Representation theory and combinatorics of diagram algebras.

Zajj Daugherty

May 15, 2016

Combinatorial representation theory

Combinatorial representation theory

Representation theory: Given an algebra A...

- What are the A-modules/representations?
- What are the simple/indecomposable A-modules/reps?
- What is the action of the center of A?
- What are their dimensions?
- How can I combine modules to make new ones, and what are they in terms of the simple modules?

Combinatorial representation theory

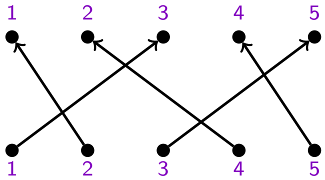
Representation theory: Given an algebra A...

- What are the A-modules/representations?
- What are the simple/indecomposable A-modules/reps?
- What is the action of the center of A?
- What are their dimensions?
- How can I combine modules to make new ones, and what are they in terms of the simple modules?

In combinatorial representation theory, we use combinatorial objects to index (construct a bijection to) modules and representations, and to encode information about them.

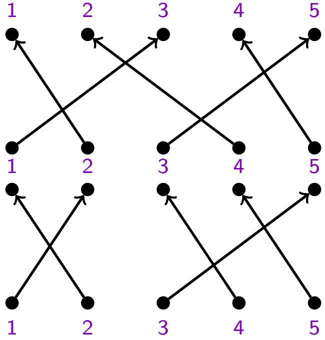
Motivating example: Schur-Weyl Duality

The **symmetric group** S_k (permutations) as diagrams:



Motivating example: Schur-Weyl Duality

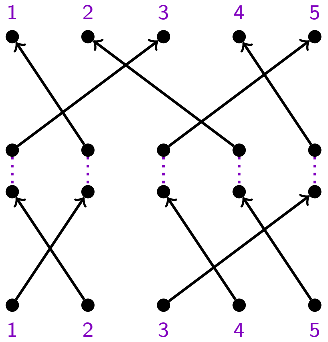
The **symmetric group** S_k (permutations) as diagrams:



(with multiplication given by concatenation)

Motivating example: Schur-Weyl Duality

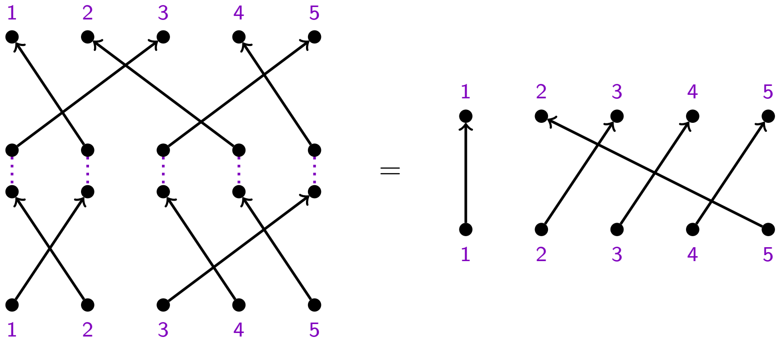
The **symmetric group** S_k (permutations) as diagrams:



(with multiplication given by concatenation)

Motivating example: Schur-Weyl Duality

The **symmetric group** S_k (permutations) as diagrams:



(with multiplication given by concatenation)

Motivating example: Schur-Weyl Duality

Motivating example: Schur-Weyl Duality

$\mathrm{GL}_n(\mathbb{C})$ acts on $\mathbb{C}^n \otimes \mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n = (\mathbb{C}^n)^{\otimes k}$ diagonally.

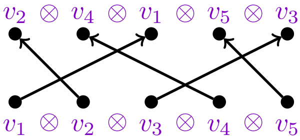
$$g \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_k) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_k.$$

Motivating example: Schur-Weyl Duality

$\mathrm{GL}_n(\mathbb{C})$ acts on $\mathbb{C}^n \otimes \mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n = (\mathbb{C}^n)^{\otimes k}$ diagonally.

$$g \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_k) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_k.$$

S_k also acts on $(\mathbb{C}^n)^{\otimes k}$ by place permutation.

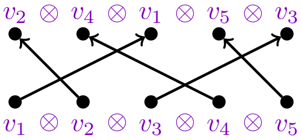


Motivating example: Schur-Weyl Duality

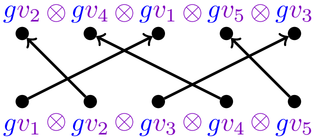
$\mathrm{GL}_n(\mathbb{C})$ acts on $\mathbb{C}^n \otimes \mathbb{C}^n \otimes \cdots \otimes \mathbb{C}^n = (\mathbb{C}^n)^{\otimes k}$ diagonally.

$$g \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_k) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_k.$$

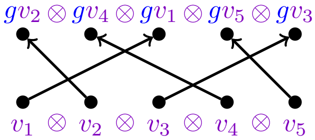
S_k also acts on $(\mathbb{C}^n)^{\otimes k}$ by place permutation.



These actions commute!



vs.



Motivating example: Schur-Weyl Duality

Schur (1901): S_k and GL_n have commuting actions on $(\mathbb{C}^n)^{\otimes k}$.

