Related rates

10/24/2011

Example

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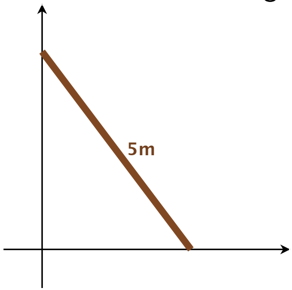

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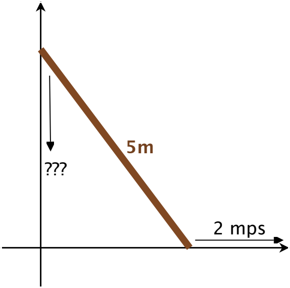

Move the base out at 2 m/s

How fast does the top move down the wall?

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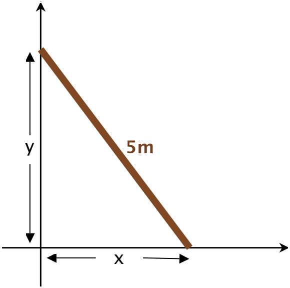

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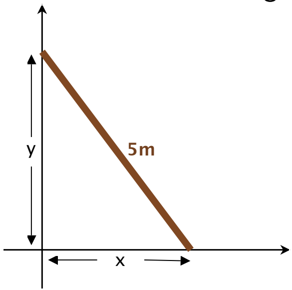

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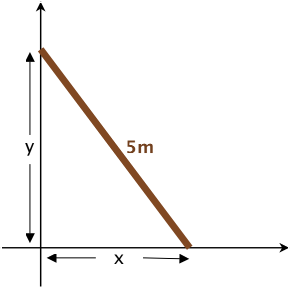

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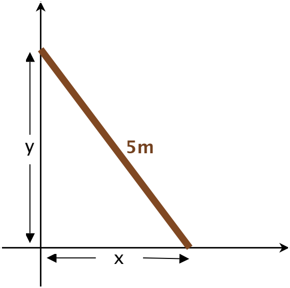

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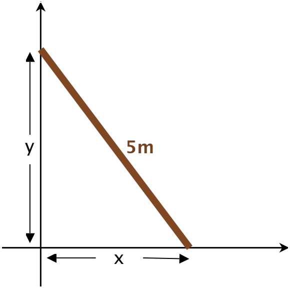

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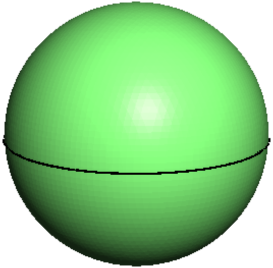
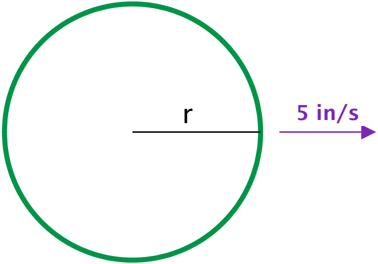
Notice:
(1) $\frac{dy}{dt} < 0$ (y is decreasing) and (2) $\lim_{x \to 5^-} \frac{dy}{dt} \to -\infty$

Example

Suppose you have a sphere whose radius is growing at a rate of 5in/s. How fast is the volume growing when the radius is 3in?





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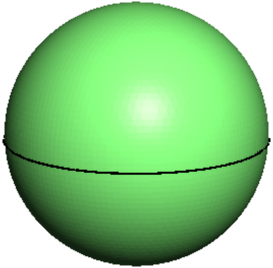

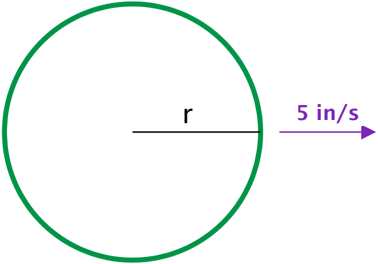

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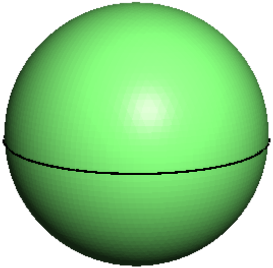

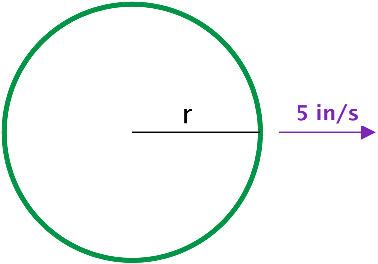

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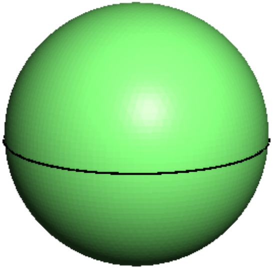

