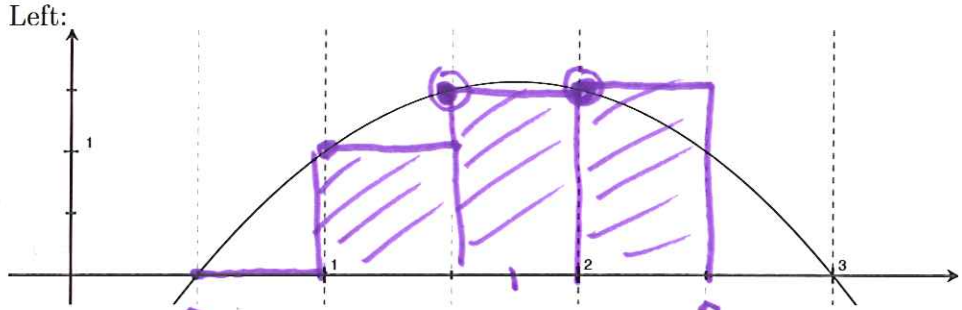
$$\int_{1/2}^{5/2} \boxed{y = \left(\tfrac{5}{4}\right)^2 - \left(x - \tfrac{7}{4}\right)^2} \, dx \qquad n = 4$$

compute estimate.

$\Delta x = \frac{1}{2}.$

Left:



$$\frac{1}{2}\Big(0+1+\underbrace{2\left(\tfrac{24}{16}\right)}_{24/8\,=\,3}\Big) = \boxed{2}\ \checkmark$$

$f(\frac{1}{2}) = 0$

$f(1) = \frac{25-9}{4^2} = \frac{16}{16} = 1$

$f(\frac{3}{2}) = \frac{25-1}{4^2} = \frac{24}{16} = 3/2$

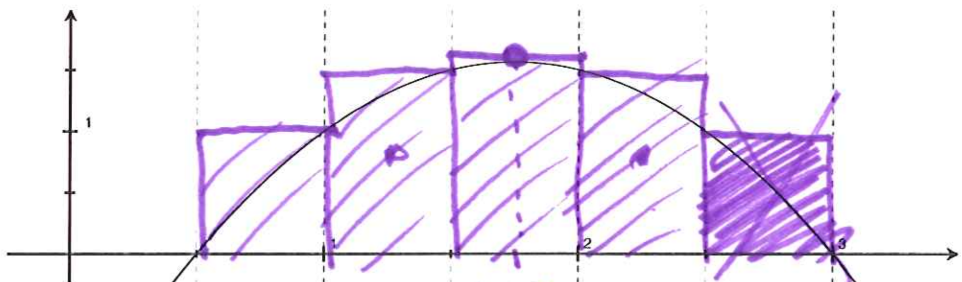
$\frac{6}{4}$

$f(2) = \frac{25-1}{4^2} = \frac{24}{16}$

$f(1.75) = \frac{25}{16}$

$f(2.5) = f(1)$

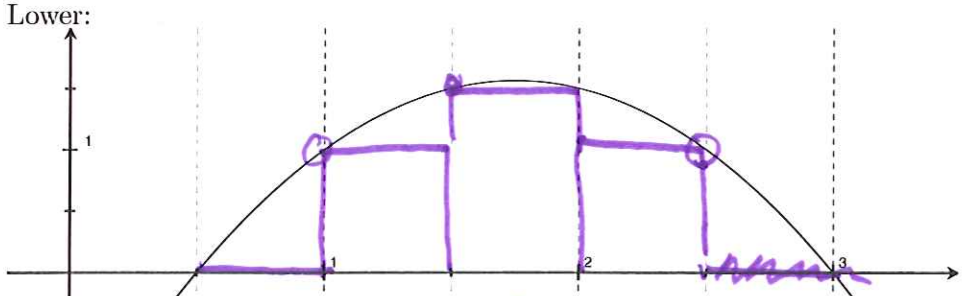
Upper:



1.75

$$\frac{1}{2}\left(1+2(3/2)+\tfrac{25}{16}\right) = \frac{89}{32} \approx 2.78\ldots$$

