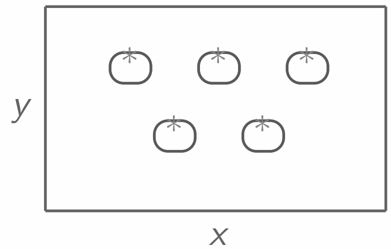
Warm up

Sketch the graph of

$$f(x) = (x-3)(x-2)(x-1) = x^3 - 6x^2 + 11x - 6$$

over the interval $[1, 4]$. Mark any critical points and inflection points. What is the absolute maximum over this interval? What is the absolute minimum over this interval?
[useful value: $\sqrt{3}/3 \approx .6$]

Suppose you want to fence off a garden, and you have 100m of fence. What is the largest area that you can fence off?



Get it into math:

Know: $2x + 2y = 100$ Want: Maximize $A = xy$

Problem: The area, xy, is a function of two variables!!
Strategy: Use the first equation to get xy into one variable:
Solve $2x + 2y = 100$ (the "constraint") and plug into xy (the function you want to optimize).

$$2x + 2y = 100 \implies y = 50 - x$$

so $xy = x(50 - x) = 50x - x^2$.

$\boxed{\text{Domain: } 0 \leq x \leq 50}$

New problem: Maximize $A(x) = 50x - x^2$ over the interval $0 < x < 50$.

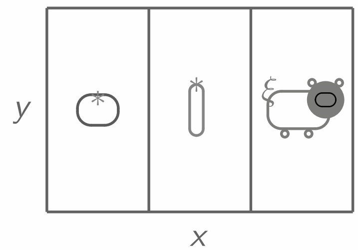
Solution. . .

Three strategies:
(1) First derivative test:

(2) Pretend we're on a closed interval, then throw out the endpoints:

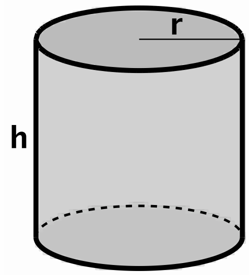
(3) Second derivative test:

Now suppose, instead, you want to divide your plot up into three equal parts:



If you still only have 100 m of fence, what is the largest area that you can fence off?

Suppose you want to make a can which holds about 16 ounces (28.875 in^3). If the material for the top and bottom of the can costs 4 ¢/in^2 and the material for the sides of the can costs 3 ¢/in^2. What is the minimum cost for the can?



Put into math:
Constraint: $V = \pi r^2 h = 28.875$.
Cost: $4 * (\text{SA of top + bottom}) + 3 * (\text{SA of side})$

Top: πr^2 Bottom: πr^2 Sides: $(2\pi r)h$

$$\boxed{\text{Total cost: } C = 4 * 2 * (\pi r^2) + 3 * ((2\pi r)h)}$$

Get into one variable: Use the constraint!

$$\pi r^2 h = 28.875 \implies h = \frac{28.875}{\pi} r^{-2} \implies C(r) = 8\pi r^2 + 6\pi r\left(\frac{28.875}{\pi} r^{-2}\right)$$

So

$$\boxed{C(r) = 8\pi r^2 + \tfrac{6*28.875}{\pi} r^{-1}}$$

(Domain: $r > 0$)

$$\boxed{\text{New problem: Minimize } C(r) = 8\pi r^2 + \tfrac{6*28.875}{\pi} r^{-1} \text{ for } r > 0.}$$

[hint: If you don't have a calculator, use the second derivative test!]