11/7: Modeling Accumulations

The purpose of calculus is twofold:

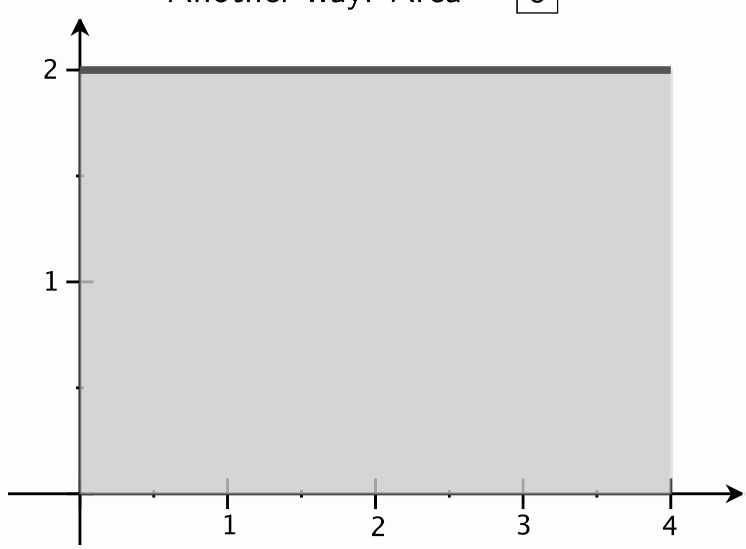
1. to find how something is changing, given what it's doing;
2. to find what something is doing, given how it's changing.

We did (1) geometrically and algebraically. We did (2) algebraically. Let's do (2) geometrically!

If you travel at 2 mph for 4 hours, how far have you gone?

Answer: 8 miles.

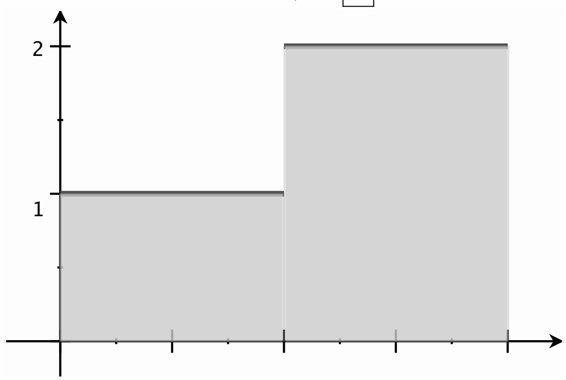
Another way: Area $= \boxed{8}$



(graph of speed, i.e. graph of derivative)

If you travel at 1 mph for 2 hours, and 2 mph for 2 hours, how far have you gone?

Area $= 2{+}4{=}\boxed{6}$



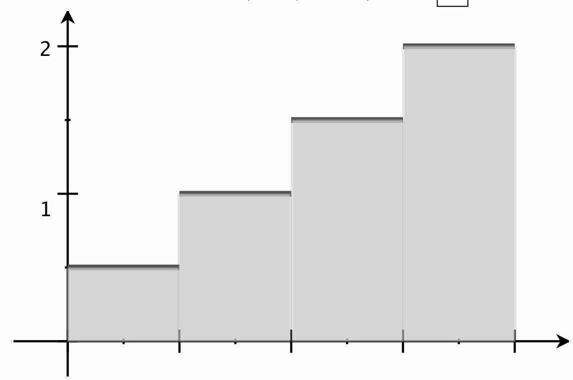
(graph of speed, i.e. graph of derivative)

If you travel at

.5 mph for 1 hour,
1 mph for 1 hour,
1.5 mph for 1 hour,
2 mph for 1 hour,

how far have you gone?

