Trigonometric function identities

Our Favorite (capitol 'F') trig identities are

1. (*symmetries*)
$$\sin(-\theta) = -\sin(\theta), \qquad \text{and} \qquad \cos(-\theta) = \cos(\theta)$$
2. (*pythagorian identity*)
$$\sin^2(\theta) + \cos^2(\theta) = 1$$
3. (*angle addition formulas*)
$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta), \qquad \text{and} \qquad \cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)$$

Using what we know about the relation between points on the unit circle and the functions $\sin(\theta)$ and $\cos(\theta)$, explain/prove the first two identities. Draw pictures.

Trigonometric function identities

For the following problems, use the three basic identities (symmetries, pythagorean, angle addition) to prove the given equalities.

1. $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$.

2. $\sin(x/2) = \pm\sqrt{\dfrac{1-\cos x}{2}}$.

3. $\cos 3x = \cos^3 x - 3\cos x \sin^2 x$.

4. $\sin 3x = 3\cos^2 x \sin x - \sin^3 x$.

5. $\sin^2 A \cot^2 A = (1 - \sin A)(1 + \sin A)$.

6. $\tan B = \dfrac{\cos B}{\sin B \cot^2 B}$.

7. $\dfrac{\tan V \cos V}{\sin V} = 1$.

8. $\sin E \cot E + \cos E \tan E = \sin E + \cos E$.

9. $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} - 1 = 0$.

10. $\dfrac{\sec A - 1}{\sec A + 1} + \dfrac{\cos A - 1}{\cos A + 1} = 0$.

11. $\sin V(1 + \cot^2 V) = \csc V$.

12. $\dfrac{\sin(\pi/2 - w)}{\cos(\pi/2 - w)} = \cot w$.

13. $\sec(\pi/2 - z) = \dfrac{1}{\sin z}$.

14. $1 + \tan^2(\pi/2 - x) = \dfrac{1}{\cos^2(\pi/2 - x)}$.

15. $\dfrac{\sin A}{\csc A} + \dfrac{\cos A}{\sec A} = 1$.

16. $\dfrac{\sec B}{\cos B} - \dfrac{\tan B}{\cot B} = 1$.

17. $\dfrac{1}{\csc^2 w} + \sec^2 w + \dfrac{1}{\sec^2 w} = 2 + \dfrac{\sec^2 w}{\csc^2 w}$.

18. $\sec^4 V - \sec^2 V = \dfrac{1}{\cot^4 V} + \dfrac{1}{\cot^2 V}$.

19. $\sin^4 x + \cos^2 x = \cos^4 x + \sin^2 x$.

20. $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$.

21. $\cot(\alpha/2) = \dfrac{\sin\alpha}{1 - \cos\alpha}$.

22. $\cos(\pi/6 - x) + \cos(\pi/6 + x) = \sqrt{3}\cos x$.

23. $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$.

24. $\sin(\pi/3 - x) + \sin(\pi/3 + x) = \sqrt{3}\cos x$.

25. $\cos(\pi/4 - x) - \cos(\pi/4 + x) = \sqrt{2}\sin x$.

26. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$.

27. $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$.

More fun with trigonometric function identities

For the following problems, use the three identities above to prove the given equalities.

1. $\cos 2\theta = 2\sin(\pi/4+\theta)\sin(\pi/4-\theta)$.

2. $(1/2)\sin 2A = \dfrac{\tan A}{1+\tan^2 A}$.

3. $\cot(x/2) = \dfrac{1+\cos x}{\sin x}$.

4. $\sin 2B(\cot B + \tan B) = 2$.

5. $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$.

6. $1+\cos 2A = \dfrac{2}{1+\tan^2 A}$.

7. $\tan 2x \tan x + 2 = \dfrac{\tan 2x}{\tan x}$.

8. $\csc A \sec A = 2\csc 2A$.

9. $\cot x = \dfrac{\sin 2x}{1-\cos 2x}$.

10. $1-\sin A = \left(\sin\dfrac{A}{2} - \cos\dfrac{A}{2}\right)^2$.

11. $\cos^4 A = \dfrac{2\cos 2A + \cos^2 2A + 1}{4}$.

12. $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \dfrac{\tan\left(\frac{A+B}{2}\right)}{\tan\left(\frac{A-B}{2}\right)}$.

13. $\dfrac{\sin\alpha + \sin 3\alpha}{\cos\alpha + \cos 3\alpha} = \tan 2\alpha$.

14. $\dfrac{\cos 2A}{1+\sin 2A} = \dfrac{\cot A - 1}{\cot A + 1}$.

15. $\dfrac{\cos A + \sin A}{\cos A - \sin A} = \dfrac{1+\sin 2A}{\cos 2A}$.

16. $\cot\alpha - \cot\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$.

17. $\tan\theta\csc\theta\cos\theta = 1$.

18. $\cos^2\theta = \dfrac{\cot^2\theta}{1+\cot^2\theta}$.

19. $\dfrac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$.

20. $(\tan A - \cot A)^2 + 4 = \sec^2 A + \csc^2 A$.

21. $\cos B\cos(A+B) + \sin B\sin(A+B) = \cos A$.

22. $\dfrac{\tan A - \sin A}{\sec A} = \dfrac{\sin^3 A}{1+\cos A}$.

23. $\dfrac{2\tan^2 A}{1+\tan^2 A} = 1-\cos 2A$.

24. $\tan 2A = \tan A + \dfrac{\tan A}{\cos 2A}$.

25. $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$.

26. $\dfrac{4\sin A}{1-\sin^2 A} = \dfrac{1+\sin A}{1-\sin A} - \dfrac{1-\sin A}{1+\sin A}$.

27. $\tan A + \sin A = \dfrac{\csc A + \cot A}{\csc A\cot A}$.