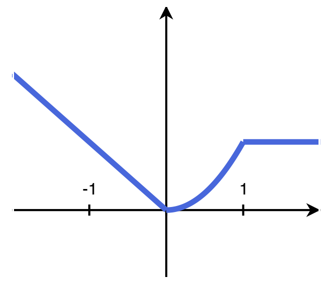
$y = x^{1/3} = \sqrt[3]{x}$



Piecewise functions

Example:

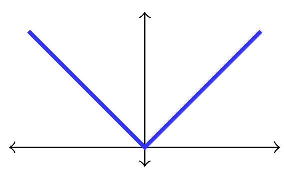
$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, x > 1 & \end{cases}$$



The absolute value of a real number x is

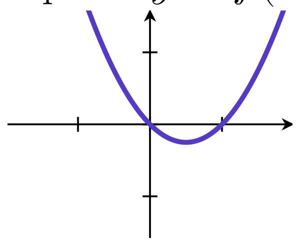
$$|x| = \begin{cases} x & \text{if } x \text{ is nonnegative,} \\ -x & \text{if } x \text{ is negative,} \end{cases}$$

so that $|x|$ is always nonnegative.

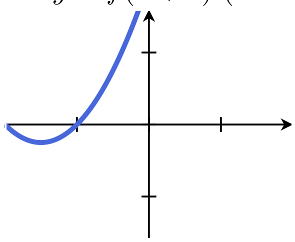


New functions from old

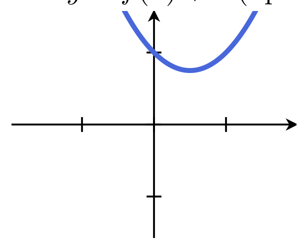
Graph of $y = f(x)$:



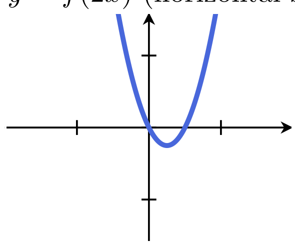
Graph of $y = f(x+2)$ (left shift) :



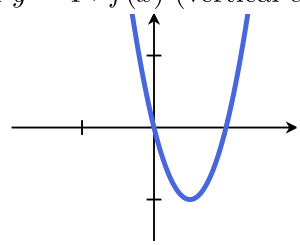
Graph of $y = f(x) + 1$ (up shift):



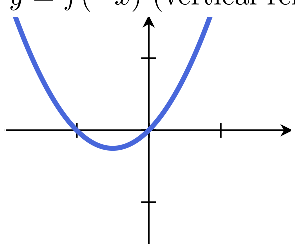
Graph of $y = f(2x)$ (horizontal squeeze) :



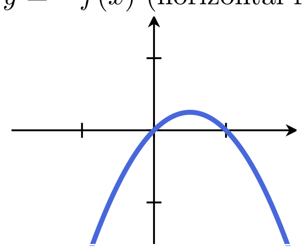
Graph of $y = 4 * f(x)$ (vertical dialation):



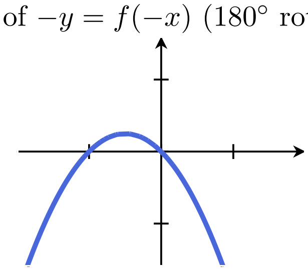
Graph of $y = f(-x)$ (vertical reflection) :



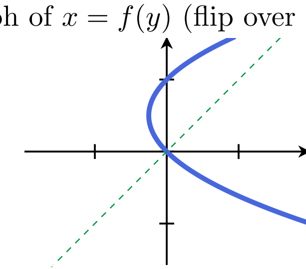
Graph of $y = -f(x)$ (horizontal reflection):



