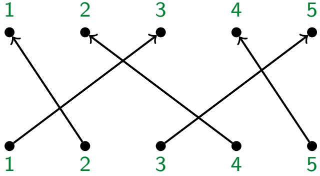
Representation theory and combinatorics of tensor power centralizer algebras

Zajj Daugherty
(Joint work in progress with Arun Ram)

December 11, 2014

Motivating example: Schur-Weyl Duality

The **symmetric group** S_k (permutations) as diagrams:



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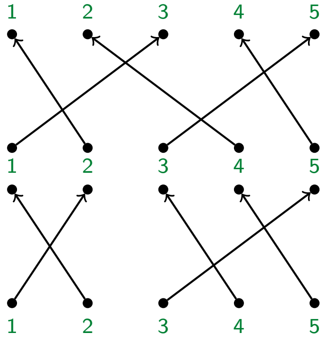

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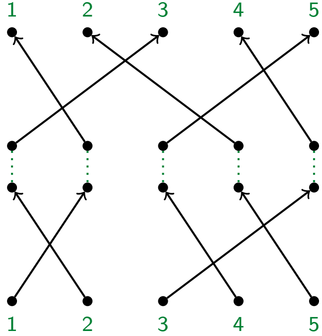

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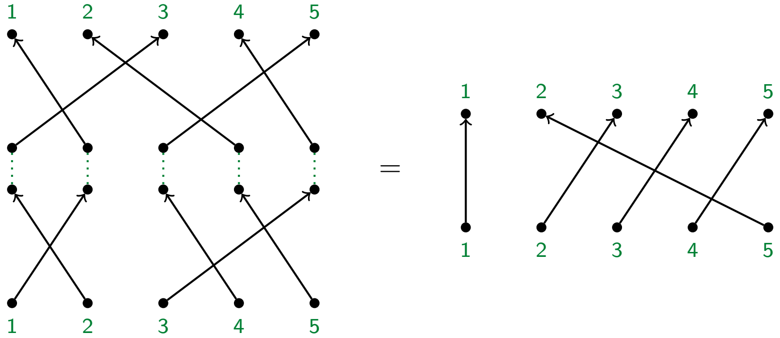

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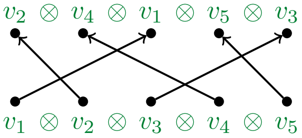

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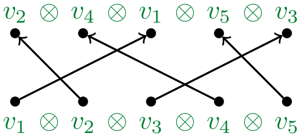
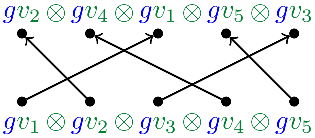
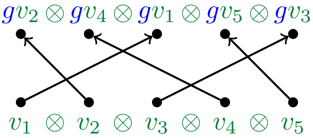


These actions commute!



vs.



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Schur (1901):

$$\underbrace{\mathrm{End}_{\mathrm{GL}_n}\left((\mathbb{C}^n)^{\otimes k}\right)}_{\substack{\text{(all linear maps that} \\ \text{commute with } \mathrm{GL}_n)}} = \underbrace{\pi(\mathbb{C}S_k)}_{\substack{\text{(img of } S_k \\ \text{action)}}} \quad \text{and} \quad \mathrm{End}_{S_k}\left((\mathbb{C}^n)^{\otimes k}\right) = \underbrace{\rho(\mathbb{C}\mathrm{GL}_n)}_{\substack{\text{(img of } \mathrm{GL}_n \\ \text{action)}}}.$$

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Powerful consequence: a duality between representations

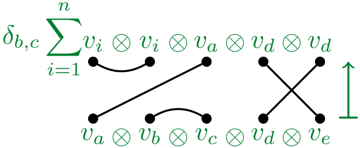
The double-centralizer relationship produces

$$(\mathbb{C}^n)^{\otimes k} \cong \bigoplus_{\lambda \vdash k} G^\lambda \otimes S^\lambda \quad \text{as a } \mathrm{GL}_n\text{-}S_k \text{ bimodule,}$$

where G^λ are distinct irreducible GL_n-modules
S^λ are distinct irreducible S_k-modules

More centralizer algebras

Brauer (1937)
Orthogonal and symplectic groups (and Lie algebras) acting on $(\mathbb{C}^n)^{\otimes k}$ diagonally centralize the **Brauer algebra**:



with  $= n$

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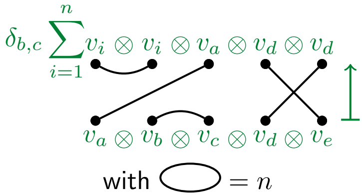
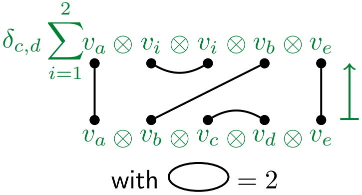


Temperley-Lieb (1971)
GL_2 and SL_2 (and $\mathfrak{gl}_2$ and $\mathfrak{sl}_2$) acting on $(\mathbb{C}^2)^{\otimes k}$ diagonally centralize the **Temperley-Lieb algebra**:



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Quantum groups and braids

Fix $q \in \mathbb{C}$, and let $\mathcal{U} = \mathcal{U}_q\mathfrak{g}$ be the Drinfeld-Jimbo quantum group associated to Lie algebra $\mathfrak{g}$.

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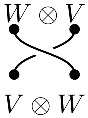
$$\check{\mathcal{R}}_{VW} \colon V \otimes W \longrightarrow W \otimes V$$



that (1) satisfies braid relations, and
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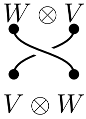

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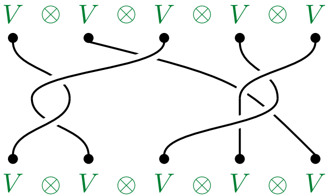

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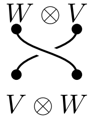

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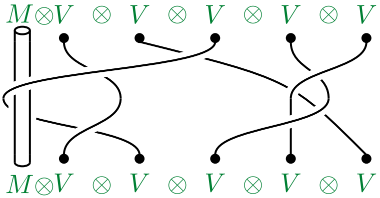
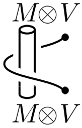
The one-pole/affine braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k}$:



Around the pole:



$$= \check{R}_{MV}\check{R}_{VM}$$

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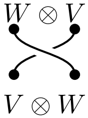

that (1) satisfies braid relations, and
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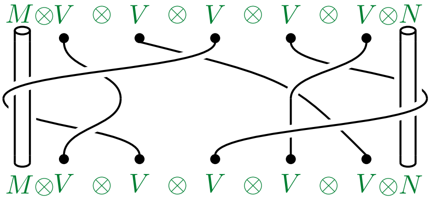
The two-pole braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k} \otimes N$:



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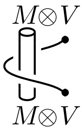
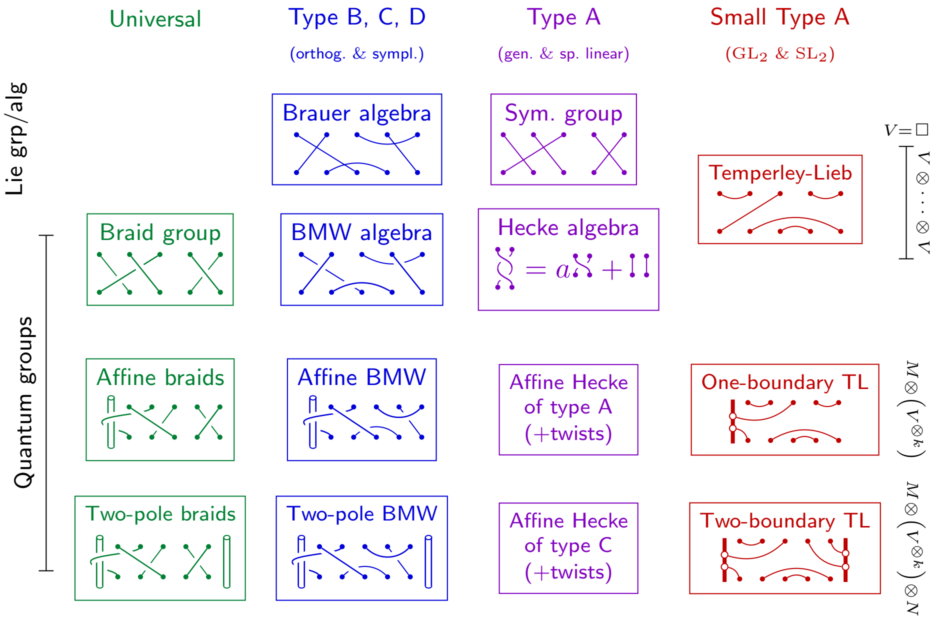
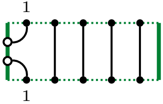
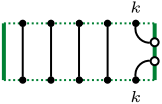
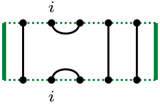


$= \check{R}_{MV}\check{R}_{VM}$


Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL_2 & SL_2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
$V = \square$
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
$= a + $
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
$\left(V^{\otimes k}\right) \otimes M$
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$N \otimes \left(V^{\otimes k}\right) \otimes M$

Two-boundary Temperley-Lieb algebras

[MNGB04] Fix $z, z_0, z_k \in \mathbb{C}$. The *two-boundary Temperley-Lieb algebra* TL_k is a diagram algebra generated over $\mathbb{C}$ by diagrams

$e_0 =$ , $e_k =$ , and $e_i =$ 

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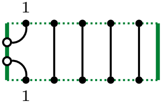
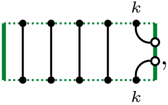
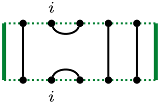
$e_0 =$ 

, $e_k =$ 

, and $e_i =$ 

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for $1 \leq i \leq k-1$,

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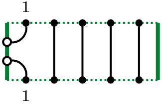
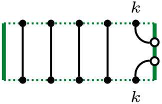
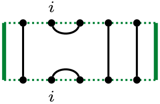
$e_0 =$ 

, $e_k =$ 

, and $e_i =$ 

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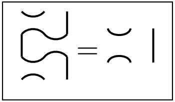
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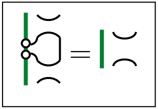
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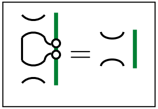

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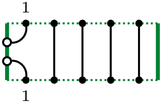
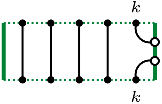
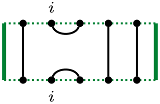
$e_0 =$ , $e_k =$ , and $e_i =$ 

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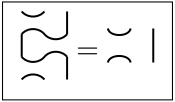
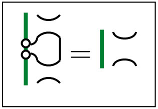
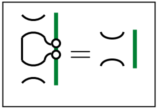
 or  or 

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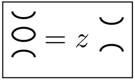

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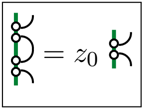

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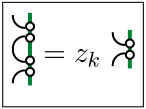

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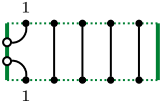
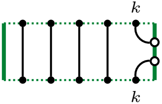
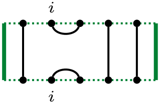
$e_0 =$ 

, $e_k =$ 

, and $e_i =$ 

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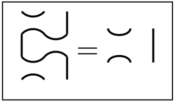
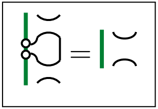
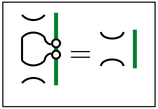
 or  or 

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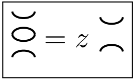
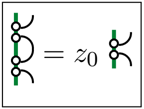
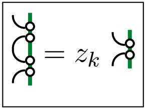


or 

or 

(Side loops are resolved with a 1 or a z_i depending on whether there are an even or odd number of connections below their lowest point.)

Diagram multiplication:

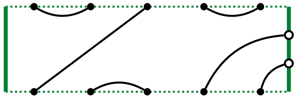


Diagram multiplication:

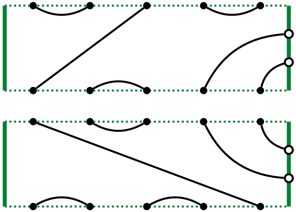


Diagram multiplication:

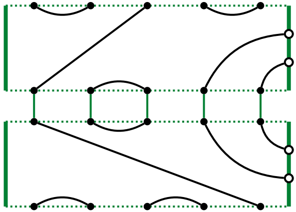


Diagram multiplication:

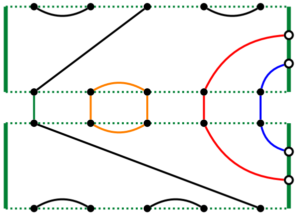
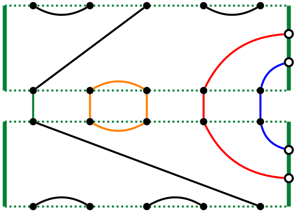
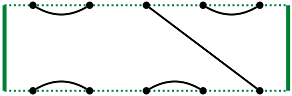


Diagram multiplication:

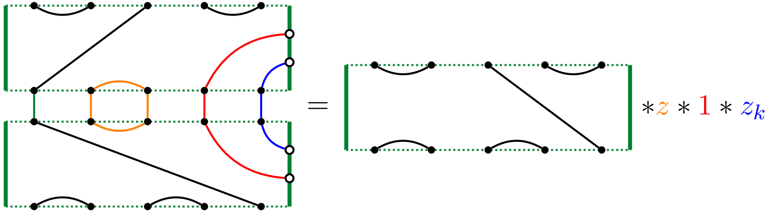


$=$



$*z*1*z_k$

Diagram multiplication:



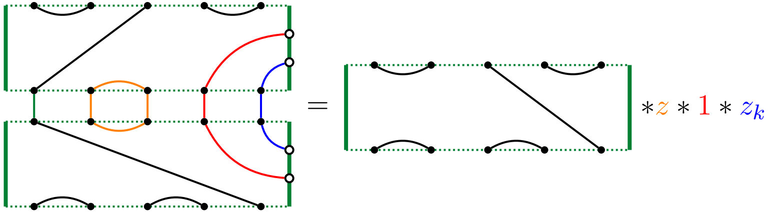
In short, TL_k has basis given by non-crossing diagrams with

(1) k connections to the top and to the bottom,

(2) an even number of connections to the right and to the left, and

(3) no edges with both ends on the left or both ends on the right.

Diagram multiplication:



In short, TL_k has basis given by non-crossing diagrams with

(1) k connections to the top and to the bottom,

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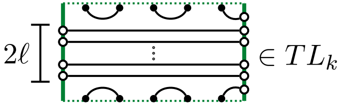
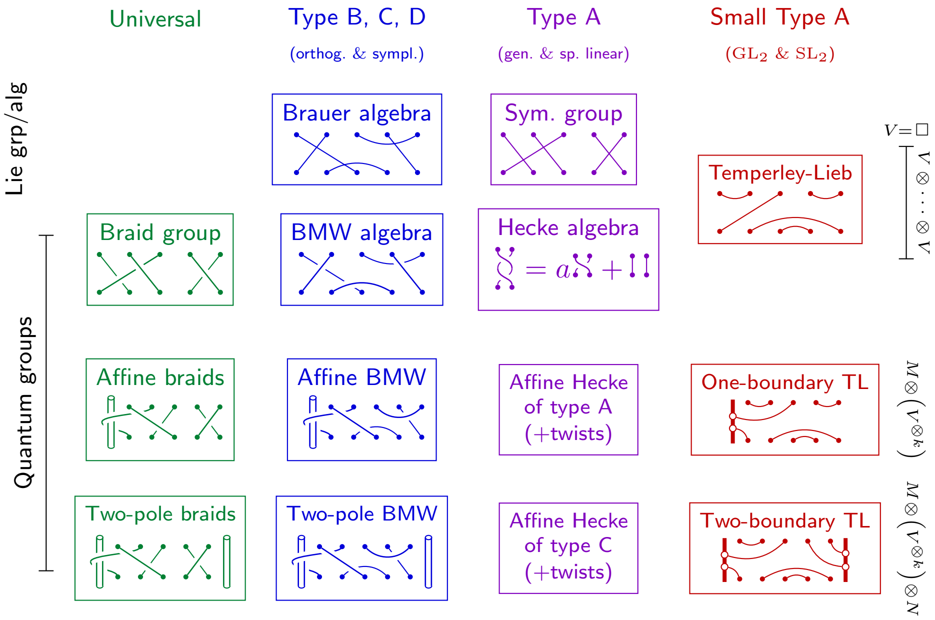
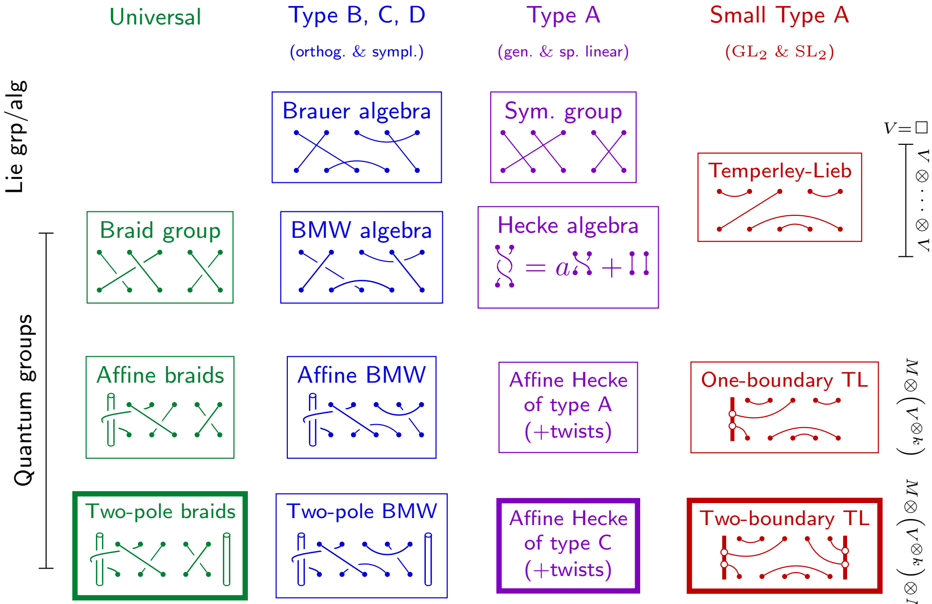
However,



So unlike the classical T-L algebras, TL_k is not finite dimensional!



Universal
Type B, C, D
(orthog. & sympl.)
Type A
(gen. & sp. linear)
Small Type A
(GL2 & SL2)
Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
Temperley-Lieb
V = □
V ⊗ · · · ⊗ V
Braid group
BMW algebra
Hecke algebra
= a +
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
(V⊗k) ⊗ M
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
N ⊗ (V⊗k) ⊗ M



Universal
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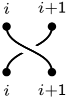
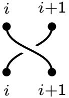
N ⊗ (V⊗k) ⊗ M

The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

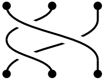
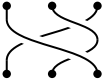
for $1 \leq i \leq k-1$,

The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

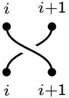
$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

subject to relations

$T_iT_{i+1}T_i =$  $=$  $= T_{i+1}T_iT_{i+1}$,

The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

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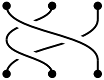
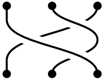
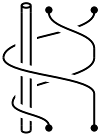
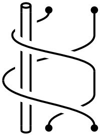
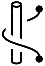
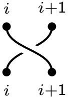
$T_i T_{i+1} T_i =$  $=$  $= T_{i+1} T_i T_{i+1}$,

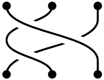
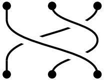
$T_1 T_0 T_1 T_0 =$  $=$  $= T_0 T_1 T_0 T_1$,

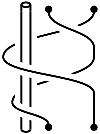
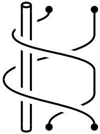
The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

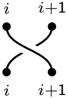
subject to relations

$T_i T_{i+1} T_i =$  $=$  $= T_{i+1} T_i T_{i+1}$,

$T_1 T_0 T_1 T_0 =$  $=$  $= T_0 T_1 T_0 T_1$,

and, similarly, $T_{k-1} T_k T_{k-1} T_k = T_k T_{k-1} T_k T_{k-1}$.

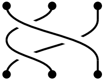
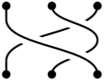
The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

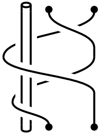
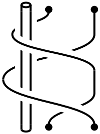
$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \leq i \leq k-1$,

subject to relations

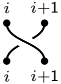
i.e.

$T_iT_{i+1}T_i =$  $=$  $= T_{i+1}T_iT_{i+1}$,

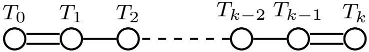
$T_1T_0T_1T_0 =$  $=$  $= T_0T_1T_0T_1$,

and, similarly, $T_{k-1}T_kT_{k-1}T_k = T_kT_{k-1}T_kT_{k-1}$.

(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

subject to relations 

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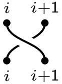
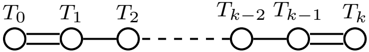
(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $\quad T_0 =$  and $\quad T_i =$ 

for $1 \le i \le k-1$,

subject to relations 

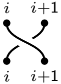
.

(2) Fix constants $t_0, t_k, t \in \mathbb{C}$.
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations

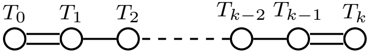
$$(T_0 - t_0^{1/2})(T_0 + t_0^{-1/2}) = 0, \quad (T_k - t_k^{1/2})(T_k + t_k^{-1/2}) = 0$$

$$\text{and} \quad (T_i - t^{1/2})(T_i + t^{-1/2}) = 0 \quad \text{for } i = 1, \ldots, k-1.$$

(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

subject to relations 

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(2) Fix constants $t_0, t_k, t = t_1 = t_2 = \cdots = t_{k-1} \in \mathbb{C}$.
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations $(T_i - t_i^{1/2})(T_i + t_i^{-1/2}) = 0$.

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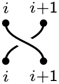
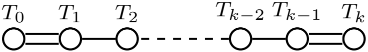
$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

subject to relations 

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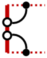
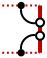
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations $(T_i - t_i^{1/2})(T_i + t_i^{-1/2}) = 0$.

(3) Set

 $= t_0^{1/2}$  $-$  $(e_0 = t_0^{1/2} - T_0)$

 $= t_k^{1/2}$  $-$  $(e_k = t_k^{1/2} - T_k)$

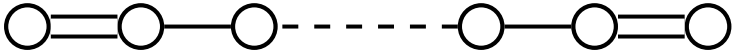
 $= t^{1/2}$  $-$  $(e_i = t^{1/2} - T_i)$

so that $e_j^2 = z_j e_j$ (for good z_j).

(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$,

subject to relations 

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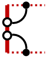
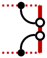
(2) Fix constants $t_0, t_k, t = t_1 = t_2 = \cdots = t_{k-1} \in \mathbb{C}$.
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations $(T_i - t_i^{1/2})(T_i + t_i^{-1/2}) = 0$.

