

## Derivatives.

- (1) Find  $\frac{dy}{dx}$  when  $y = (2x + 3)(5x + 6)$ .
- (2) Find  $\frac{dy}{dx}$  when  $y = \left(x + \frac{1}{x}\right) \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ .
- (3) Find  $\frac{dy}{dx}$  when  $y = (2x - 5)^2(3x - 4)^3$ .
- (4) Find  $\frac{dy}{dx}$  when  $y = \left(ex^2 + \frac{\pi}{x^3} + x^{7/2}\right)$ .
- (5) Find  $\frac{dy}{dx}$  when  $y = \left(\frac{x - 3}{x - 4}\right)^2$ .
- (6) Find  $\frac{dy}{dx}$  when  $y = \frac{3x + 5}{4 - x^2}$ .
- (7) Find  $\frac{dy}{dx}$  when  $y = \frac{x}{\sqrt{1 - 2x}}$ .
- (8) Find  $\frac{dy}{dx}$  when  $y = \frac{1 + \sqrt{x}}{1 - \sqrt{x}}$ .
- (9) Find  $\frac{dy}{dx}$  when  $y = \frac{2(x + 1)}{x^2 + 2x - 3}$ .
- (10) Find  $\frac{dy}{dx}$  when  $y = \frac{\sqrt{a + x} - \sqrt{a - x}}{\sqrt{a + x} + \sqrt{a - x}}$ .
- (11) Find  $\frac{dy}{dx}$  when  $y = \frac{x^2 - 2}{x + 1}$ .
- (12) Find  $\frac{dy}{dx}$  when  $y = \frac{\sqrt{x}}{\sqrt{x - 3}}$ .
- (13) Find  $\frac{dy}{dx}$  when  $y = \frac{x^n + 1}{x^n - 1}$ .
- (14) Find  $\frac{dy}{dx}$  when  $y = \frac{\sqrt{1 + x^2}}{\sqrt{1 - x^2}}$ .

(15) Find  $\frac{dy}{dx}$  when  $y = \frac{2x^2 - 1}{x\sqrt{1+x^2}}$

(16) Find  $\frac{dy}{dx}$  when  $y = u^n$ .

(17) Find  $\frac{dy}{dx}$  when  $y = \sqrt{1-x^2}$ .

**Derivatives with exponentials, logs and trig functions.**

(1) Find  $\frac{dy}{dx}$  when  $y = a^{\cos x}$ .

(2) Find  $\frac{dy}{dx}$  when  $y = \ln \frac{\sin^m x}{\cos^n x}$ .

(3) Find  $\frac{dy}{dx}$  when  $y = e^{ax} \sin bx$ .

(4) Find  $\frac{dy}{dx}$  when  $y = \ln \left( \frac{1 - \cos x}{1 + \cos x} \right)$ .

(5) Find  $\frac{dy}{dx}$  when  $y = \ln \sqrt{\frac{1 - \tan x}{1 + \tan x}}$ .

(6) Find  $\frac{dy}{dx}$  when  $y = e^{ax} \cos(bx + c)$ .

(7) Find  $\frac{dy}{dx}$  when  $y = \frac{\sqrt{x + \ln \tan x}}{xe^{2x}}$ .

(8) Find  $\frac{dy}{dx}$  when  $y = \ln \frac{1 + x \sin x}{1 - x \sin x}$ .

(9) Find  $\frac{dy}{dx}$  when  $y = \ln \left( \frac{1 - \cos x}{1 + \cos x} \right)^{1/2}$ .

(10) Find  $\frac{dy}{dx}$  when  $y = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}}$ .

(11) If  $y = \ln(\sin x)$  show that  $\frac{d^3 y}{dx^3} = 2 \csc^2 x \cot x$ .

(12) If  $y = e^{ax} \cos bx$  show that  $\frac{d^2 y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$ .

(13) If  $y = a \cos(\ln x) + b \sin(\ln x)$  show that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ .

(14) If  $y = Ae^{-kt} \cos(pt + c)$  show that  $\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2y = 0$ , where  $n^2 = p^2 + k^2$ .

(15) If  $y = e^{-x} \cos x$  prove that  $\frac{d^4y}{dx^4} + 4y = 0$ .