Lower:



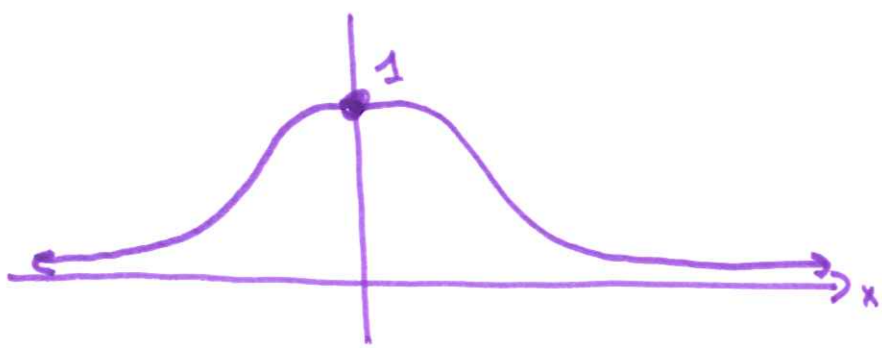
$$\frac{1}{2}\Big(1+\underbrace{\tfrac{24}{16}}_{3/2}+1\Big) = \frac{1}{2}\Big(\frac{32+24}{\underbrace{16}_{7/2}}\Big) = \boxed{7/4} = 1.75$$

Today: more numerical integration

why: some functions don't have nice antiderivatives

ex $\int_{-1}^{1} e^{-x^2}\,dx$ (think: let $u = -x^2$ so $du = -2x\,dx$ no luck)

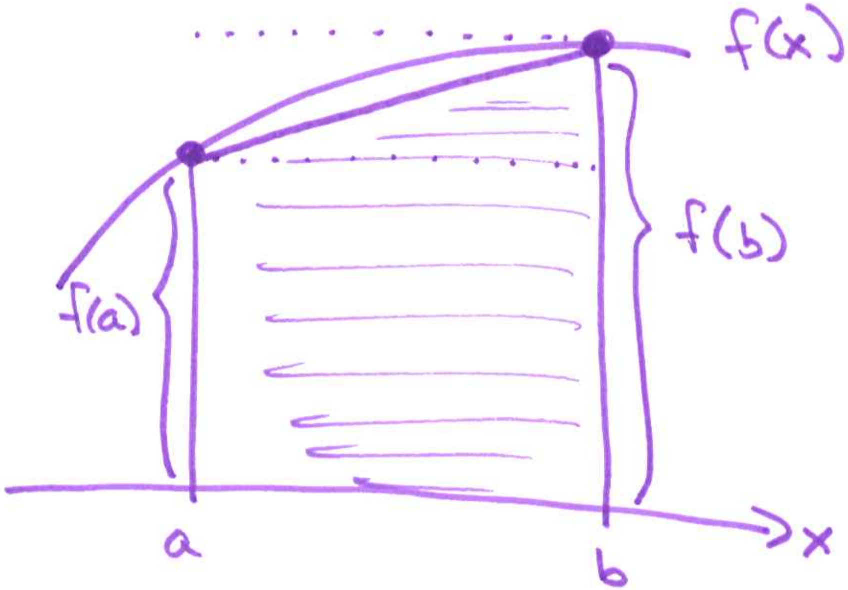
e^{-x^2}: $= \frac{1}{e^{x^2}}$



$\text{erf}(x) = c\int_0^x e^{-t^2}\,dt$ where $c = \frac{2}{\sqrt{\pi}}$

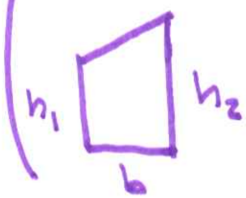
Can only approximate (take a limit)