Area $= \;.5 + 1 + 1.5 + 2 = \boxed{5}$



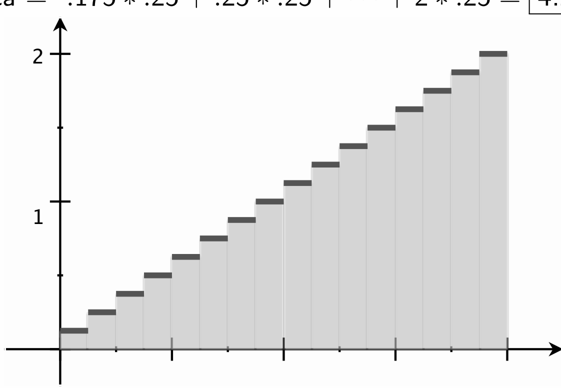
(graph of speed, i.e. graph of derivative)

If you travel at

.175 mph for 1/4 hour,
.25 mph for 1/4 hour,
...
2 mph for 1/4 hour,

how far have you gone?

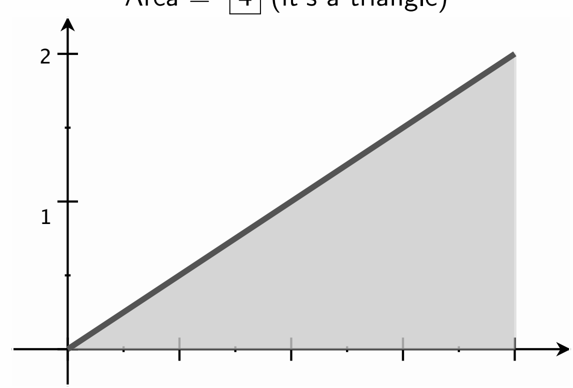
Area $= \;.175 * .25 + .25 * .25 + \cdots + 2 * .25 = \boxed{4.25}$



(graph of speed, i.e. graph of derivative)

If you travel at $\frac{1}{2}t$ mph for 2 hours, how far have you gone?

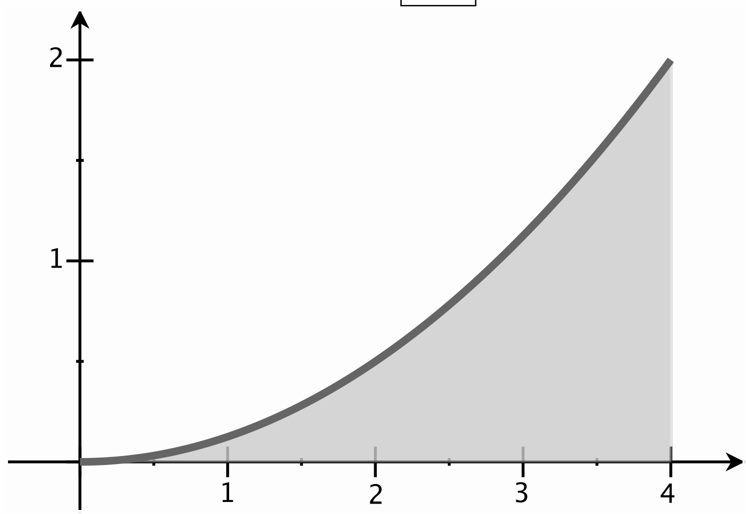
Area $= \;\boxed{4}$ (it's a triangle)



(graph of speed, i.e. graph of derivative)

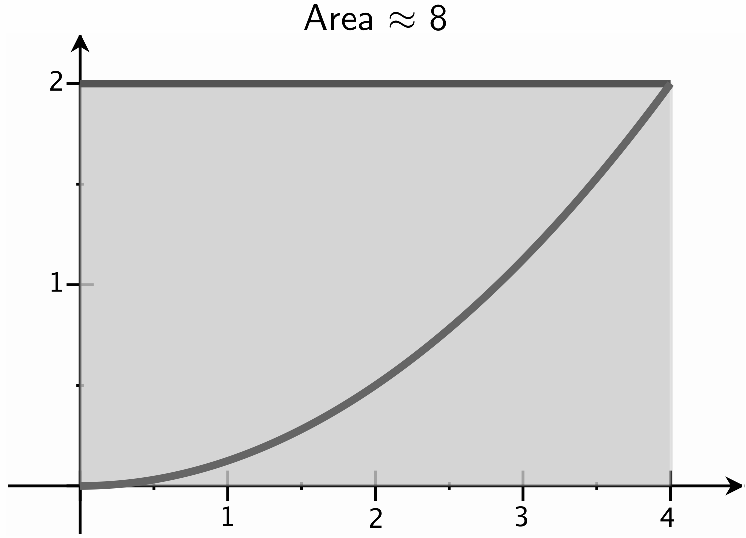
Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

