20. If $f'(x) = g'(x)$ for all x and $f(5) = g(5)$, then $f(x) = g(x)$ for all x.

21. If f is differentiable and $f(0) < f(1)$, then there is a number c, with $0 < c < 1$, such that $f'(c) > 0$.

22. The position of a particle on the x-axis is given by $s = f(t)$; its initial position and velocity are $f(0) = 3$ and $f'(0) = 4$. The acceleration is bounded by $5 \le f''(t) \le 7$ for $0 \le t \le 2$. What can we say about the position $f(2)$ of the particle at $t = 2$?

23. Suppose that g and h are continuous on $[a, b]$ and differentiable on (a, b). Prove that if $g'(x) \le h'(x)$ for $a < x < b$ and $g(b) = h(b)$, then $h(x) \le g(x)$ for $a \le x \le b$.

24. Deduce the Constant Function Theorem from the Increasing Function Theorem and the Decreasing Function Theorem. (See Problem 17.)

25. Prove that if $f'(x) = g'(x)$ for all x in (a, b), then there is a constant C such that $f(x) = g(x) + C$ on (a, b). [Hint: Apply the Constant Function Theorem to $h(x) = f(x) - g(x)$.]

26. Suppose that $f'(x) = f(x)$ for all x. Prove that $f(x) = Ce^x$ for some constant C. [Hint: Consider $f(x)/e^x$.]

27. Suppose that f is continuous on $[a, b]$ and differentiable on (a, b) and that $m \le f'(x) \le M$ on (a, b). Use the Racetrack Principle to prove that $f(x) - f(a) \le M(x - a)$ for all x in $[a, b]$, and that $m(x - a) \le f(x) - f(a)$ for all x in $[a, b]$. Conclude that $m \le (f(b) - f(a))/(b - a) \le M$. This is called the Mean Value Inequality. In words: If the instantaneous rate of change of f is between m and M on an interval, so is the average rate of change of f over the interval.

28. Suppose that $f''(x) \ge 0$ for all x in (a, b). We will show the graph of f lies above the tangent line at $(c, f(c))$ for any c with $a < c < b$.

(a) Use the Increasing Function Theorem to prove that $f'(c) \le f'(x)$ for $c \le x < b$ and that $f'(x) \le f'(c)$ for $a < x \le c$.
(b) Use (a) and the Racetrack Principle to conclude that $f(c) + f'(c)(x - c) \le f(x)$, for $a < x < b$.

CHAPTER SUMMARY (see also Ready Reference at the end of the book)

- **Derivatives of elementary functions**
  Power, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and hyperbolic functions.
- **Derivatives of sums, differences, and constant multiples**
- **Product and quotient rules**
- **Chain rule**
  Differentiation of implicitly defined functions, inverse functions.
- **Tangent line approximation, local linearity**
- **Hyperbolic functions**
- **Theorems about differentiable functions**
  Mean value theorem, increasing function theorem, constant function theorem, Racetrack Principle.

REVIEW EXERCISES AND PROBLEMS FOR CHAPTER THREE

Exercises

Find derivatives for the functions in Exercises 1–74. Assume a, b, c, and k are constants.

1. $w = (t^2 + 1)^{100}$

2. $y = e^{3w/2}$

3. $f(t) = 2te^t - \dfrac{1}{\sqrt{t}}$

4. $g(t) = \dfrac{4}{3 + \sqrt{t}}$

5. $h(t) = \dfrac{4 - t}{4 + t}$

6. $f(x) = \dfrac{x^3}{9}(3 \ln x - 1)$

7. $f(x) = \dfrac{x^2 + 3x + 2}{x + 1}$

8. $g(\theta) = e^{\sin\theta}$

9. $h(\theta) = \theta(\theta^{-1/2} - \theta^{-2})$

10. $f(\theta) = \ln(\cos\theta)$

11. $f(y) = \ln\left(\ln(2y^3)\right)$

12. $g(x) = x^k + k^x$

13. $y = e^{-\pi} + \pi^{-e}$

14. $z = \sin^3\theta$

15. $f(t) = \cos^2(3t + 5)$

16. $M(\alpha) = \tan^2(2 + 3\alpha)$

17. $s(\theta) = \sin^2(3\theta - \pi)$

18. $h(t) = \ln\left(e^{-t} - t\right)$

19. $p(\theta) = \dfrac{\sin(5 - \theta)}{\theta^2}$

20. $w(\theta) = \dfrac{\theta}{\sin^2\theta}$

21. $f(\theta) = \dfrac{1}{1 + e^{-\theta}}$

22. $g(w) = \dfrac{1}{2^w + e^w}$

23. $g(x) = \dfrac{x^2 + \sqrt{x} + 1}{x^{3/2}}$

24. $h(z) = \sqrt{\dfrac{\sin(2z)}{\cos(2z)}}$

25. $q(\theta) = \sqrt{4\theta^2 - \sin^2(2\theta)}$

26. $w = 2^{-4z}\sin(\pi z)$

27. $s(x) = \arctan(2 - x)$

28. $r(\theta) = e^{(e^\theta + e^{-\theta})}$

29. $m(n) = \sin(e^n)$

30. $k(\alpha) = e^{\tan(\sin \alpha)}$

31. $g(t) = t\cos(\sqrt{t}e^t)$

32. $f(r) = (\tan 2 + \tan r)^e$

33. $h(x) = xe^{\tan x}$

34. $y = e^{2x}\sin^2(3x)$

35. $g(x) = \tan^{-1}(3x^2+1)$

36. $y = 2^{\sin x}\cos x$

37. $h(x) = \ln e^{ax}$

38. $k(x) = \ln e^{ax} + \ln b$

39. $f(\theta) = e^{k\theta} - 1$

40. $f(t) = e^{-4kt}\sin t$

41. $H(t) = (at^2 + b)e^{-ct}$

42. $g(\theta) = \sqrt{a^2 - \sin^2\theta}$

43. $f(x) = a^{5x}$

44. $f(x) = \dfrac{a^2 - x^2}{a^2 + x^2}$

45. $w(r) = \dfrac{ar^2}{b + r^3}$

46. $f(s) = \dfrac{a^2 - s^2}{\sqrt{a^2 + s^2}}$

47. $y = \arctan\left(\dfrac{2}{x}\right)$

48. $r(t) = \ln\left(\sin\left(\dfrac{t}{k}\right)\right)$

49. $g(w) = \dfrac{5}{(a^2 - w^2)^2}$

50. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

51. $g(u) = \dfrac{e^{au}}{a^2 + b^2}$

52. $y = \dfrac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$

53. $g(t) = \dfrac{\ln(kt) + t}{\ln(kt) - t}$

54. $z = \dfrac{e^{t^2} + t}{\sin(2t)}$

55. $f(t) = \sin\sqrt{e^t + 1}$

56. $g(y) = e^{2e^{(y^3)}}$

57. $g(x) = -\frac{1}{2}(x^5 + 2x - 9)$

58. $y = -3x^4 - 4x^3 - 6x + 2$

59. $g(z) = \dfrac{z^7 + 5z^6 - z^3}{z^2}$

60. $f(z) = (\ln 3)z^2 + (\ln 4)e^z$

61. $g(x) = 2x - \dfrac{1}{\sqrt[3]{x}} + 3^x - e$

62. $f(x) = (3x^2 + \pi)(e^x - 4)$

63. $f(\theta) = \theta^2\sin\theta + 2\theta\cos\theta - 2\sin\theta$

64. $y = \sqrt{\cos(5\theta)} + \sin^2(6\theta)$

65. $r(\theta) = \sin\left((3\theta - \pi)^2\right)$

66. $y = \left(x^2 + 5\right)^3\left(3x^3 - 2\right)^2$

67. $N(\theta) = \tan(\arctan(k\theta))$

68. $h(t) = e^{kt}(\sin at + \cos bt)$

69. $f(x) = (2 - 4x - 3x^2)(6x^e - 3\pi)$

70. $f(t) = (\sin(2t) - \cos(3t))^4$

71. $s(y) = \sqrt[3]{(\cos^2 y + 3 + \sin^2 y)}$

72. $f(x) = (4 - x^2 + 2x^3)(6 - 4x + x^7)$

73. $h(x) = \left(\dfrac{1}{x} - \dfrac{1}{x^2}\right)\left(2x^3 + 4\right)$

74. $f(z) = \sqrt{5z} + 5\sqrt{z} + \dfrac{5}{\sqrt{z}} - \sqrt{\dfrac{5}{z}} + \sqrt{5}$

For Exercises 75–76, assume that y is a differentiable function of x and find dy/dx.