### Derivatives at a point.

(1) Let  $y = \tan 2x - 2 \tan x + 2$ . Find  $\frac{dy}{dx}$  at  $x = \pi/4$ .

(2) Let  $y = \frac{\sin^2 x + \cos x}{1 + x^2}$ . Find  $\frac{dy}{dx}\Big|_{x=0}$  and  $\frac{dy}{dx}\Big|_{x=\pi/2}$ .

(3) Let  $y = \cot x + \sec^2 x + 5$ . Find  $\frac{dy}{dx}$  at  $x = \pi/6$ .

(4) Let  $y = \cos(\sin x^2)$ . Find  $\frac{dy}{dx}\Big|_{x=\pi/3}$ .

(5) Let  $y = (\cot \sqrt{x} + 5 \sin^2 \sqrt{x})^2$ . Find  $\frac{dy}{dx}$  at  $x = \pi^2/16$ .

(6) Let  $y = \frac{1 - \sin x}{1 + \cos x}$ . Find  $\frac{dy}{dx}$  at  $x = \pi/2$ .

(7) Let  $y = x^2 \sin x + 2x \cos x - 2x$ . Find  $\frac{dy}{dx}$  at  $x = 0$  and  $x = \pi/2$ .

(8) Let  $y = \frac{\sin x^2}{\sqrt{1 + x^2}}$ . Find  $\frac{dy}{dx}\Big|_{x=0}$  and  $\frac{dy}{dx}\Big|_{x=\sqrt{\pi/2}}$ .

(9) Let  $y = (\csc x + \sin x + \tan x)^3$ . Find  $\frac{dy}{dx}$  at  $x = \pi/4$ .

### Differential equations.

(1) If  $y = x + \tan x$  show that  $\cos^2 x \cdot \frac{d^2y}{dx^2} - 2y + 2x = 0$ .

(2) If  $y = A \cos nx + B \sin nx$  show that  $\frac{d^2y}{dx^2} + n^2y = 0$ .

(3) If  $y = 2 \sin x + 3 \cos x$  show that  $y + \frac{d^2y}{dx^2} = 0$ .

(4) If  $y = a \sin x + b \cos x$  show that  $\frac{d^2y}{dx^2} + y = 0$ .

(5) If  $y = \sin(\sin x)$  show that  $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ .

(6) If  $y = a \sin x + b \cos x$  prove that  $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$ .

(7) If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\dots}}}}$  show that  $(2y - 1)\frac{dy}{dx} = \cos x$ .

### Parametric equations.

(1) Find  $\frac{dy}{dx}$  when  $x = a \cos \theta$  and  $y = b \sin \theta$ .

(2) Find  $\frac{dy}{dx}$  when  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$ .

(3) Find  $\frac{dy}{dx}$  when  $x = a \sec^2 \theta$  and  $y = b \tan^3 \theta$ .

(4) Find  $\frac{dy}{dx}$  when  $x = b \sin^3 \phi$  and  $y = b \cos^3 \phi$ .

(5) If  $x = a(t - \sin t)$  and  $y = a(1 - \cos t)$  find  $\frac{d^2y}{dx^2}$ .

(6) If  $x = a(\theta + \sin \theta)$  and  $y = a(1 - \cos \theta)$  find  $\frac{d^2y}{dx^2}$  at  $\theta = \pi/2$ .

(7) If  $x = 2 \cos \theta - \cos 2\theta$  and  $y = 2 \sin \theta - \sin 2\theta$  find  $\frac{d^2y}{dx^2}$  at  $\theta = \pi/2$ .

(8) If  $x = \sin t$  and  $y = \sin mt$  prove that  $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$ .

### Implicit differentiation.

(1) Find  $\frac{dy}{dx}$  when  $y^2 \sin x + y \tan x + (1 + x^2) \cos x = 0$ .

(2) Find  $\frac{dy}{dx}$  when  $\sin(xy) + \frac{x}{y} = x^2 - y$ .

(3) Find  $\frac{dy}{dx}$  when  $2y^2 + \frac{y}{1+x^2} + \tan^2 x + \sin y = 0$ .

(4) Find  $\frac{dy}{dx}$  when  $\tan(x+y) + \tan(x-y) = 1$ .