Area = $\boxed{???}$



Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

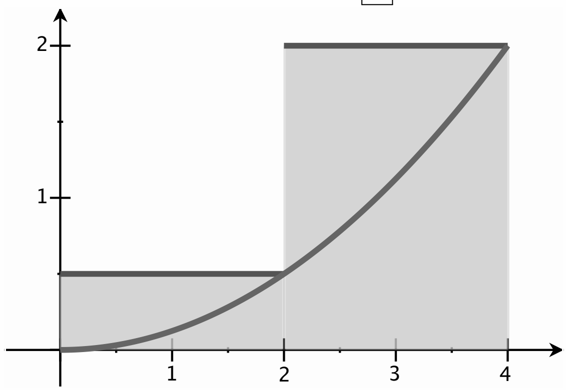
Estimate 1: pick the highest point
Area ≈ 8



Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

Estimate 2: pick two points

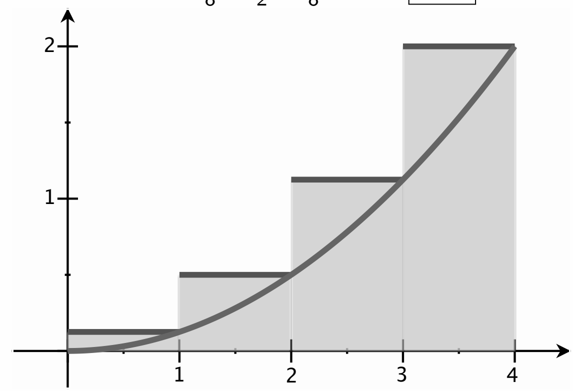
Area $\approx 1+4 = \boxed{5}$



Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

Estimate 3: pick four points

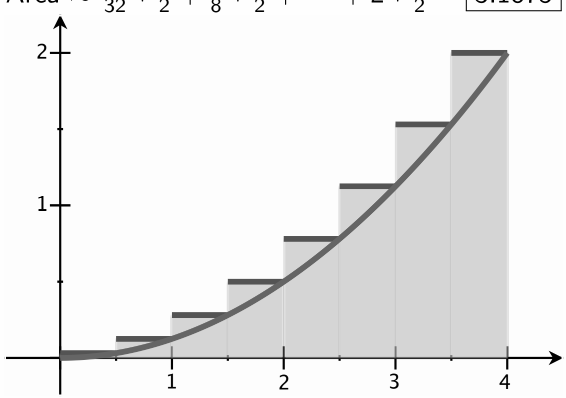
Area $\approx \frac{1}{8} + \frac{1}{2} + \frac{9}{8} + 2 = \boxed{3.75}$



Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

Estimate 4: pick eight points

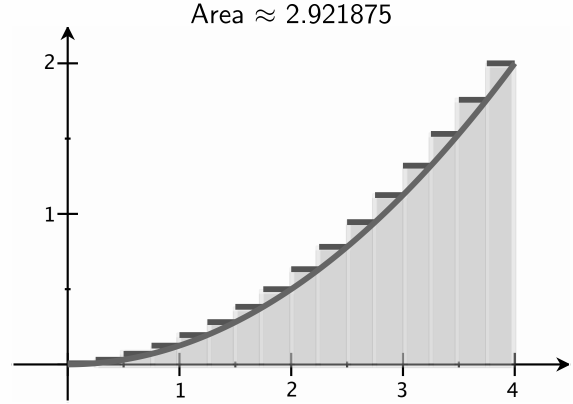
Area $\approx \frac{1}{32} * \frac{1}{2} + \frac{1}{8} * \frac{1}{2} + \cdots + 2 * \frac{1}{2} = \boxed{3.1875}$



Estimate the area under the curve
$y = \frac{1}{8}x^2$ between $x = 0$ and $x = 4$:

