Extra limits with trigonometric functions

1. Show $\lim_{x\to 0} \dfrac{\sin 3x}{4x} = 3/4.$

2. Show $\lim_{x\to 0} \dfrac{\sin x \cos x}{3x} = 1/3.$

3. Show $\lim_{x\to 0} \dfrac{\tan x}{x} = 1.$

4. Show $\lim_{x\to 0} \dfrac{1-\cos x}{\sin^2 x} = 1/2.$

5. Show $\lim_{x\to 0} \dfrac{\tan ax}{\tan bx} = a/b.$

6. Show $\lim_{x\to 0} \dfrac{\sin(x/4)}{x} = 1/4.$

7. Show $\lim_{x\to 0} \dfrac{\sin mx}{\tan nx} = m/n.$

8. Show $\lim_{\theta\to 0} \dfrac{1-\cos 6\theta}{\theta} = 0.$

9. Show $\lim_{x\to 0} \dfrac{1-\cos 2x}{3\tan^2 x} = 2/3.$

10. Show $\lim_{x\to 0} \dfrac{\cos^2 x}{1-\sin x} = 1.$

11. Show $\lim_{x\to 0} \dfrac{\tan 2x - x}{3x - \sin x} = 1/2.$

12. Show $\lim_{x\to a} \dfrac{\sin x - \sin a}{x - a} = \cos(a).$

13. Show $\lim_{x\to 0} \dfrac{\sin 5x - \sin 3x}{\sin x} = 2.$

14. Show $\lim_{x\to 0} \dfrac{\tan 3x - 2x}{3x - \sin^2 x} = 2.$

15. Show $\lim_{x\to 0} \dfrac{x^2 - \tan 2x}{\tan x} = 1/3.$

16. Show $\lim_{x\to \pi/4} \dfrac{1-\tan x}{x - \pi/4} = -2.$

17. Show $\lim_{x\to 0} \dfrac{\tan(x/2)}{3x} = -2.$

18. Show $\lim_{x\to 0} \dfrac{1-\cos 2x + \tan^2 x}{x \sin x} = 1/6.$

19. Show $\lim_{h\to 0} \dfrac{\sin(a+h) - \sin a}{h} = \cos(a).$

20. Show $\lim_{h\to \infty} \dfrac{\cos(\pi/h)}{h-2} = 0.$