Trapezoids



area under curve

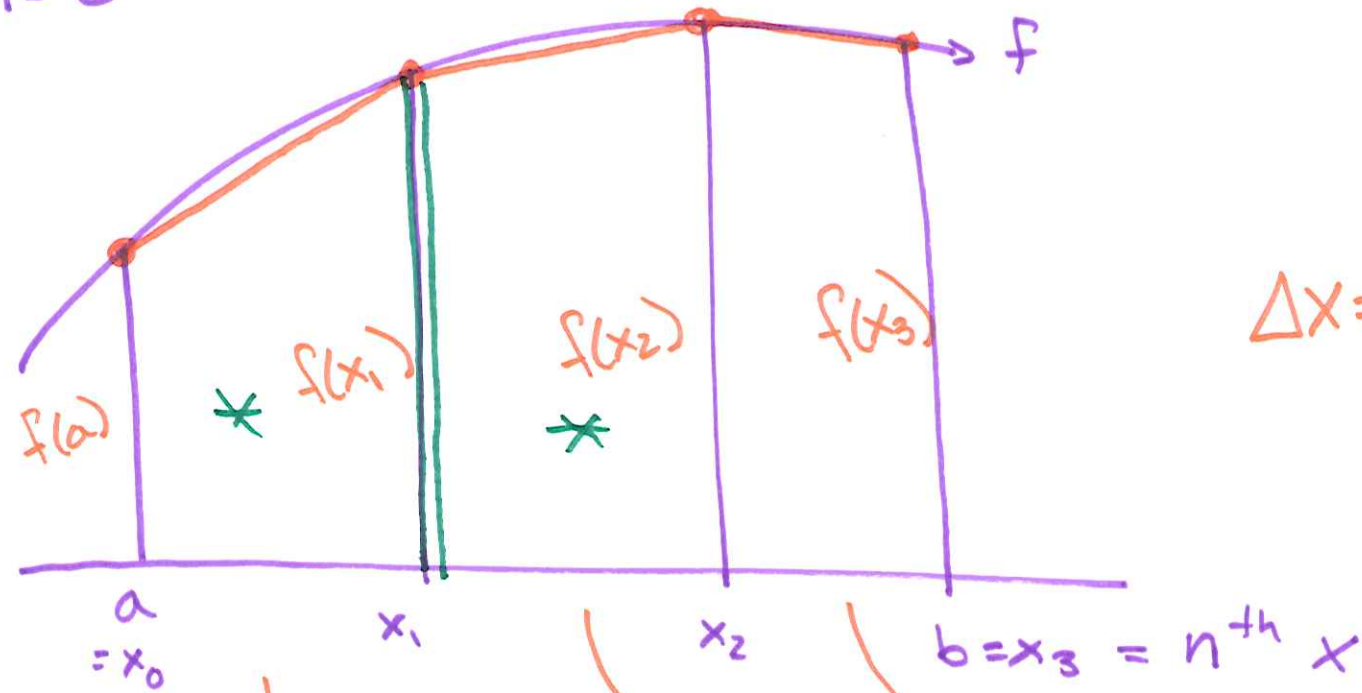
$$\approx \frac{1}{2}\left(f(a)+f(b)\right)(b-a)$$

area (

) $= \dfrac{h_1+h_2}{2} \cdot b$

↑ average value of height

$n = 3$



$$\Delta x = \frac{b-a}{3}$$

$$\frac{1}{2}\Delta x\left(\cancel{2}\left(\underline{f(x_1)}+f(x_0)\right) + \left(\underline{f(x_1)}+\underline{f(x_2)}\right) + \left(\underline{f(x_2)}+f(x_3)\right) \right)$$