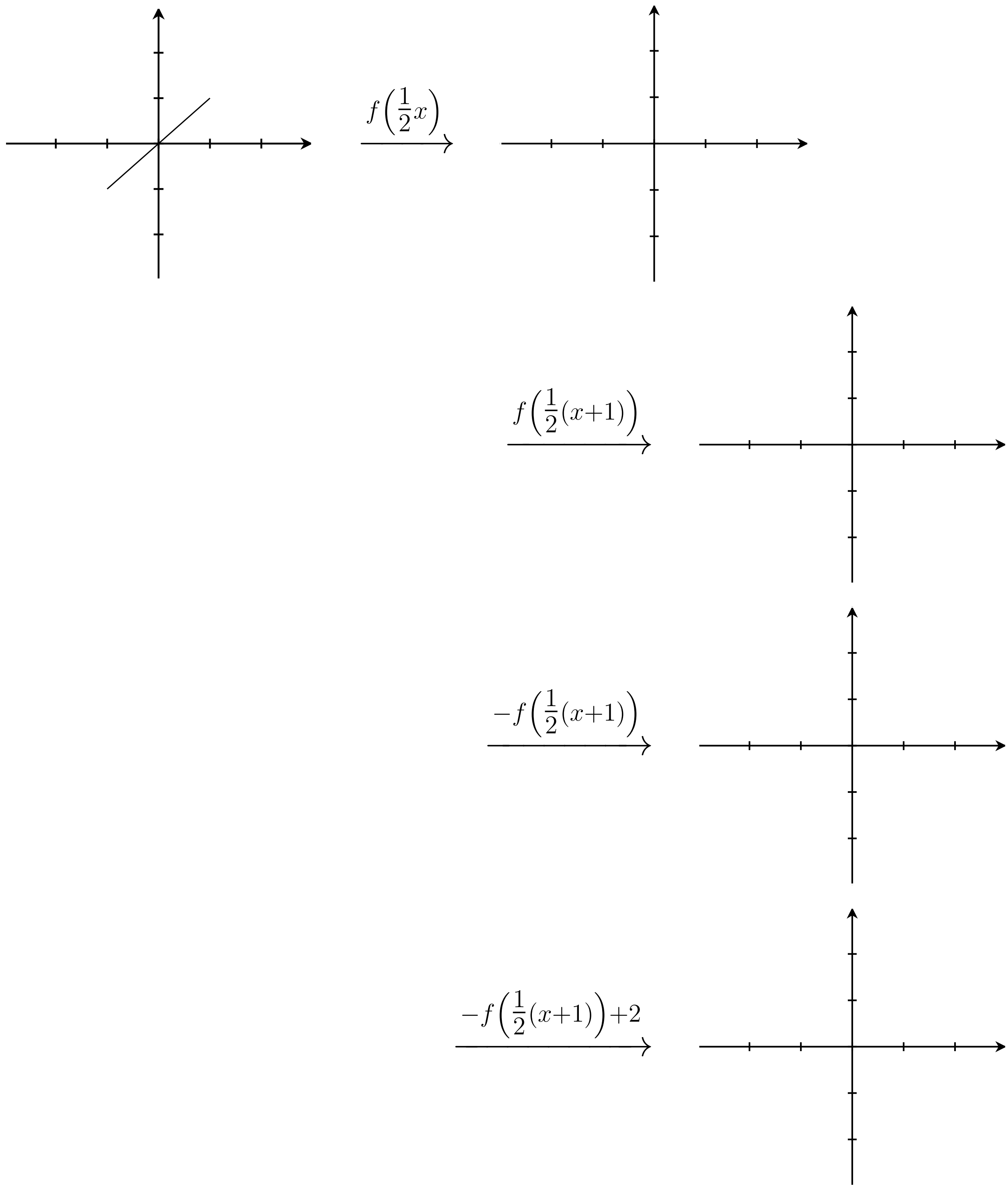
Graph of $-y = f(-x)$ (180° rotation) :



Graph of $x = f(y)$ (flip over $y = x$):



Ex: Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:

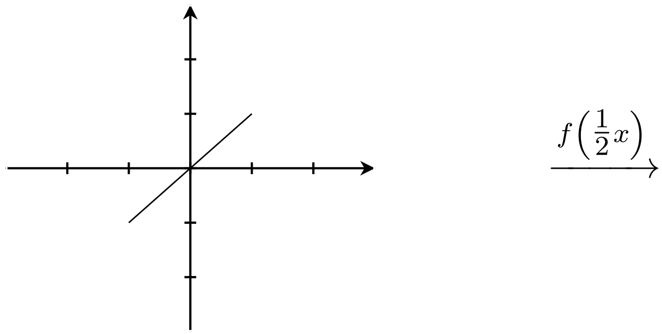


The *domain* of a function f is the set of x over which $f(x)$ is defined.

The *range* of a function f is the set of y which satisfy $y = f(x)$ for some x.

You try: (see notes)

Transform the graph of $f(x)$ into the graph of $-f\left(\frac{1}{2}(x+1)\right)+2$:



...

The domain of a function f is the set of x over which $f(x)$ is defined.
The range of a function f is the set of y which satisfy
$y = f(x)$ for some x.

You try: If we set the domain of $f(x)$ to be $-1 \leq x \leq 1$, compute the domain and range of the functions at each step of computing the example above.