75. $x^3 + y^3 - 4x^2y = 0$

76. $\sin(ay) + \cos(bx) = xy$

77. Find the slope of the curve $x^2 + 3y^2 = 7$ at $(2, -1)$.

78. Assume y is a differentiable function of x and that $y + \sin y + x^2 = 9$. Find dy/dx at the point $x = 3, y = 0$.

79. Find the equations for the lines tangent to the graph of $xy + y^2 = 4$ where $x = 3$.

Problems

80. If $f(t) = 2t^3 - 4t^2 + 3t - 1$, find $f'(t)$ and $f''(t)$.

81. If $f(x) = 13 - 8x + \sqrt{2}x^2$ and $f'(r) = 4$, find r.

82. If $f(x) = 4x^3 + 6x^2 - 23x + 7$, find the intervals on which $f'(x) \geq 1$.

83. If $f(x) = (3x + 8)(2x - 5)$, find $f'(x)$ and $f''(x)$.

For Problems 84–89, use Figure 3.47.

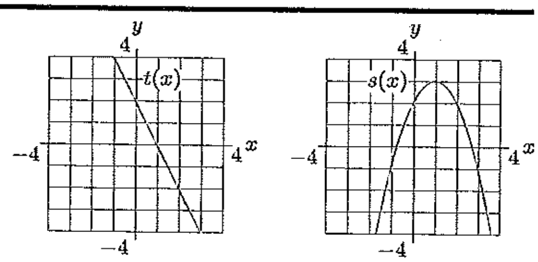

Figure 3.47

84. Let $h(x) = t(x)s(x)$ and $p(x) = t(x)/s(x)$. Estimate:

(a) $h'(1)$ (b) $h'(0)$ (c) $p'(0)$

85. Let $r(x) = s(t(x))$. Estimate $r'(0)$.

(b) Overestimate
(c) $|\text{Error}| < 0.3$.

15 $a = 1$; $f(a) = 1$
Underestimate
$f(1.2) \approx 1.4$

17 0.1

19 $331.3 + 0.606T$ m/sec

21 (b) 1% increase

23 $f(1 + \Delta x) \geq f(1) + f'(1)\Delta x$

25 (a) 16,398 m
(b) $16{,}398 + 682(\theta - 20)$ m
(c) True: 17,070 m
Approx: 17,080 m

27 (a) 1492 m
(b) $1492 + 143(\theta - 20)$ m
(c) True: 1638 m
Approx: 1635 m

29 $E(x) = \cos x - 1$
$k = -1/2$; $f''(0) = -1$
$E(x) \approx -(1/2)x^2$

31 $E(x) = \sqrt{x} - (1 + (1/2)(x - 1))$
$k = -1/8$; $f''(1) = -1/4$
$E(x) \approx -(1/8)(x - 1)^2$

33 Method 1:
$e^{2x} = (e^x)^2 \approx (1 + x)^2 = 1 + 2x + x^2$
Method 2:
$e^{2x} \approx 1 + 2x$

35 $e^x \sin x \approx x$
$\frac{d}{dx}(e^x \sin x)|_{x=0} = 1$
$e^x \sin x/(1 + x) \approx x - x^3$
$\frac{d}{dx}(\frac{e^x \sin x}{1+x})|_{x=0} = 1$

39 $0.9 < x < 1.1$

Section 3.10

1 False

3 True

5 False

7 Yes; yes

9 No; yes

11 $f'(c) = -0.5$, $f'(x_1) > -0.5$, $f'(x_2) < -0.5$

13 6 distinct zeros

19 Racetrack

21 Mean Value

Chapter 3 Review

1 $200t(t^2 + 1)^{99}$

3 $2e^t + 2te^t + 1/(2t^{3/2})$

5 $-8/(4 + t)^2$

7 $1, x \neq -1$

9 $(1/2)\theta^{-1/2} + \theta^{-2}$

11 $3/(y \ln(2y^3))$

13 0

15 $-6\cos(3t + 5) \cdot \sin(3t + 5)$

17 $6\cos(3\theta - \pi)\sin(3\theta - \pi)$

19 $-\theta^{-3}(\theta\cos(5 - \theta) + 2\sin(5 - \theta))$

21 $e^{-\theta}/(1 + e^{-\theta})^2$

23 $\frac{1}{2}x^{-1/2} - x^{-2} - \frac{3}{2}x^{-5/2}$

25 $\frac{(4\theta - 2\sin(2\theta)\cos(2\theta))}{\sqrt{4\theta^2 - \sin^2(2\theta)}}$

27 $-1/(1 + (2 - x)^2)$

29 $e^{\eta}\cos(e^{\eta})$

31 $\cos(\sqrt{t}e^t) - t\sin(\sqrt{t}e^t) \cdot (\sqrt{t}e^t + e^t/(2\sqrt{t}))$

33 $e^{\tan x} + xe^{\tan x}/\cos^2 x$

35 $6x/\left(9x^4 + 6x^2 + 2\right)$

37 a

39 $ke^{k\theta}$

41 $(-cat^2 + 2at - bc)e^{-ct}$

43 $5\ln(a)a^{5x}$

45 $(2abr - ar^4)/(b + r^3)^2$

47 $-2/(x^2 + 4)$

49 $20w/(a^2 - w^2)^3$

51 $ae^{au}/(a^2 + b^2)$

53 $(2\ln(kt) - 2)/(\ln(kt) - t)^2$

55 $e^t\cos\sqrt{e^t + 1}/(2\sqrt{e^t + 1})$

57 $-(5x^4 + 2)/2$

59 $5z^4 + 20z^3 - 1$

61 $2 + 1/(3x^{4/3}) + 3^x\ln 3$

63 $\theta^2\cos\theta$

65 $6(3\theta - \pi)\cos[(3\theta - \pi)^2]$

67 k

69 $(-4 - 6x)(6x^e - 3\pi) + (2 - 4x - 3x^2)(6ex^{e-1})$

71 0

73 $4x - 2 - 4x^{-2} + 8x^{-3}$

75 $(8xy - 3x^2)/(3y^2 - 4x^2)$

77 2/3

79 $(y - 1) = -(1/5)(x - 3)$, $(y + 4) = -(4/5)(x - 3)$

81 $r = 3\sqrt{2}$

83 $f'(x) = 12x + 1$ and $f''(x) = 12$

85 4

87 $x \approx 1, -0.4$, or 2.4

89 $y = -4x + 6$

91 1.6

93 Proportional to r^2

95 (a) $H'(2) = 11$
(b) $H'(2) = -1/4$
(c) $H'(2) = r'(1) \cdot 3$
(we don't know $r'(1)$)
(d) $H'(2) = -3$

97 (a) $y = 20x - 48$
(b) $y = 11x/9 - 16/9$
(c) $y = -4x + 20$
(d) $y = -24x + 57$
(e) $y = 8.06x - 15.84$
(f) $y = -0.94x + 6.27$

99 1.909 radians (109.4°) or 1.231 radians (70.5°)

101 Not perpendicular; $x \approx 1.3$

103 0

105 1

107 (a) $2, 1, -3$
(b) $f(2.1) \approx 0.7$, overestimate
$f(1.98) \approx 1.06$, underestimate
Second better

109 (a) $dg/dr = -2GM/r^3$
(b) dg/dr is rate of change of acceleration due to pull of gravity
The further away from the earth's center, the weaker gravity's pull
(c) -3.05×10^{-6}
(d) Reasonable because magnitude of dg/dr is so small (compared to $g = 9.8$) that for r near 6400 km, g not varying much

111 (a) Falling, 0.38 m/hr
(b) Rising, 3.76 m/hr
(c) Rising, 0.75 m/hr
(d) Falling, 1.12 m/hr

113 (a) $v = -2\pi\omega y_0\sin(2\pi\omega t)$
$a = -4\pi^2\omega^2 y_0\cos(2\pi\omega t)$
(b) Amplitudes: different
$(y_0,\ 2\pi\omega y_0,\ 4\pi^2\omega^2 y_0)$
Periods $= 1/\omega$

115 (a) $0.40048/k$ years
(b) 33.4 years
(c) $33.4 - 2781(k - 0.012)$ years
(d) True: 40.0 years
Approx 39.0 years

117 (a) 4023 gals sold at \$2
(b) 2 gal/\$
(c) Sales drop by 1250 gal/\$1 incr
(d) -0.0008\$/gal

119 (b)

123 (a) $x(x + 1)^{x-1} + (x + 1)^x\ln(x + 1)$
$x\cos x(\sin x)^{x-1} + (\sin x)^x\ln(\sin x)$
(b) $xf'(x)\,(f(x))^{x-1} + (f(x))^x\ln(f(x))$
(c) $(\ln x)^{x-1} + (\ln x)^x\ln(\ln x)$

125 (a) 0
(b) 0
(c) $2^{-2r}4^r = 1$

Ch. 3 Understanding

1 True

3 True

5 True

7 False

9 True

11 False

13 False

15 False; $f(x) = |x|$

17 False; $\cos t + t^2$

19 False; $f(x) = 6, g(x) = 10$

21 False; $f(x) = 5x + 7, g(x) = x + 2$

23 False; $f(x) = x^2, g(x) = x^2 - 1$

25 False; $f(x) = e^{-x}, g(x) = x^2$

27 (a) Not a counterexample
(b) Not a counterexample
(c) Not a counterexample
(d) Counterexample

29 False

31 False

33 Possible answer
$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 2 \\ 19 & \text{if } x = 2 \end{cases}$$

Section 4.1

1 Critical points:
$x = -3$ and $x = 2$
Extrema:
$f(-3)$ local maximum
$f(2)$ local minimum

3 Critical points:
$x = 0$ and $x = \pm 2$
Extrema:
$f(0)$ local minimum
$f(-2)$ and $f(2)$ are not local extrema.

Notice that the substitution in the preceding example again converts the inside of the messiest function into something simple. In addition, since the derivative of the inside function is not waiting for us, we have to solve for x so that we can get dx entirely in terms of w and dw.

Example 14 Find $\int (x+7)\sqrt[3]{3-2x}\,dx$.

Solution Here, instead of the derivative of the inside function (which is -2), we have the factor $(x+7)$. However, substituting $w = 3-2x$ turns out to help anyway. Then $dw = -2\,dx$, so $(-1/2)\,dw = dx$. Now we must convert everything to w, including $x+7$. If $w = 3-2x$, then $2x = 3-w$, so $x = 3/2 - w/2$, and therefore we can write $x+7$ in terms of w. Thus

$$\begin{aligned}
\int (x+7)\sqrt[3]{3-2x}\,dx &= \int \left(\frac{3}{2}-\frac{w}{2}+7\right)\sqrt[3]{w}\left(-\frac{1}{2}\right)dw \\
&= -\frac{1}{2}\int\left(\frac{17}{2}-\frac{w}{2}\right)w^{1/3}\,dw \\
&= -\frac{1}{4}\int (17-w)w^{1/3}\,dw \\
&= -\frac{1}{4}\int (17w^{1/3}-w^{4/3})\,dw \\
&= -\frac{1}{4}\left(17\frac{w^{4/3}}{4/3}-\frac{w^{7/3}}{7/3}\right)+C. \\
&= -\frac{1}{4}\left(\frac{51}{4}(3-2x)^{4/3}-\frac{3}{7}(3-2x)^{7/3}\right)+C.
\end{aligned}$$

Looking back over the solution, the reason this substitution works is that it converts $\sqrt[3]{3-2x}$, the messiest part of the integrand, to $\sqrt[3]{w}$, which can be combined with the other term and then integrated.

Exercises and Problems for Section 7.1

Exercises

1. Use substitution to express each of the following integrals as a multiple of $\int_a^b (1/w)\,dw$ for some a and b. Then evaluate the integrals.

   (a) $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx$ (b) $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$

2. (a) Find the derivatives of $\sin(x^2+1)$ and $\sin(x^3+1)$.
   (b) Use your answer to part (a) to find antiderivatives of:
   (i) $x\cos(x^2+1)$ (ii) $x^2\cos(x^3+1)$
   (c) Find the general antiderivatives of:
   (i) $x\sin(x^2+1)$ (ii) $x^2\sin(x^3+1)$

Find the integrals in Exercises 3–46. Check your answers by differentiation.

3. $\int te^{t^2}\,dt$
4. $\int e^{3x}\,dx$
5. $\int e^{-x}\,dx$
6. $\int 25e^{-0.2t}\,dt$
7. $\int t\cos(t^2)\,dt$
8. $\int \sin(2x)\,dx$
9. $\int \sin(3-t)\,dt$
10. $\int xe^{-x^2}\,dx$
11. $\int (r+1)^3\,dr$
12. $\int y(y^2+5)^8\,dy$
13. $\int t^2(t^3-3)^{10}\,dt$
14. $\int x^2(1+2x^3)^2\,dx$
15. $\int x(x^2+3)^2\,dx$
16. $\int x(x^2-4)^{7/2}\,dx$
17. $\int y^2(1+y)^2\,dy$
18. $\int (2t-7)^{73}\,dt$
19. $\int \frac{dy}{y+5}$
20. $\int \frac{1}{\sqrt{4-x}}\,dx$
21. $\int (x^2+3)^2\,dx$
22. $\int x^2e^{x^3+1}\,dx$

23. $\int \sin\theta(\cos\theta + 5)^7\,d\theta$
24. $\int \sqrt{\cos 3t}\sin 3t\,dt$
25. $\int \sin^6\theta\cos\theta\,d\theta$
26. $\int \sin^3\alpha\cos\alpha\,d\alpha$
27. $\int \sin^6(5\theta)\cos(5\theta)\,d\theta$
28. $\int \tan(2x)\,dx$
29. $\int \frac{(\ln z)^2}{z}\,dz$
30. $\int \frac{e^t+1}{e^t+t}\,dt$
31. $\int \frac{y}{y^2+4}\,dy$
32. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$
33. $\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$
34. $\int \frac{1+e^x}{\sqrt{x+e^x}}\,dx$
35. $\int \frac{e^x}{2+e^x}\,dx$
36. $\int \frac{x+1}{x^2+2x+19}\,dx$
37. $\int \frac{t}{1+3t^2}\,dt$
38. $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$
39. $\int \frac{(t+1)^2}{t^2}\,dt$
40. $\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx$
41. $\int \cosh x\,dx$
42. $\int \sinh 3t\,dt$
43. $\int (\sinh z)e^{\cosh z}\,dz$
44. $\int \cosh(2w+1)\,dw$
45. $\int x\cosh x^2\,dx$
46. $\int \cosh^2 x\sinh x\,dx$

For the functions in Exercises 47–54, find the general antiderivative. Check your answers by differentiation.

47. $p(t) = \pi t^3 + 4t$
48. $f(x) = \sin 3x$
49. $f(x) = 2x\cos(x^2)$
50. $r(t) = 12t^2\cos(t^3)$
51. $f(x) = \sin(2-5x)$
52. $f(x) = e^{\sin x}\cos x$
53. $f(x) = \frac{x}{x^2+1}$
54. $f(x) = \frac{1}{3\cos^2(2x)}$

For Exercises 55–62, use the Fundamental Theorem to calculate the definite integrals.

55. $\int_0^{\pi} \cos(x+\pi)\,dx$
56. $\int_0^{1/2} \cos(\pi x)\,dx$
57. $\int_0^{\pi/2} e^{-\cos\theta}\sin\theta\,d\theta$
58. $\int_1^2 2xe^{x^2}\,dx$
59. $\int_1^8 \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx$
60. $\int_{-1}^{e-2} \frac{1}{t+2}\,dt$
61. $\int_1^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$
62. $\int_0^2 \frac{x}{(1+x^2)^2}\,dx$

For Exercises 63–68, evaluate the definite integrals. Whenever possible, use the Fundamental Theorem of Calculus, perhaps after a substitution. Otherwise, use numerical methods.

63. $\int_{-1}^3 (x^3+5x)\,dx$
64. $\int_{-1}^1 \frac{1}{1+y^2}\,dy$
65. $\int_1^3 \frac{1}{x}\,dx$
66. $\int_1^3 \frac{dt}{(t+7)^2}$
67. $\int_{-1}^2 \sqrt{x+2}\,dx$
68. $\int_1^2 \frac{\sin t}{t}\,dt$

Find the integrals in Exercises 69–76.

69. $\int y\sqrt{y+1}\,dy$
70. $\int z(z+1)^{1/3}\,dz$
71. $\int \frac{t^2+t}{\sqrt{t+1}}\,dt$
72. $\int \frac{dx}{2+2\sqrt{x}}$
73. $\int x^2\sqrt{x-2}\,dx$
74. $\int (z+2)\sqrt{1-z}\,dz$
75. $\int \frac{t}{\sqrt{t+1}}\,dt$
76. $\int \frac{3x-2}{\sqrt{2x+1}}\,dx$

Problems

In Problems 77–80, show the two integrals are equal using a substitution.

77. $\int_0^{\pi/3} 3\sin^2(3x)\,dx = \int_0^{\pi} \sin^2(y)\,dy$
78. $\int_1^2 2\ln(s^2+1)\,ds = \int_1^4 \frac{\ln(t+1)}{\sqrt{t}}\,dt$
79. $\int_1^e (\ln w)^3\,dw = \int_0^1 z^3e^z\,dz$
80. $\int_0^{\pi} (\pi-x)\cos x\,dx = \int_0^{\pi} x\cos(\pi-x)\,dx$
81. Using the substitution $w = x^2$, find a function $g(w)$ such that $\int_{\sqrt{a}}^{\sqrt{b}} dx = \int_a^b g(w)\,dw$ for all $0 < a < b$.
82. Using the substitution $w = e^x$, find a function $g(w)$ such that $\int_a^b e^{-x}dx = \int_{e^a}^{e^b} g(w)\,dw$ for all $a < b$.