$$= \frac{1}{2}\Delta x\left(f(a)+f(b) + 2\left(f(x_1)+f(x_2)\right)\right)$$

In general, using trapezoids,

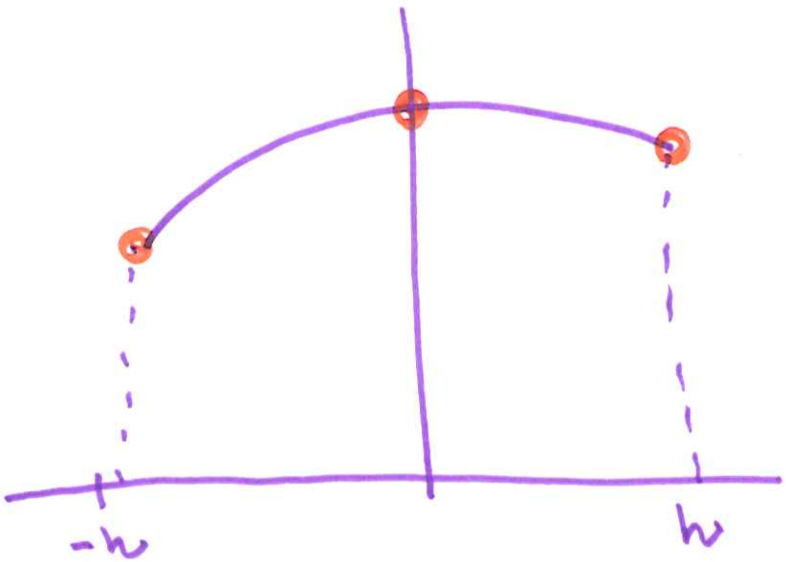
$$\cancel{\text{Area}} \approx \frac{1}{2}\,\frac{(b-a)}{n}\left(f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_i)\right)$$

def integral

Simpson's:

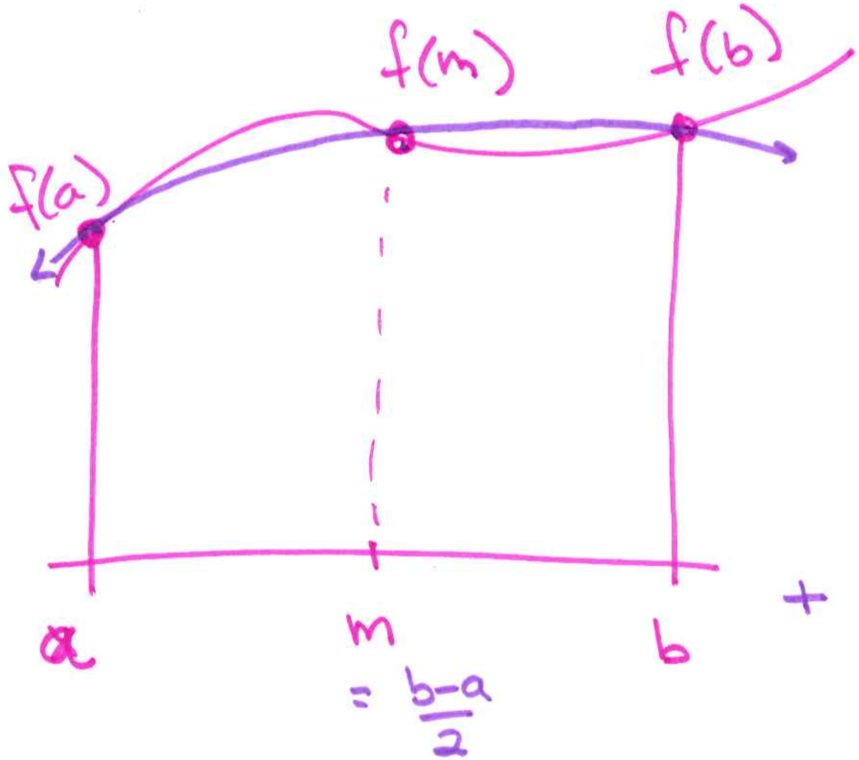
model curve as a quadratic function.

new: need 3 points.



fit a quadratic to these three pts.

idea: I <u>can</u> integrate polynomials even if I can't do f(x).



$$p(x) = (x-m)(x-b)\,\frac{f(a)}{(a-m)(a-b)}$$

$$+ (x-a)(x-b)\,\frac{f(m)}{(m-a)(m-b)}$$

$$+ (x-a)(x-m)\,\frac{f(b)}{(b-a)(b-m)}$$

want $\displaystyle\int_a^b P(x)\,dx = \underline{\underline{\frac{b-a}{6}}}\,\big(f(a) + 4f(m) + f(b)\big)$

