

Where is a function continuous?

In general: What does it mean for a function $f(x)$ to be continuous at $x = a$? Explain how to test if a function is continuous at $x = a$.

Specifically:

1. For which values of x is the function $f(x) = x^2 + 3x + 4$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

2. For which values of x is the function $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

3. For which values of x is the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

4. For which values of x is the function $f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, & \text{if } x \neq 0, \\ 1, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

5. Determine the value of k for which the function $f(x) = \begin{cases} \frac{\sin 2x}{5x}, & \text{if } x \neq 0, \\ k, & \text{if } x = 0, \end{cases}$ is continuous at $x = 0$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

6. For which values of x is the function $f(x) = \begin{cases} x - 1, & \text{if } 1 \leq x < 2, \\ 2x - 3, & \text{if } 2 \leq x \leq 3, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

7. For which values of x is the function $f(x) = \begin{cases} \cos x, & \text{if } x \geq 0, \\ -\cos x, & \text{if } x < 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

8. For which values of x is the function $f(x) = \begin{cases} \sin(1/x), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

9. Find the value of a for which the function $f(x) = \begin{cases} ax + 5, & \text{if } x \leq 2, \\ x - 1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
10. For which values of x is the function $f(x) = \begin{cases} 1 + x^2, & \text{if } 0 \leq x \leq 1, \\ 2 - x, & \text{if } x > 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
11. For which values of x is the function $f(x) = 2x - |x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
12. Find the value of a for which the function $f(x) = \begin{cases} 2x - 1, & \text{if } x < 2, \\ a, & \text{if } x = 2, \\ x + 1, & \text{if } x > 2, \end{cases}$ is continuous at $x = 2$. Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
13. For which values of x is the function $f(x) = \begin{cases} \frac{|x - a|}{x - a}, & \text{if } x \neq a, \\ 1, & \text{if } x = a, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
14. For which values of x is the function $f(x) = \begin{cases} \frac{x - |x|}{2}, & \text{if } x \neq 0, \\ 2, & \text{if } x = 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
15. For which values of x is the function $f(x) = \begin{cases} \sin x, & \text{if } x < 0, \\ x, & \text{if } x \geq 0, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
16. For which values of x is the function $f(x) = \begin{cases} \frac{x^n - 1}{x - 1}, & \text{if } x \neq 1, \\ n, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
17. Explain how you know that $f(x) = \sec x$ is continuous for all values of x . Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
18. For which values of x is the function $f(x) = \cos |x|$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

19. For which values of x is the function $f(x) = \lfloor x \rfloor$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
20. For which values of x is the function $f(x) = \begin{cases} x^3 - x^2 + 2x - 2, & \text{if } x \neq 1, \\ 4, & \text{if } x = 1, \end{cases}$ continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.
21. For which values of x is the function $f(x) = |x| + |x - 1|$, $-1 \leq x \leq 2$, continuous? Justify your answer with limits if necessary and draw a graph of the function to illustrate your answer.

Answers

- | | | | |
|------------------------|--------------------------|------------------------|----------------|
| 1. all x | 2. all x | 3. $x \neq 0$ | 4. $x \neq 0$ |
| 5. $k = 2/4$ | 6. $1 \leq x \leq 3$ | 7. $x \neq 0$ | 8. $x \neq 0$ |
| 9. $a = -2$ | 10. $x \geq 0, x \neq 1$ | 11. all x | 12. $a = 3$ |
| 13. $x \neq a$ | 14. $x \neq 0$ | 15. all x | 16. all x |
| 17. | 18. all x | 19. x not an integer | 20. $x \neq 1$ |
| 21. $-1 \leq x \leq 2$ | | | |