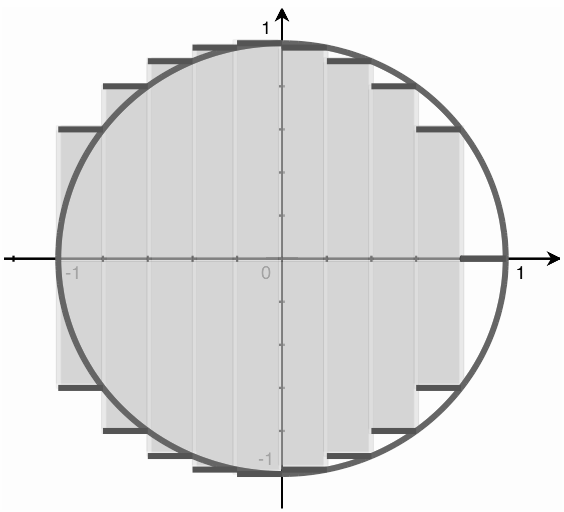
Estimate 5: pick sixteen points

Area ≈ 2.921875



Estimating the Area of a Circle with $r = 1$

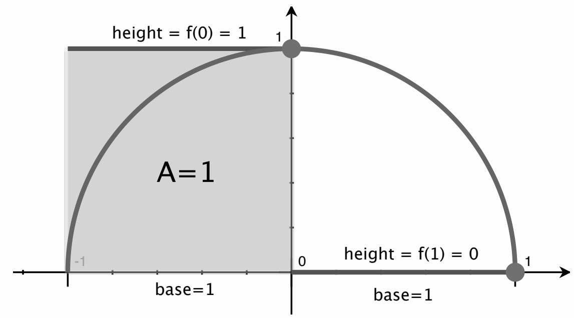
Divide it up into rectangles:



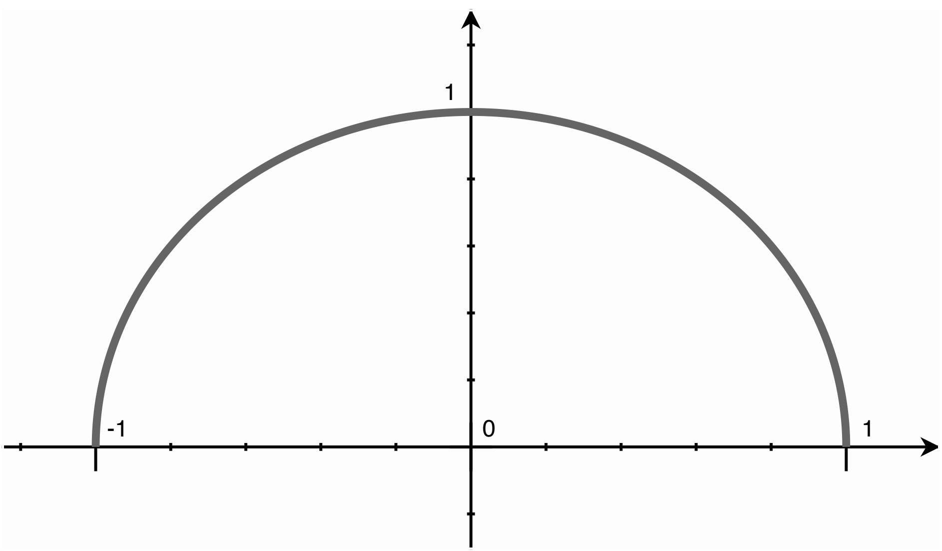
Estimating the Area of a Circle with $r = 1$