You try: Find the natural domain and range of each:

$$a(x) = 1 - \sqrt{x} \qquad b(x) = \frac{9}{1-x^2} \qquad c(x) = 1/|x-3|$$

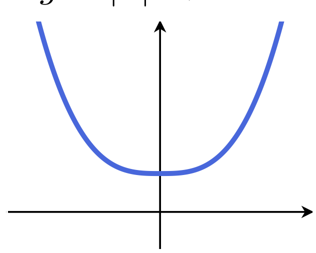
Function composition

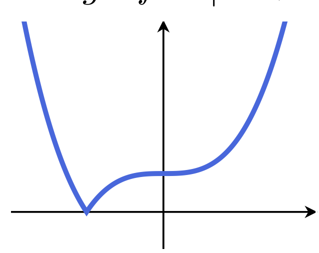
If f and g are functions, the composite function $f \circ g$ (f composed with g) is defined by

$$(f \circ g)(x) = f(g(x))$$

Example: Let $f(x) = x^3 + 1$ and let $g(x) = |x|$. We have

$f \circ g = |x|^3 + 1$ and $g \circ f = |x^3 + 1|$.



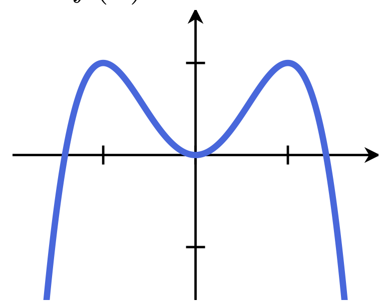


Symmetries

A function $f(x)$ is even if it satisfies

$$f(-x) = f(x)$$

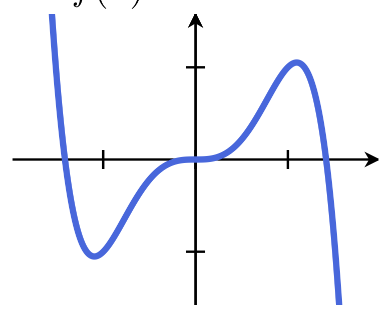
ex: $f(x) = 2x^2 - x^4$



A function $f(x)$ is odd if it satisfies

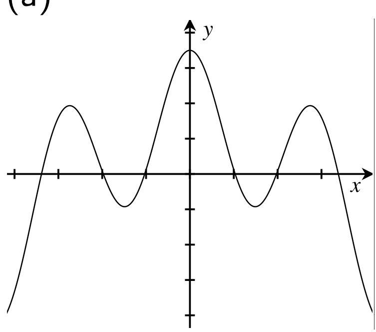
$$f(-x) = -f(x)$$

ex: $f(x) = 2x^3 - x^5$

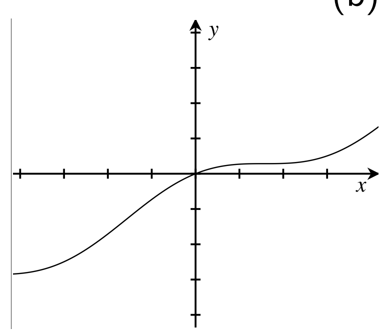


Examples: Even, odd, or neither?

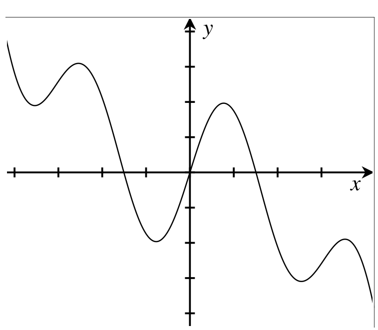
(a)



(b)



(c)

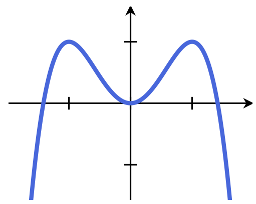


(d) $f(x) = \dfrac{x^3 + x}{x + \frac{1}{x}}$

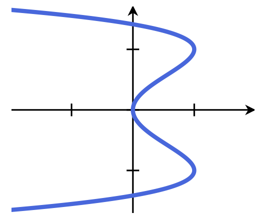
(for this one:
actually plug in $-x$
and see what happens
algebraically)

A graph is a graph of a function if for every x in its domain, there is exactly one y on the graph which is mapped to by that x:

Function:

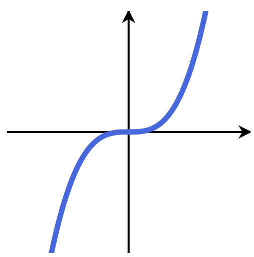


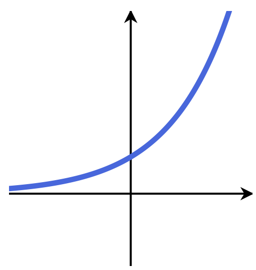
Not a function:



A function is additionally one-to-one if for every y, there is at most one x which maps to that y.

A 1-to-1 functions:





Function that's not 1-to-1: