Derivatives of Exponential and Logarithm Functions

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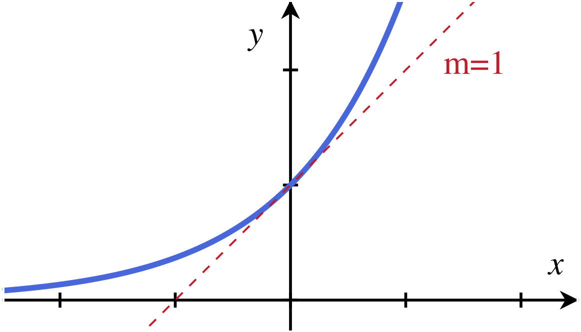
The Derivative of $y = e^x$

Recall!

e^x is the unique exponential function whose slope at $x = 0$ is 1:



$$\lim_{h \to 0} \frac{e^{0+h} - e^0}{h} = \boxed{\lim_{h \to 0} \frac{e^h - 1}{h} = 1}$$

The Derivative of $y = e^x$...

$$\boxed{\lim_{h\to 0} \frac{e^h - 1}{h} = 1}$$

$$\begin{aligned}\frac{d}{dx}e^x &= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} \\ &= \lim_{h\to 0} \frac{e^x\left(e^h - 1\right)}{h} \\ &= e^x \lim_{h\to 0} \frac{e^h - 1}{h} \\ &= e^x * 1\end{aligned}$$

So

$$\boxed{\frac{d}{dx}e^x = e^x}$$

Examples

Calculate...

1. $\frac{d}{dx} e^{17x}$

2. $\frac{d}{dx} e^{\sin x}$

3. $\frac{d}{dx} e^{\sqrt{x^2+x}}$

Notice, every time:

$$\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$$

The Derivative of $y = \ln x$

To find the derivative of $\ln(x)$, use implicit differentiation!
Remember:

$$y = \ln x \quad \Longrightarrow \quad e^y = x$$

Take a derivative of both sides of $e^y = x$ to get

$$\frac{dy}{dx} e^y = 1$$

So

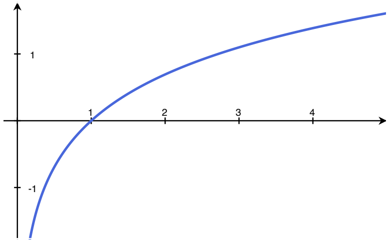
$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$$

$$\boxed{\frac{d}{dx} \ln(x) = \frac{1}{x}}$$

Does it make sense?

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$f(x) = \ln(x)$



$f(x) = \frac{1}{x}$

Examples

Calculate

1. $\frac{d}{dx} \ln x^2$

2. $\frac{d}{dx} \ln(\sin(x^2))$

Notice, every time:

$$\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$$

The Calculus Standards: e^x and $\ln x$

To get the other derivatives:

$$a^x = e^{x \ln a}$$
$$\log_a x = \frac{\ln x}{\ln a}$$

For example:

$$\frac{d}{dx} 2^x = \frac{d}{dx} e^{x \ln(2)} = \ln(2) * e^{x \ln(2)} = \ln(2) * 2^x$$

(ln(2) is a constant!!!)

You try: $\frac{d}{dx} \log_2(x)$

Differential equations

Suppose y is some mystery function of x and satisfies the equation

$$y' = ky$$

Goal: What is y??

1. If $k = 1$, then $y = e^x$ has this property and thus solves the equation.
2. For *any* k, $y = e^{kx}$ solves the equation too!

This equation, $\frac{d}{dx}y = ky$ is an example of a *differential equation*.