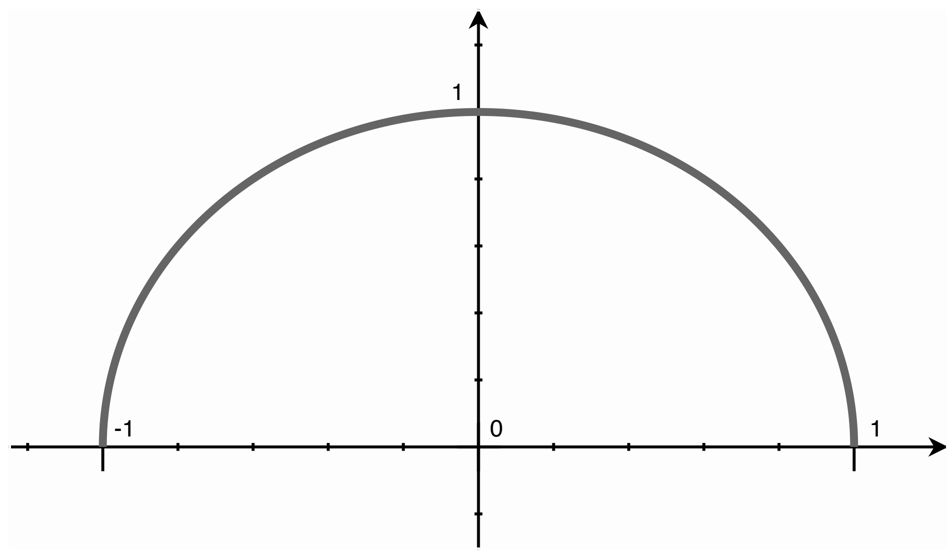
Divide it up into rectangles:

Estimate area of the half circle ($f(x) = \sqrt{1 - x^2}$) and mult. by 2.



# rect.	Area
4	2*1 = 2
4*2	
4*3	
4*4	
4*5	





The Method of Accumulations

Big idea: Estimating, and then taking a limit.

Let the number of pieces go to ∞
i.e. let the base of the rectangle for to 0.

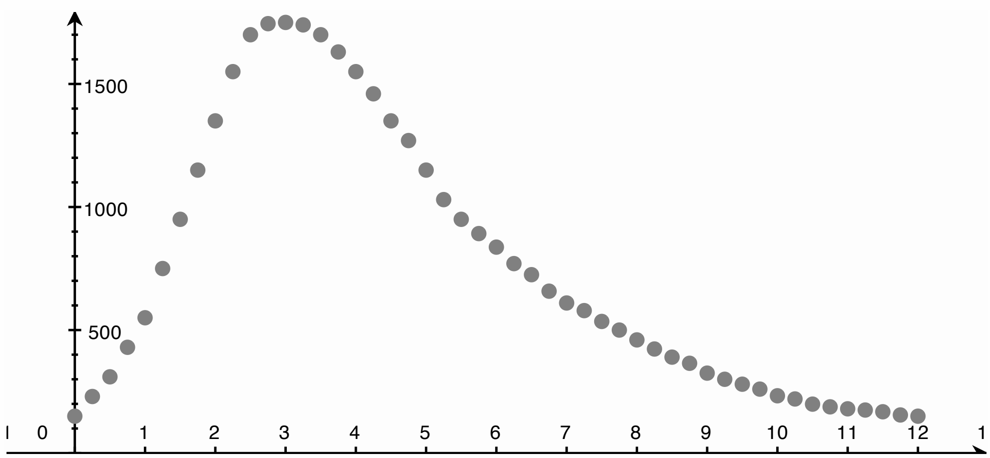
This not only gives us a way to calculate, but gives us a proper definition of what we mean by area!

Also good for volumes and lengths...

A small dam breaks on a river. The average flow out of the stream is given by the following:

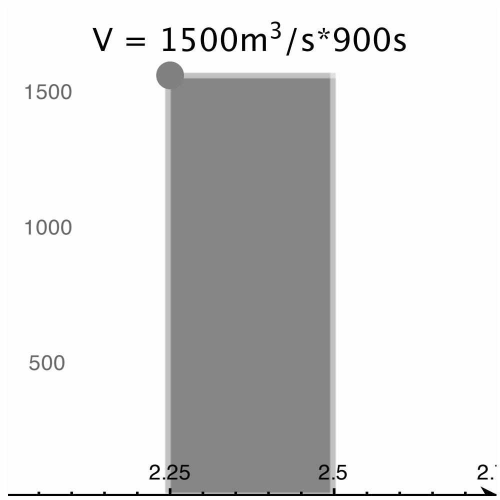
hours	m^3/s	hours	m^3/s	hours	m^3/s
0	150	4.25	1460	8.25	423
0.25	230	4.5	1350	8.5	390
0.5	310	4.75	1270	8.75	365
0.75	430	5	1150	9	325
1	550	5.25	1030	9.25	300
1.25	750	5.5	950	9.5	280
1.5	950	5.75	892	9.75	260
1.75	1150	6	837	10	233
2	1350	6.25	770	10.25	220
2.25	1550	6.5	725	10.5	199
2.5	1700	6.75	658	10.75	188
2.75	1745	7	610	11	180
3	1750	7.25	579	11.25	175
3.25	1740	7.5	535	11.5	168
3.5	1700	7.75	500	11.75	155
3.75	1630	8	460	12	150
4	1550				