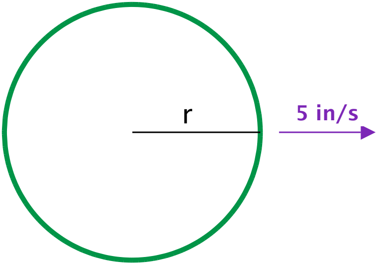

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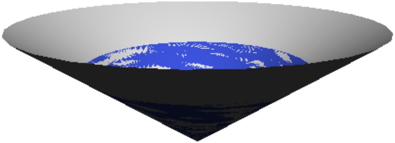
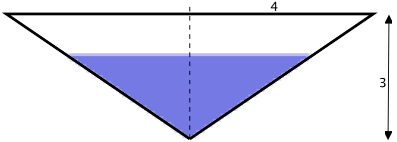
Substitute in the known values:
$\frac{dV}{dt} = 4\pi * 3^2 * 5 = \boxed{4 * 9 * 5\pi \text{ in}^3/\text{s}}$

Take an upside-down cone-shaped bowl, with a radius of 4in at the top and a total height of 3in fill it with water at a rate of 1/2 in^3/min. How fast is the height of water increasing when h=2in?





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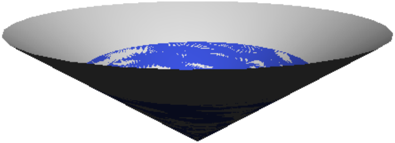

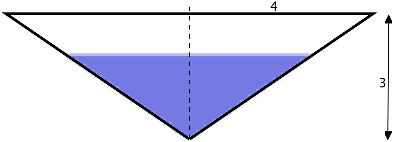

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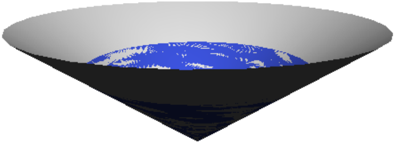

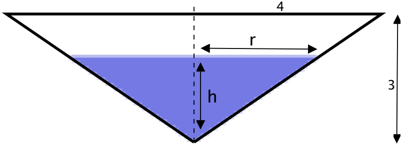

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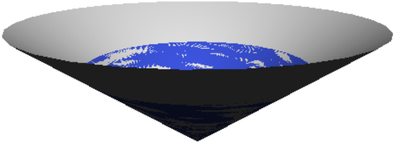

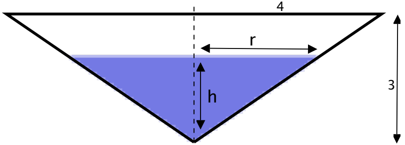

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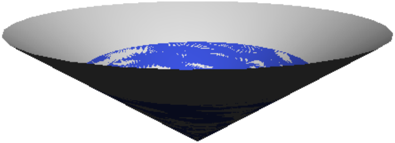

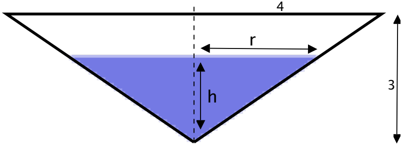

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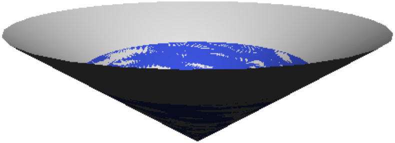

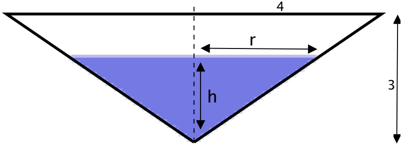

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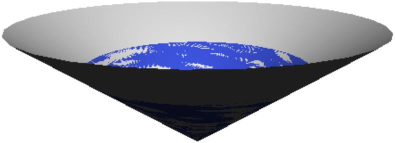

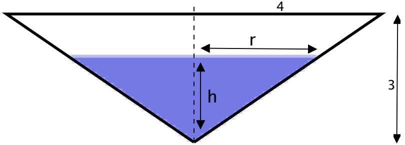

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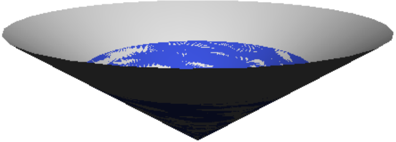

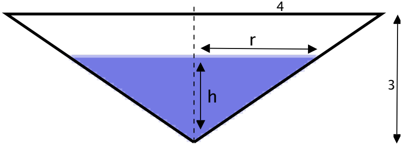

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So $\boxed{\left.\frac{dh}{dt}\right|_{h=2} = \frac{9}{128\pi}}$

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