Optimization

11/4/2011

Warm up

Sketch the graph of

$$f(x) = (x-3)(x-2)(x-1) = x^3 - 6x^2 + 11x - 6$$

over the interval $[1, 4]$. Mark any critical points and inflection points. What is the absolute maximum over this interval? What is the absolute minimum over this interval?
[useful value: $\sqrt{3}/3 \approx .6$]

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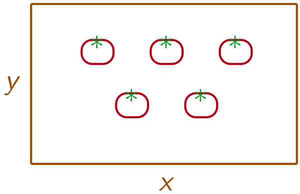

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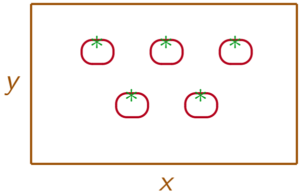

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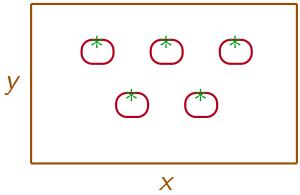

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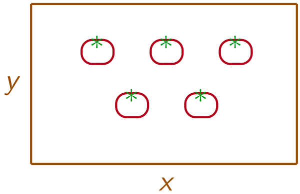

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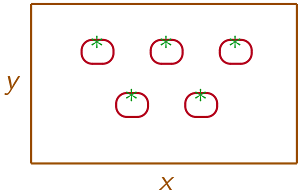

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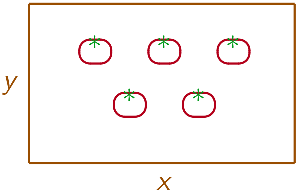

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$\boxed{\text{Domain: } 0 \le x \le 50}$

New problem: Maximize $A(x) = 50x - x^2$ over the interval $0 < x < 50$.

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Solution...

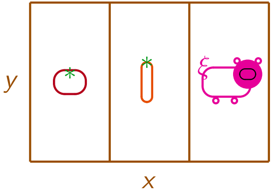
Three strategies:

(1) First derivative test:

(2) Pretend we're on a closed interval, then throw out the endpoints:

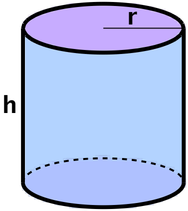
(3) Second derivative test:

Now suppose, instead, you want to divide your plot up into three equal parts:



If you still only have 100 m of fence, what is the largest area that you can fence off?

Suppose you want to make a can which holds about 16 ounces (28.875 in^3). If the material for the top and bottom of the can costs 4 ¢/in^2 and the material for the sides of the can costs 3 ¢/in^2. What is the minimum cost for the can?



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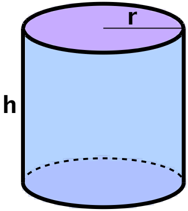

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Constraint: $V = \pi r^2 h = 28.875$.

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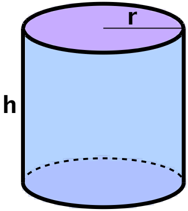

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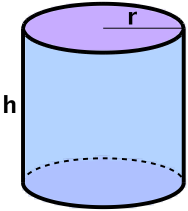

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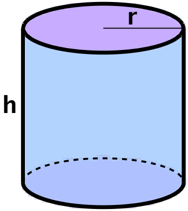

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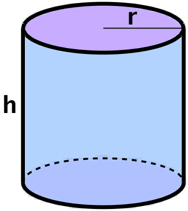

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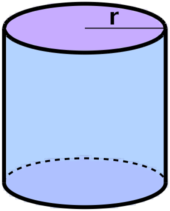

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(Domain: $r > 0$)

New problem: Minimize $C(r) = 8\pi r^2 + \frac{6*28.875}{\pi} r^{-1}$ for $r > 0$.

[hint: If you don't have a calculator, use the second derivative test!]