A small dam breaks on a river. The average flow out of the stream is given by the following:



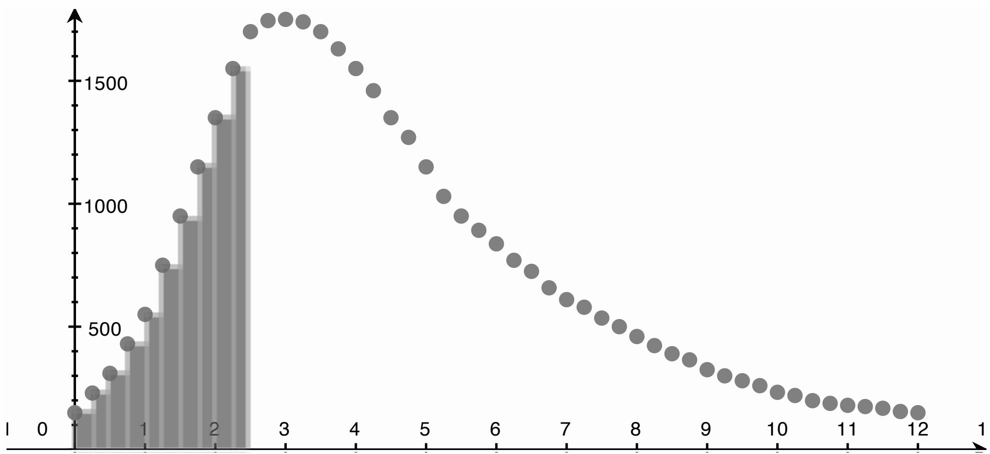
Over each time interval, we estimate the volume of water by