$a = \frac{1}{2}$, $b = 5/2$, $n = 4$

$f(\frac{1}{2}) = 0$

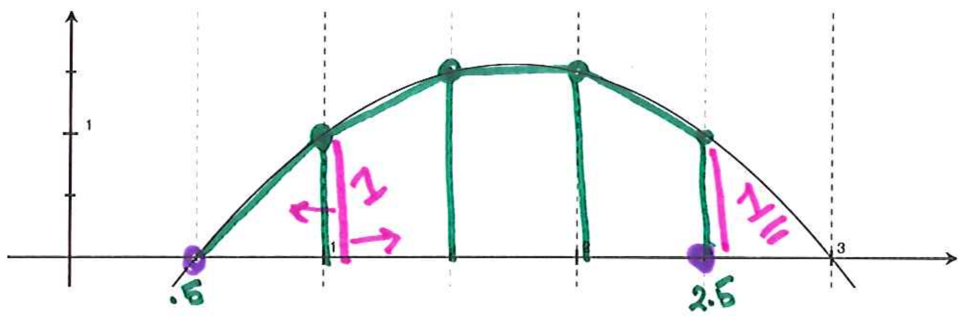
$f(1) = 1$

$f(1.5) = 3/2$

$f(2) = 3/2$

$f(2.5) = 1$

* Trapezoid:

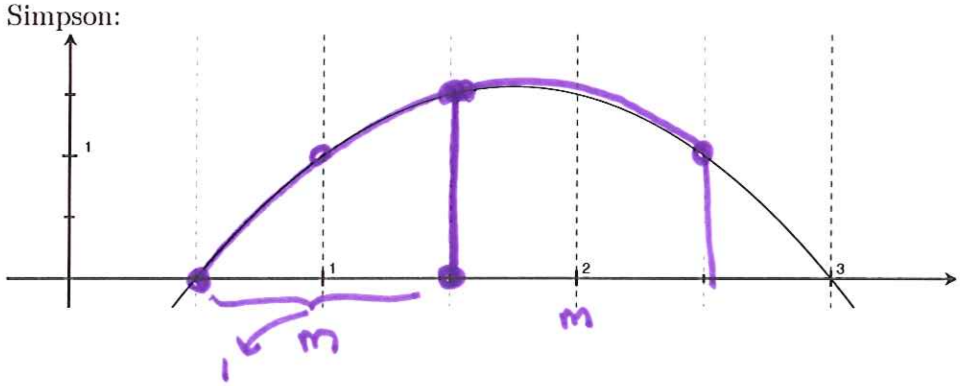


$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(0 + 1 + 2\left(1 + 3/2 + 3/2\right)\right) = 9/4 = 2.25$$

$\frac{1}{2}$ ↓ $1/2$, $\frac{1}{2}$ ↓ Δx

$n = \#$ partition intervals.

Simpson:



$$A \approx \frac{1}{6}\left(f(1.5) + 4f(1) + f(.5)\right) + \frac{1}{6}\left(f(2.5) + 4f(2) + f(1.5)\right)$$

$\stackrel{?}{=} 2.33\bar{3}$.

Actual: $$\int_{1/2}^{5/2} (5/4)^2 - (x - 7/4)^2\, dx = \left(\frac{5}{4}\right)^2 x - \frac{1}{3}\left(x - \frac{7}{4}\right)^3 \Big|_{x=1/2}^{5/2}$$

$$= \left(\frac{5}{4}\right)^2\left(\frac{5}{2} - \frac{1}{2}\right) - \frac{1}{3}\left(\left(\frac{3}{4}\right)^3 - \left(-\frac{5}{4}\right)^3\right) = \frac{7}{3} = 2.\bar{3}$$

on ww/book n is even

on applet plug in $n/2$ for n.

for simpsons.

Read about errors.

M_2 is max of $f^{''}(x)$ (2)

over interval $[a,b]$

M_4 is max of $f^{(4)}(x)$

over interval $[a,b]$.