Even better,

$$\underbrace{\mathrm{End}_{\mathrm{GL}_n}\left((\mathbb{C}^n)^{\otimes k}\right)}_{\text{(all linear maps that commute with } \mathrm{GL}_n)} = \underbrace{\pi(\mathbb{C}S_k)}_{\text{(img of } S_k \text{ action)}} \quad \text{and} \quad \mathrm{End}_{S_k}\left((\mathbb{C}^n)^{\otimes k}\right) = \underbrace{\rho(\mathbb{C}\mathrm{GL}_n)}_{\text{(img of } \mathrm{GL}_n \text{ action)}}.$$

Motivating example: Schur-Weyl Duality

Schur (1901): S_k and GL_n have commuting actions on $(\mathbb{C}^n)^{\otimes k}$.
Even better,

$$\underbrace{\mathrm{End}_{\mathrm{GL}_n}\left((\mathbb{C}^n)^{\otimes k}\right)}_{\substack{\text{(all linear maps that}\\ \text{commute with } \mathrm{GL}_n)}} = \underbrace{\pi(\mathbb{C}S_k)}_{\substack{\text{(img of } S_k\\ \text{action)}}} \quad \text{and} \quad \mathrm{End}_{S_k}\left((\mathbb{C}^n)^{\otimes k}\right) = \underbrace{\rho(\mathbb{C}\mathrm{GL}_n)}_{\substack{\text{(img of } \mathrm{GL}_n\\ \text{action)}}}.$$

Why this is exciting:

The double-centralizer relationship produces

$$(\mathbb{C}^n)^{\otimes k} \cong \bigoplus_{\lambda \vdash k} G^\lambda \otimes S^\lambda \quad \text{as a } \mathrm{GL}_n\text{-}S_k \text{ bimodule,}$$

where G^λ are distinct irreducible GL_n-modules
S^λ are distinct irreducible S_k-modules

Motivating example: Schur-Weyl Duality

Schur (1901): S_k and GL_n have commuting actions on $(\mathbb{C}^n)^{\otimes k}$.
Even better,

$$\underbrace{\mathrm{End}_{\mathrm{GL}_n}\left((\mathbb{C}^n)^{\otimes k}\right)}_{\substack{\text{(all linear maps that}\\ \text{commute with } \mathrm{GL}_n\text{)}}} = \underbrace{\pi(\mathbb{C}S_k)}_{\substack{\text{(img of } S_k\\ \text{action)}}} \quad \text{and} \quad \mathrm{End}_{S_k}\left((\mathbb{C}^n)^{\otimes k}\right) = \underbrace{\rho(\mathbb{C}\mathrm{GL}_n)}_{\substack{\text{(img of } \mathrm{GL}_n\\ \text{action)}}}.$$

Why this is exciting:

The double-centralizer relationship produces

$$(\mathbb{C}^n)^{\otimes k} \cong \bigoplus_{\lambda \vdash k} G^\lambda \otimes S^\lambda \quad \text{as a } \mathrm{GL}_n\text{-}S_k \text{ bimodule,}$$

where G^λ are distinct irreducible GL_n-modules
S^λ are distinct irreducible S_k-modules

For example,

$$\mathbb{C}^n \otimes \mathbb{C}^n \otimes \mathbb{C}^n = \left(G^{\ydiagram{3}} \otimes S^{\ydiagram{3}}\right) \oplus \left(G^{\ydiagram{2,1}} \otimes S^{\ydiagram{2,1}}\right) \oplus \left(G^{\ydiagram{1,1,1}} \otimes S^{\ydiagram{1,1,1}}\right)$$

Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\Box)$$

Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square)$$

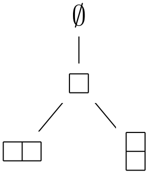
$$\emptyset$$

$$|$$

$$\square$$

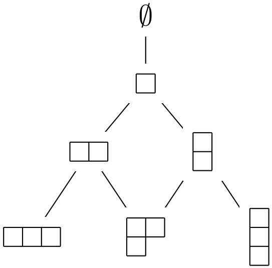
Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square)$$



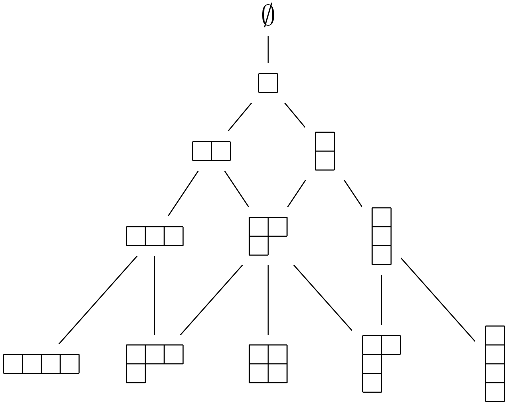
Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square)$$



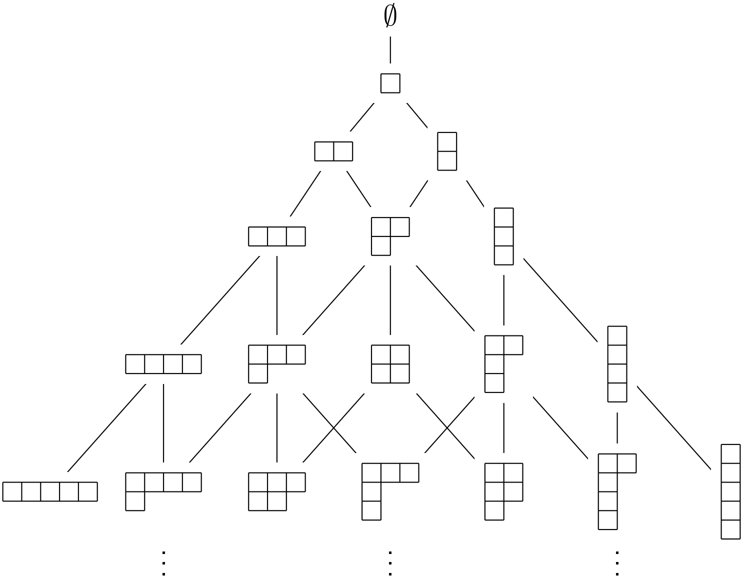
Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



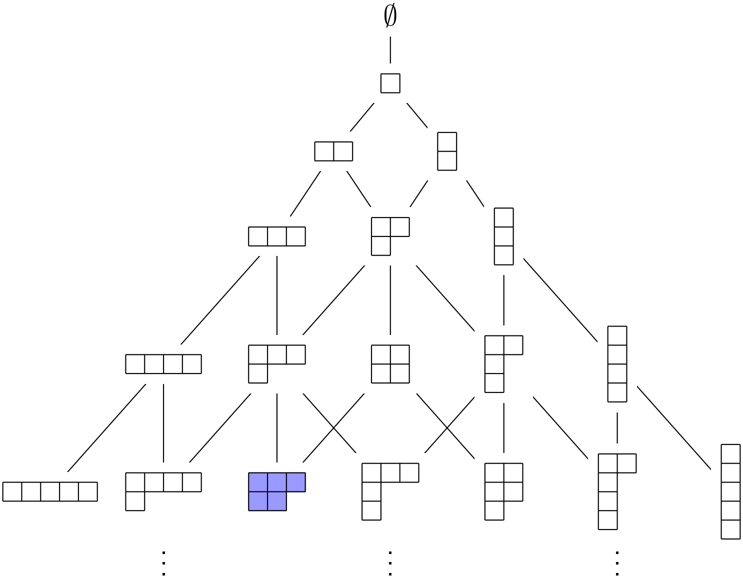
Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \cdots$$



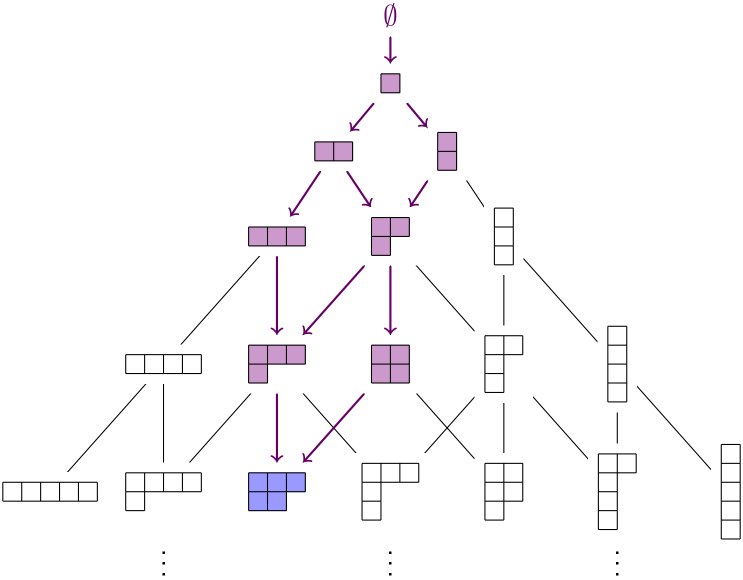
Representation theory of $V^{\otimes k}$

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \cdots$$



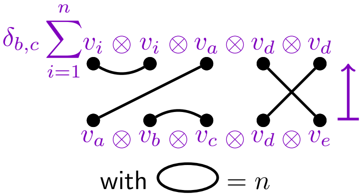
Representation theory of $V^{\otimes k}$

$V = \mathbb{C} = L(\square)$, $\qquad L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \cdots$



More centralizer algebras

Brauer (1937)
Orthogonal and symplectic groups (and Lie algebras) acting on $(\mathbb{C}^n)^{\otimes k}$ diagonally centralize the **Brauer algebra**:



More centralizer algebras

Rep theory of $V^{\otimes k}$, orthogonal and symplectic:

$$V = \mathbb{C} = L(\square)$$

More centralizer algebras

Rep theory of $V^{\otimes k}$, orthogonal and symplectic:

$V = \mathbb{C} = L(\square), \qquad L(\square)$

$$\emptyset$$

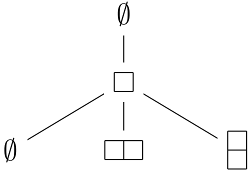
$$|$$

$$\square$$

More centralizer algebras

Rep theory of $V^{\otimes k}$, orthogonal and symplectic:

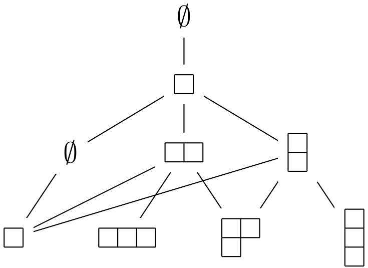
$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square)$$



More centralizer algebras

Rep theory of $V^{\otimes k}$, orthogonal and symplectic:

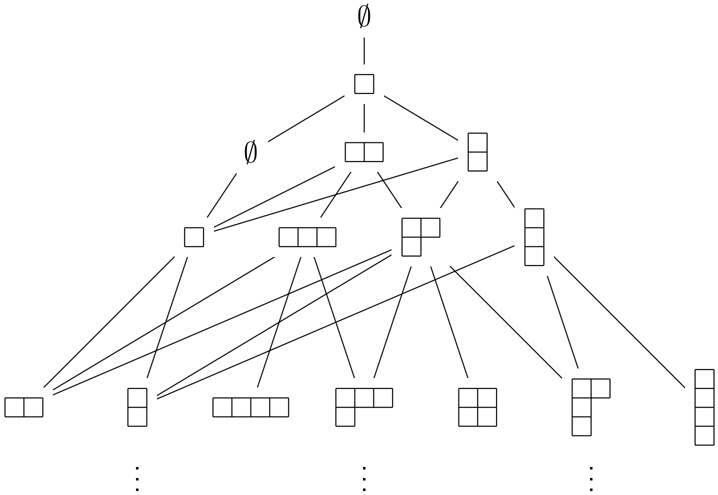
$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square)$$



More centralizer algebras

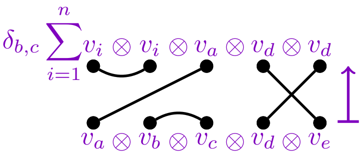
Rep theory of $V^{\otimes k}$, orthogonal and symplectic:

$$V = \mathbb{C} = L(\square), \qquad L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \cdots$$



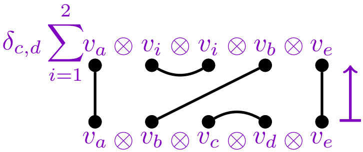
More centralizer algebras

Brauer (1937)
Orthogonal and symplectic groups (and Lie algebras) acting on $(\mathbb{C}^n)^{\otimes k}$ diagonally centralize
the **Brauer algebra**:



with  $= n$

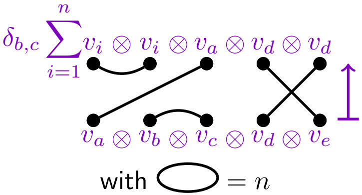
Temperley-Lieb (1971)
GL_2 and SL_2 (and $\mathfrak{gl}_2$ and $\mathfrak{sl}_2$) acting on $(\mathbb{C}^2)^{\otimes k}$ diagonally centralize
the **Temperley-Lieb algebra**:



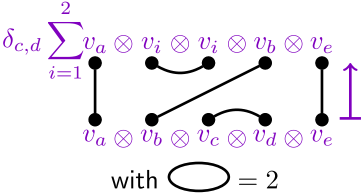
with  $= 2$

More centralizer algebras

Brauer (1937)
Orthogonal and symplectic groups (and Lie algebras) acting on $(\mathbb{C}^n)^{\otimes k}$ diagonally centralize the **Brauer algebra**:



Temperley-Lieb (1971)
GL_2 and SL_2 (and $\mathfrak{gl}_2$ and $\mathfrak{sl}_2$) acting on $(\mathbb{C}^2)^{\otimes k}$ diagonally centralize the **Temperley-Lieb algebra**:



Either way:
Diagrams encoding maps $V^{\otimes k} \to V^{\otimes k}$ that commute with the action of some classical algebra.

Quantum groups and braids

Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

Quantum groups and braids

Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

$\mathcal{U} \otimes \mathcal{U}$ has an invertible element $\mathcal{R} = \sum_{\mathcal{R}} R_1 \otimes R_2$ that yields a map

$$\check{\mathcal{R}}_{VW} \colon V \otimes W \longrightarrow W \otimes V$$

that (1) satisfies braid relations, and
(2) commutes with the action on $V \otimes W$

for any $\mathcal{U}$-module V.

Quantum groups and braids

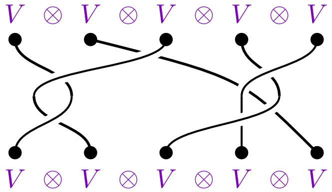
Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

$\mathcal{U} \otimes \mathcal{U}$ has an invertible element $\mathcal{R} = \sum_{\mathcal{R}} R_1 \otimes R_2$ that yields a map

$$\check{\mathcal{R}}_{VW} \colon V \otimes W \longrightarrow W \otimes V$$

that (1) satisfies braid relations, and
(2) commutes with the action on $V \otimes W$
for any $\mathcal{U}$-module V.

The braid group shares a commuting action with $\mathcal{U}$ on $V^{\otimes k}$:



Quantum groups and braids

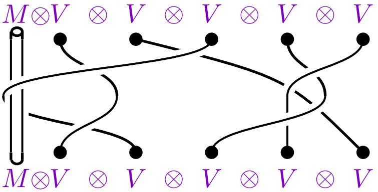
Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

$\mathcal{U} \otimes \mathcal{U}$ has an invertible element $\mathcal{R} = \sum_{\mathcal{R}} R_1 \otimes R_2$ that yields a map

$$\check{\mathcal{R}}_{VW} \colon V \otimes W \longrightarrow W \otimes V$$

that (1) satisfies braid relations, and
(2) commutes with the action on $V \otimes W$
for any $\mathcal{U}$-module V.

The one-pole/affine braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k}$:



Around the pole:

$$= \check{R}_{MV} \check{R}_{VM}$$

Quantum groups and braids

Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

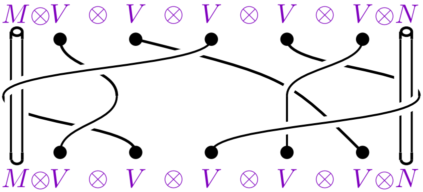
$\mathcal{U} \otimes \mathcal{U}$ has an invertible element $\mathcal{R} = \sum_{\mathcal{R}} R_1 \otimes R_2$ that yields a map

$$\check{\mathcal{R}}_{VW} : V \otimes W \longrightarrow W \otimes V$$

that (1) satisfies braid relations, and
(2) commutes with the action on $V \otimes W$

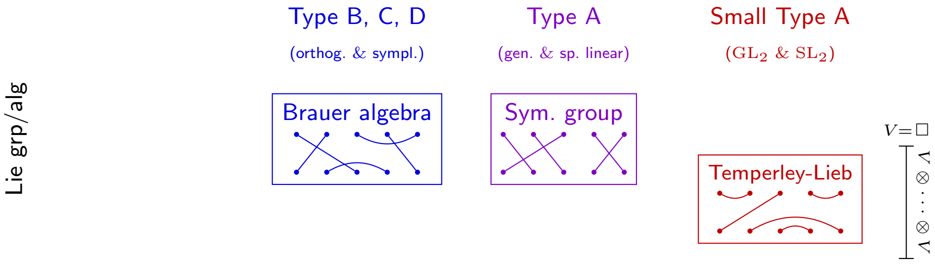
for any $\mathcal{U}$-module V.

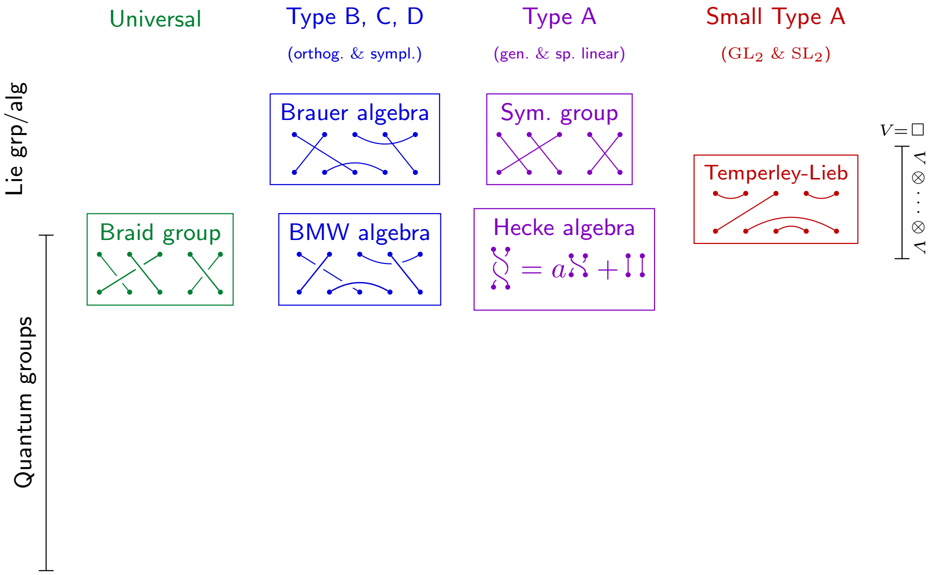
The two-pole braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k} \otimes N$:

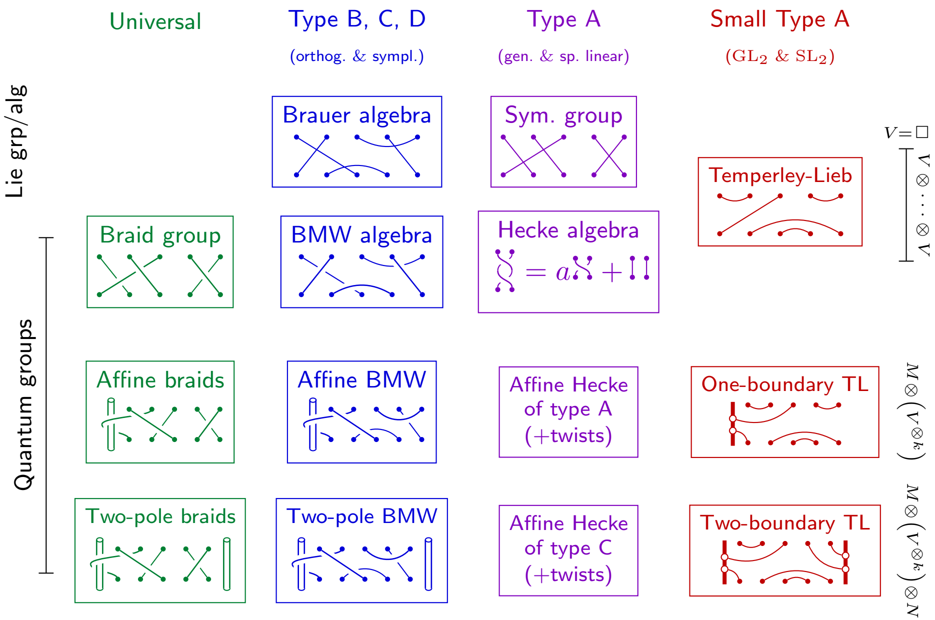


Around the pole:

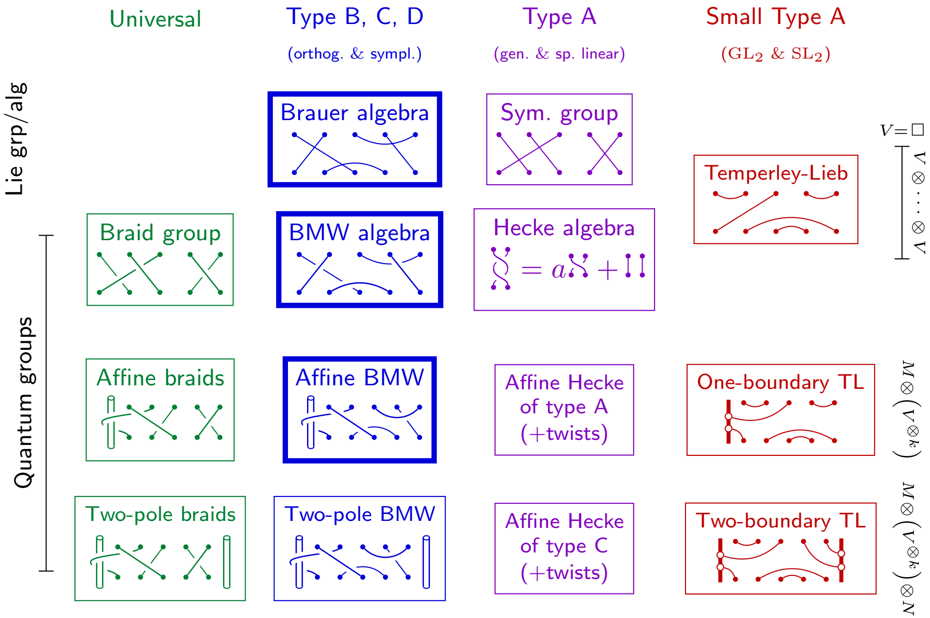
$= \check{R}_{MV}\check{R}_{VM}$


Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Brauer algebra
Sym. group
Temperley-Lieb
$V = \square$
$V \otimes \cdots \otimes V$

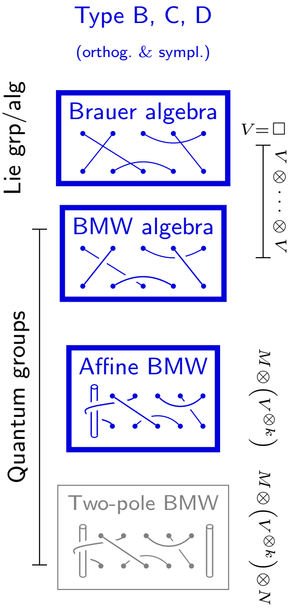

Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
Braid group
BMW algebra
Hecke algebra
$V = \square$
$V \otimes \cdots \otimes V$



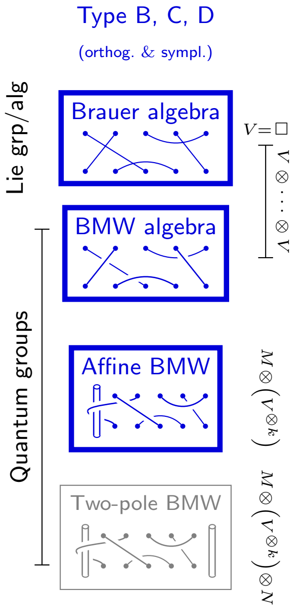
Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
$V = \square$
Temperley-Lieb
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
$= a$
$+$
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
$\left(\otimes^k V\right) \otimes M$
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$N \otimes \left(\otimes^k V\right) \otimes M$



Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
$V = \square$
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
Affine braids
Affine BMW
Affine Hecke
of type A
(+twists)
One-boundary TL
$M \otimes \left(V^{\otimes k}\right)$
Two-pole braids
Two-pole BMW
Affine Hecke
of type C
(+twists)
Two-boundary TL
$N \otimes \left(V^{\otimes k}\right) \otimes M$

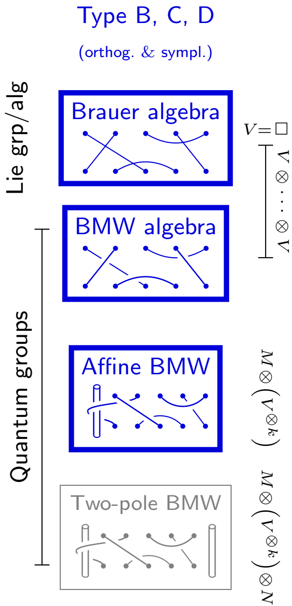


Type B, C, D
(orthog. & sympl.)
Lie grp/alg
Brauer algebra
BMW algebra
Quantum groups
Affine BMW
Two-pole BMW
$V = \square$
$V \otimes \cdots \otimes V$
$\left(V^{\otimes k}\right) \otimes M$
$N \otimes \left(V^{\otimes k}\right) \otimes M$



Nazarov (95): degenerate affine BMW algebras

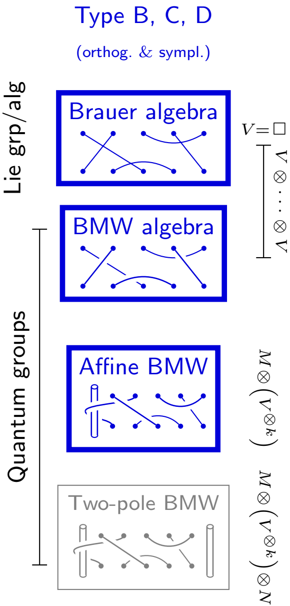
act on $M \otimes V^{\otimes k}$, commuting with the action of the Lie algebras of types B, C, D.



Nazarov (95): degenerate affine BMW algebras

act on $M \otimes V^{\otimes k}$, commuting with the action of the Lie algebras of types B, C, D.

Häring-Oldenburg (98) and Orellana-Ram (04): affine BMW algebras act on $M \otimes V^{\otimes k}$, commuting with the action of the quantum groups of types B, C, D.



Nazarov (95): degenerate affine BMW algebras

act on $M \otimes V^{\otimes k}$, commuting with the action of the Lie algebras of types B, C, D.

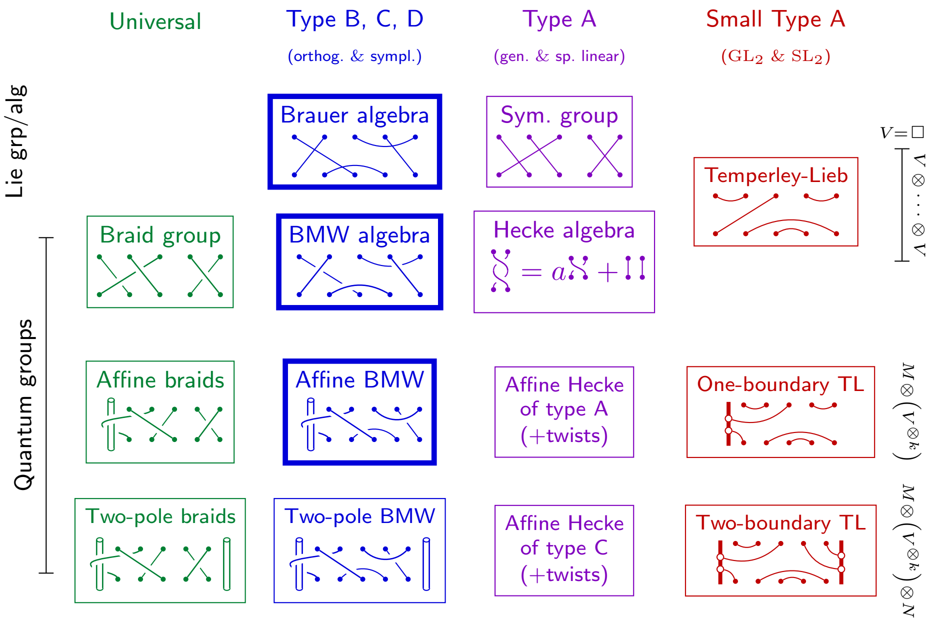
Häring-Oldenburg (98) and Orellana-Ram (04): affine BMW algebras act on $M \otimes V^{\otimes k}$, commuting with the action of the quantum groups of types B, C, D.

Centralizer perspective: (D.-Ram-Virk) Use centralizer relationships to study these the affine and degenerate affine algebras simultaneously (representation theory of the quantum groups and the Lie algebras are basically the same). Some results:

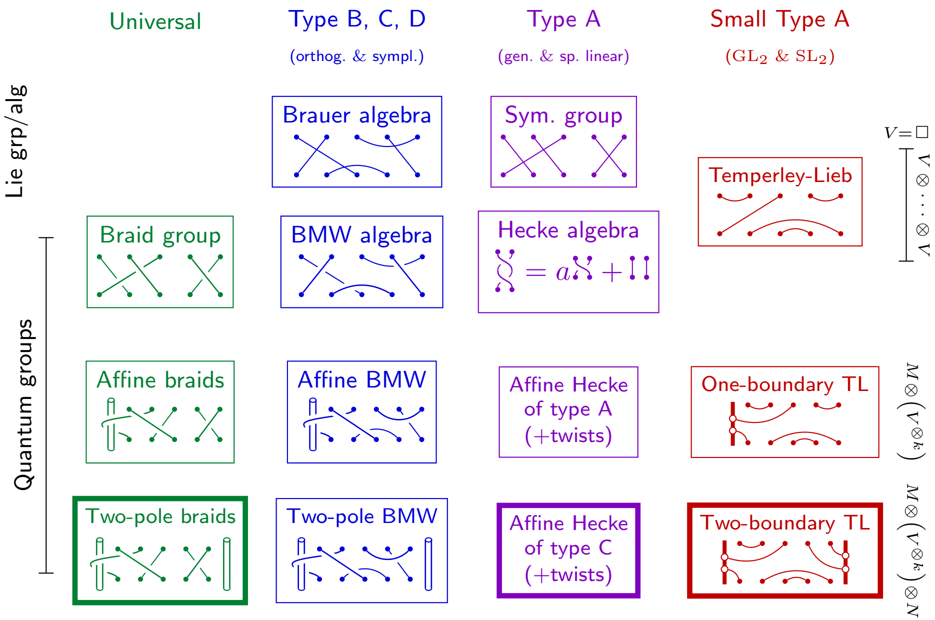
(a) The center of each algebra.

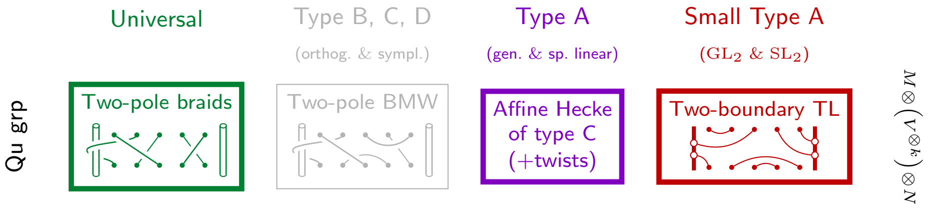
(b) Difficult "admissibility conditions" handled.

(c) Powerful "intertwiner" operators.



Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
$V=\square$
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
$= a + $
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
$M \otimes (V^{\otimes k})$
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$N \otimes (V^{\otimes k}) \otimes M$


Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
$V = \square$
Temperley-Lieb
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
$= a \; + $
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
$(V^{\otimes k}) \otimes M$
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$N \otimes (V^{\otimes k}) \otimes M$


Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Qu grp
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$M \otimes (V^{\otimes k}) \otimes N$

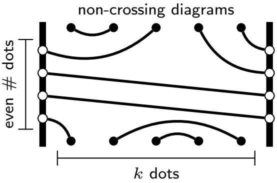
Universal | Type B, C, D (orthog. & sympl.) | Type A (gen. & sp. linear) | Small Type A (GL_2 & SL_2)

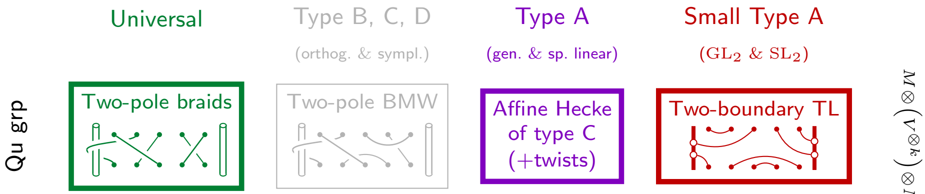
Qu grp

$N \otimes \left(V^{\otimes k}\right) \otimes M$

Two boundary algebras:

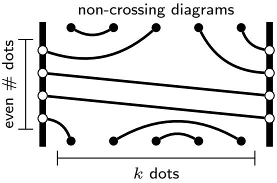
Mitra, Nienhuis, De Gier, Batchelor (2004): Studying the six-vertex model with additional integrable boundary terms, introduced the two-boundary Temperley-Lieb algebra TL_k:





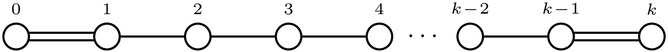
Two boundary algebras:

Mitra, Nienhuis, De Gier, Batchelor (2004): Studying the six-vertex model with additional integrable boundary terms, introduced the two-boundary Temperley-Lieb algebra TL_k:



De Gier, Nichols (2008): Explored representation theory of TL_k using diagrams and established a connection to the affine Hecke algebras of type A and C.

Affine type C Hecke algebra and two-boundary braids

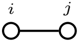


Fix constants t_0, t_k, and $t = t_1 = \cdots = t_{k-1}$. The affine Hecke algebra of type C, $\mathcal{H}_k$, is generated by $T_0, T_1, \ldots, T_k$ with relations

$$\underbrace{T_i T_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{T_j T_i \ldots}_{m_{i,j} \text{ factors}}$$

where $m_{i,j} =$

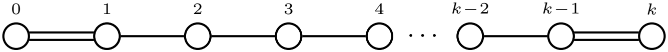
2 if 

3 if 

4 if 

and $T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1$.

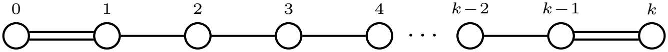
Affine type C Hecke algebra and two-boundary braids



Fix constants t_0, t_k, and $t = t_1 = \cdots = t_{k-1}$. The affine Hecke algebra of type C, $\mathcal{H}_k$, is generated by $T_0, T_1, \ldots, T_k$ with relations

$$\underbrace{T_i T_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{T_j T_i \ldots}_{m_{i,j} \text{ factors}} \qquad \text{and} \qquad T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1.$$

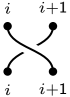
Affine type C Hecke algebra and two-boundary braids



Fix constants t_0, t_k, and $t = t_1 = \cdots = t_{k-1}$. The affine Hecke algebra of type C, $\mathcal{H}_k$, is generated by $T_0, T_1, \ldots, T_k$ with relations

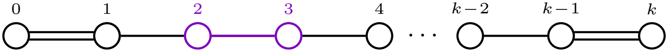
$$\underbrace{T_iT_j\ldots}_{m_{i,j}\text{ factors}} = \underbrace{T_jT_i\ldots}_{m_{i,j}\text{ factors}} \qquad \text{and} \qquad T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1.$$

The two-boundary (two-pole) braid group B_k is generated by

$T_k =$  $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$.

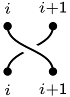
Affine type C Hecke algebra and two-boundary braids



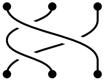
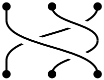
Fix constants t_0, t_k, and $t = t_1 = \cdots = t_{k-1}$. The affine Hecke algebra of type C, $\mathcal{H}_k$, is generated by $T_0, T_1, \ldots, T_k$ with relations

$$\underbrace{T_iT_j\ldots}_{m_{i,j}\text{ factors}} = \underbrace{T_jT_i\ldots}_{m_{i,j}\text{ factors}} \qquad \text{and} \qquad T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1.$$

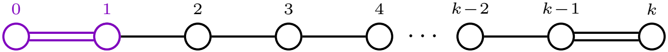
The two-boundary (two-pole) braid group B_k is generated by

$T_k =$  $T_0 =$  and $T_i =$  for $1 \le i \le k-1$.

Relations:

$T_iT_{i+1}T_i =$  $=$  $= T_{i+1}T_iT_{i+1}$

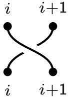
Affine type C Hecke algebra and two-boundary braids



Fix constants t_0, t_k, and $t = t_1 = \cdots = t_{k-1}$. The affine Hecke algebra of type C, $\mathcal{H}_k$, is generated by $T_0, T_1, \ldots, T_k$ with relations

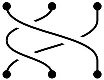
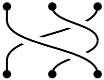
$$\underbrace{T_i T_j \ldots}_{m_{i,j}\text{ factors}} = \underbrace{T_j T_i \ldots}_{m_{i,j}\text{ factors}} \qquad \text{and} \qquad T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1.$$

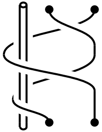
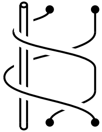
The two-boundary (two-pole) braid group B_k is generated by

$T_k =$  $T_0 =$  and $T_i =$ 

for $1 \leq i \leq k-1$.

Relations:

$T_i T_{i+1} T_i =$  $=$  $= T_{i+1} T_i T_{i+1}$

$T_1 T_0 T_1 T_0 =$  $=$  $= T_0 T_1 T_0 T_1$

Affine type C Hecke algebra and two-boundary braids

Punchline:

- For any for any complex reductive Lie algebras $\mathfrak{g}$, the quantum group $\mathcal{U}_q\mathfrak{g}$ and the two-boundary braid group B_k have commuting actions on $M \otimes (V)^{\otimes k} \otimes N$.
- When $\mathfrak{g} = \mathfrak{gl}_n$, for good choices of M, N, and V, the action of the two-boundary braid group factors to an action of the affine Hecke algebra of type C.

Affine type C Hecke algebra and two-boundary braids

Punchline:

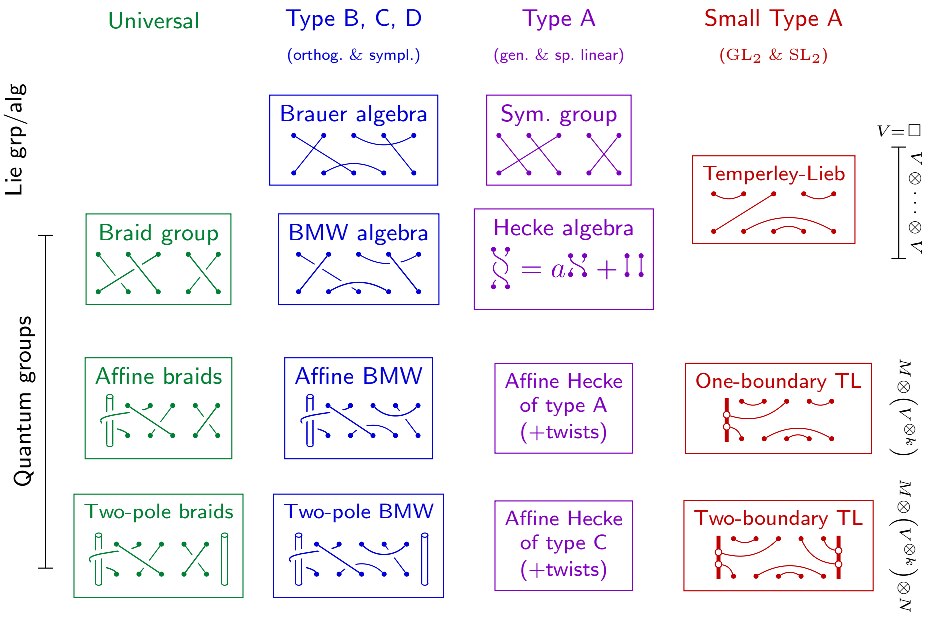
- For any for any complex reductive Lie algebras $\mathfrak{g}$, the quantum group $\mathcal{U}_q\mathfrak{g}$ and the two-boundary braid group B_k have commuting actions on $M \otimes (V)^{\otimes k} \otimes N$.
- When $\mathfrak{g} = \mathfrak{gl}_n$, for good choices of M, N, and V, the action of the two-boundary braid group factors to an action of the affine Hecke algebra of type C.

Some consequences:

(a) A combinatorial classification and construction of irreducible representations of H_k (type C with distinct parameters).

(b) A diagrammatic intuition for H_k.

(c) A classification of the representations of TL_k via the action of its center.


Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
Braid group
BMW algebra
Hecke algebra
$= a$ $+$
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$V = \square$
$V \otimes \cdots \otimes V$
$M \otimes (V^{\otimes k})$
$N \otimes (V^{\otimes k}) \otimes M$