Average rate $\times$ 900 s



Over each time interval, we estimate the volume of water by

Average rate $\times$ 900 s

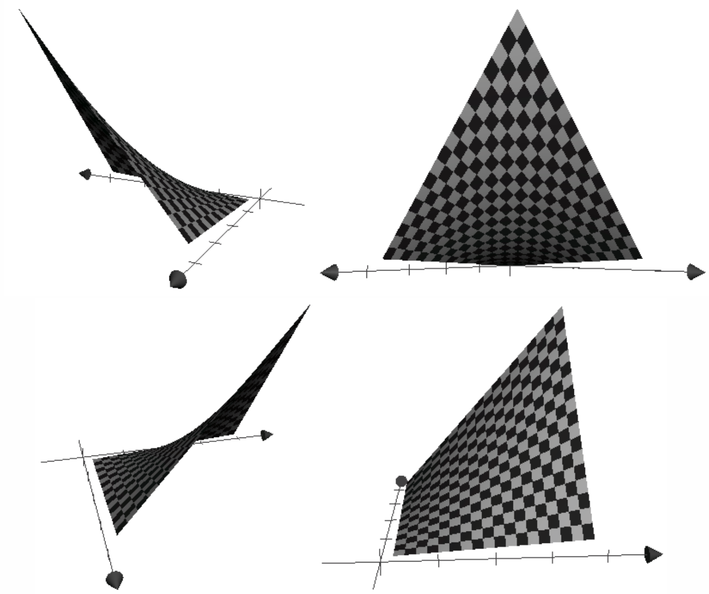


Over each time interval, we estimate the volume of water by

Average rate $\times$ 900 s

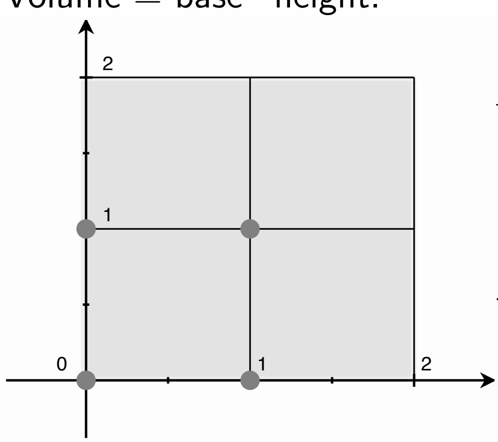
hours	m^3	hours	m^3	hours	m^3
0	135000	4.25	1314000	8.25	380700
0.25	207000	4.5	1215000	8.5	351000
0.5	279000	4.75	1143000	8.75	328500
0.75	387000	5	1035000	9	292500
1	495000	5.25	927000	9.25	270000
1.25	675000	5.5	855000	9.5	252000
1.5	855000	5.75	802800	9.75	234000
1.75	1035000	6	753300	10	209700
2	1215000	6.25	693000	10.25	198000
2.25	1395000	6.5	652500	10.5	179100
2.5	1530000	6.75	592200	10.75	169200
2.75	1570500	7	549000	11	162000
3	1575000	7.25	521100	11.25	157500
3.25	1566000	7.5	481500	11.5	151200
3.5	1530000	7.75	450000	11.75	139500
3.75	1467000	8	414000	12	135000
4	1395000			total=33,319,800	

A tent is raised and has height given by xy over the 2×2 grid where $0 < x < 2$ and $0 < y < 2$. What is the volume of the tent?



A tent is raised and has height given by xy over the 2×2 grid where $0 < x < 2$ and $0 < y < 2$. What is the volume of the tent?

Estimate via boxes!
Volume = base *height.

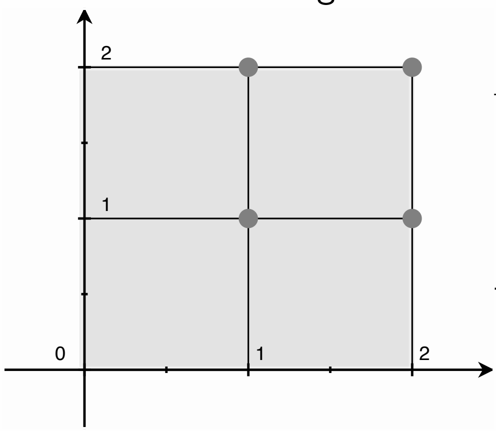


x	y	height $= xy$	volume
0	0	0	0 * 1
0	1	0	0 * 1
1	0	0	0 * 1
1	1	1	1 * 1

total volume ≈ 1

A tent is raised and has height given by xy over the 2×2 grid where $0 < x < 2$ and $0 < y < 2$. What is the volume of the tent?

Estimate via boxes!
Volume = base *height.

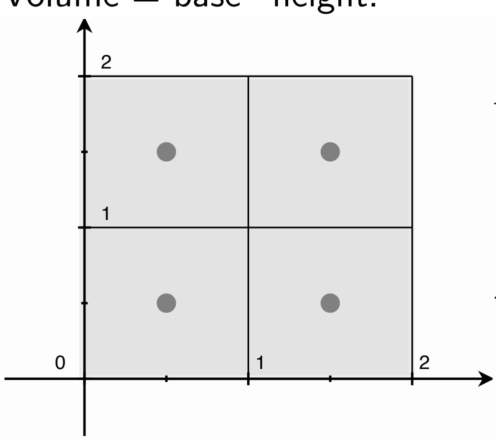


x	y	height $= xy$	volume
1	1	1	1 * 1
1	2	2	2 * 1
2	1	2	2 * 1
2	2	4	4 * 1

total volume ≈ 9

A tent is raised and has height given by xy over the 2×2 grid where $0 < x < 2$ and $0 < y < 2$. What is the volume of the tent?

Estimate via boxes!
Volume = base *height.



x	y	height $= xy$	volume
.5	.5	.25	.5 * 1
.5	1.5	.75	.75 * 1
1.5	.5	.75	.75 * 1
1.5	1.5	2.25	2.25 * 1

total volume ≈ 4.25