Derivatives

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Definition

The derivative of a function f is a new function defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

We will say that a function f is differentiable at a point $x = a$ if the derivative function f' exists at a.

Suppose we consider the piecewise defined function

$$f(x) = \begin{cases} x & x \leq 1 \\ 1 & 1 < x < 3 \\ -x + 4 & 3 \leq x \end{cases}$$

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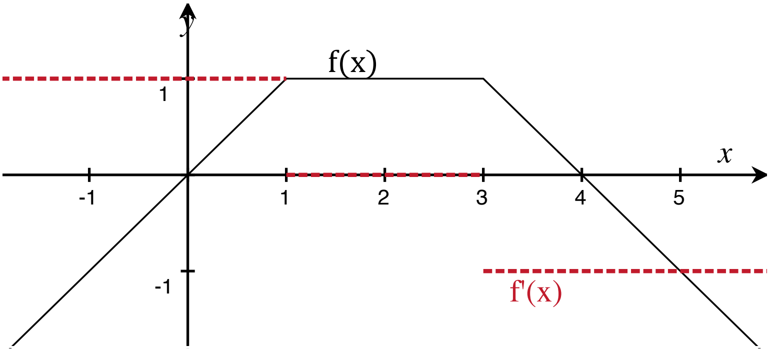
It's derivative is:

$$f(x) = \begin{cases} 1 & x < 1 \\ 0 & 1 < x < 3 \\ -1 & 3 < x \end{cases}$$



y
f(x)
1
x
-1
1
2
3
4
5
-1
f'(x)

More examples

If a and b are constants:

$$f(x) = a \qquad\qquad f(x) = ax + b$$

Derivative "rule"

$f(x)$	$f'(x)$
$x^0 = 1$	0
$x^1 = x$	1
x^2	$2x$
x^3	$3x^2$
x^{-1}	$-x^{-2}$
x^{-2}	$-2x^{-3}$
$x^{1/2}$	$(1/2)x^{-1/2}$
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If $f(x) = x^a$ then $f'(x) = ax^{a-1}$

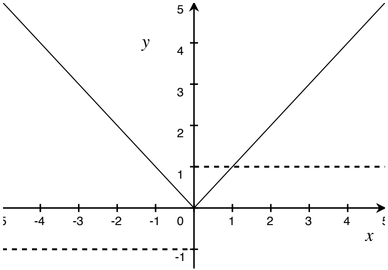
Find an equation of the tangent line to the graph of $f(x) = x^{4/3}$ at the point where $x = 1$.

$$y = f(1) + f'(1)(x - 1).$$

Find the derivative of $f(x) = |x|$.

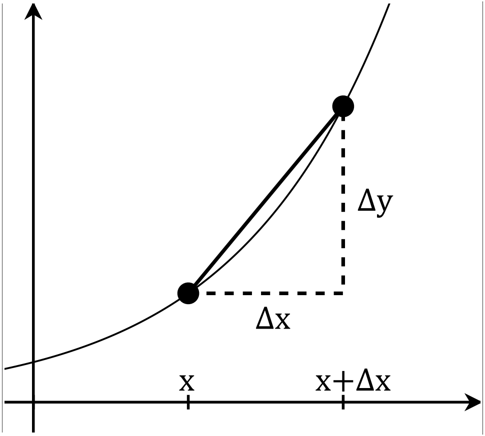
$$\lim_{h \to 0^+} \frac{|0 + h| - 0}{h} = 1$$

$$\lim_{h \to 0^-} \frac{|0 + h| - 0}{h} = -1$$



Notation for the Derivative

$$y' = D_x y = \frac{dy}{dx} = \frac{d}{dx} f(x) = f'(x).$$



Example

For the function $y = f(x) = 1/x$, find the slope of its tangent line at $x = 2$. Compare it with the average rate of change over the interval $[2, 3]$.

Higher Order Derivatives

When we differentiate a function $f(x)$ we obtain a new function $f'(x)$.
The derivative is again a candidate for differentiation, and we call its derivative *the second derivative* of $f(x)$.
So long as the derivatives exist we can continue this process to obtain a succession of higher derivatives.

Higher Order Derivatives ...

$$y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\frac{d}{dx}f(x) = \frac{d^2}{dx^2}f(x) = D_x^2 y = D_x^2 f(x).$$

The *n*th derivative, where n is a positive integer

$$y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n} = \frac{d^n}{dx^n} f(x) = D_x^n y = D_x^n f(x).$$