(5) Find  $\frac{dy}{dx}$  when  $a \sin(xy) + b \cos(x/y) = 0$ .

(6) If  $x = \ln(\tan(y/x))$  find  $\frac{dy}{dx}$ .

**Derivatives with inverse trig functions.**

- (1) Find  $\frac{dy}{dx}$  when  $y = \sin^{-1} x^3$ .
- (2) Find  $\frac{dy}{dx}$  when  $y = \sin^{-1} x^4$ .
- (3) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \sqrt{x}$ .
- (4) Find  $\frac{dy}{dx}$  when  $y = \sin^{-1} 3x$ .
- (5) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} 5x$ .
- (6) Find  $\frac{dy}{dx}$  when  $y = \sec^{-1} x^2$ .
- (7) Find  $\frac{dy}{dx}$  when  $y = \csc^{-1} x^2$ .
- (8) Find  $\frac{dy}{dx}$  when  $y = \cos^{-1} \sqrt{x}$ .
- (9) Find  $\frac{dy}{dx}$  when  $y = \sin^{-1} 2x^2$ .
- (10) Find  $\frac{dy}{dx}$  when  $y = \csc^{-1}(\sin x)$ .
- (11) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \sqrt{x-1}$ .
- (12) Find  $\frac{dy}{dx}$  when  $y = \sin(\tan^{-1} x)$ .
- (13) Find  $\frac{dy}{dx}$  when  $y = x \cos^{-1} x$ .
- (14) Find  $\frac{dy}{dx}$  when  $y = x \sin^{-1} x$ .
- (15) Find  $\frac{dy}{dx}$  when  $y = x \tan^{-1} x$ .
- (16) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \sqrt{x} - \tan^{-1} x$ .
- (17) Find  $\frac{dy}{dx}$  when  $y = (1+x^2) \tan^{-1} x$ .
- (18) Find  $\frac{dy}{dx}$  when  $y = \tan x \cos^{-1} x$ .

- (19) Find  $\frac{dy}{dx}$  when  $y = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) + \tan^{-1} x$ .
- (20) Find  $\frac{dy}{dx}$  when  $y = (1-x^2) \cos^{-1} x$ .
- (21) Find  $\frac{dy}{dx}$  when  $y = \tan x \cdot \tan^{-1} x$ .
- (22) Find  $\frac{dy}{dx}$  when  $y = \sec^{-1} x + \csc^{-1} x$ .
- (23) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1}(a/x) \cdot \cot^{-1}(x/a)$ .
- (24) Find  $\frac{dy}{dx}$  when  $y = (\tan^{-1} 2x)^3$ .
- (25) Find  $\frac{dy}{dx}$  when  $y = \cos^{-1}(\tan x^2)$ .
- (26) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \left( \frac{2x}{1-x^2} \right)$ .
- (27) Find  $\frac{dy}{dx}$  when  $y = \sec^{-1} \left( \frac{1-x^2}{1+x^2} \right)$ .
- (28) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right)$ .
- (29) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \left( \frac{1+x^2}{1-x^2} \right)$ .
- (30) Find  $\frac{dy}{dx}$  when  $y = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$ .
- (31) Find  $\frac{dy}{dx}$  when  $y = \cot^{-1} \left( \frac{1+\cos x}{1-\cos x} \right)^{1/2}$ .
- (32) Find  $\frac{dy}{dx}$  when  $y = \cot^{-1} \left( \frac{1+\cos 3x}{1-\cos 3x} \right)^{1/2}$ .
- (33) Find  $\frac{dy}{dx}$  when  $y = \sin^{-1} \left( \frac{a+b \cos x}{b+a \cos x} \right)$ .
- (34) Find  $\frac{dy}{dx}$  when  $y = \cos^{-1} \left( \frac{1+2 \cos x}{2+\cos x} \right)$ .
- (35) Find  $\frac{dy}{dx}$  when  $y = \tan^{-1} \left( \frac{1-\cos x}{\sin x} \right)$ .
- (36) Differentiate  $\sin^{-1} \left( \frac{x^2-1}{1+x^2} \right)$  with respect to  $\cos^{-1} \left( \frac{1-x^2}{1+x^2} \right)$ .
- (37) If  $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$  prove that  $(1-x^2) \frac{dy}{dx} - xy = 1$ .