(3) Set

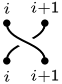
 $= t_0^{1/2}$  $-$  $\qquad (e_0 = t_0^{1/2} - T_0)$

 $= t_k^{1/2}$  $-$  $\qquad (e_k = t_k^{1/2} - T_k)$

 $= t^{1/2}$  $-$  $\qquad (e_i = t^{1/2} - T_i)$

so that $e_j^2 = z_j e_j$ (for good z_j).
The two-boundary Temperley-Lieb algebra is the quotient of $\mathcal{H}_k$ by the relations $e_i e_{i\pm1} e_i = e_i$ for $i = 1, \ldots, k-1$.

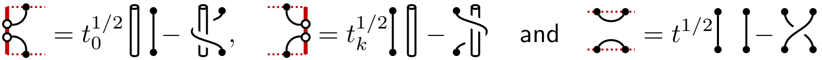
(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

for $1 \le i \le k-1$.

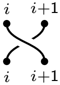
(2) Fix constants $t_0, t_k, t = t_1 = t_2 = \cdots = t_{k-1} \in \mathbb{C}$.
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations $(T_i - t_i^{1/2})(T_i + t_i^{-1/2}) = 0$.

(3) Set



so that $e_j^2 = z_j e_j$. The two-boundary Temperley-Lieb algebra is the quotient of $\mathcal{H}_k$ by the relations $e_i e_{i\pm1} e_i = e_i$ for $i = 1, \ldots, k-1$.

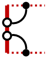
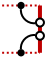
(1) The two-boundary (two-pole) braid group $\mathcal{B}_k$ is generated by

$T_k =$ , $T_0 =$  and $T_i =$ 

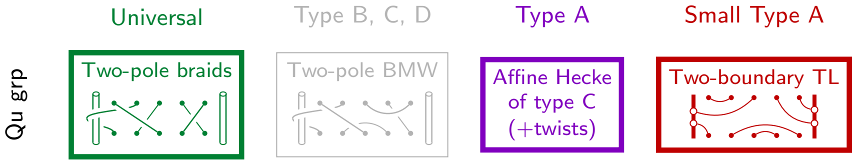
for $1 \leq i \leq k-1$.

(2) Fix constants $t_0, t_k, t = t_1 = t_2 = \cdots = t_{k-1} \in \mathbb{C}$.
The affine type C Hecke algebra $\mathcal{H}_k$ is the quotient of $\mathbb{C}\mathcal{B}_k$ by the relations $(T_i - t_i^{1/2})(T_i + t_i^{-1/2}) = 0$.

(3) Set

 $= t_0^{1/2}$  $-$ ,  $= t_k^{1/2}$  $-$  and  $= t^{1/2}$  $-$ 

so that $e_j^2 = z_j e_j$. The two-boundary Temperley-Lieb algebra is the quotient of $\mathcal{H}_k$ by the relations $e_i e_{i\pm1} e_i = e_i$ for $i = 1, \ldots, k-1$.



Representation theory of $\mathcal{H}_k$

The representations of $\mathcal{H}_k$ are indexed by pairs $(\mathbf{c}, J)$, where

$\mathbf{c}$ is a point in the fundamental chamber of the (finite) type C hyperplane system, and

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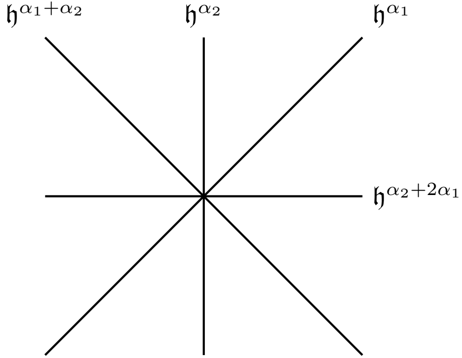
Example: $k = 2$



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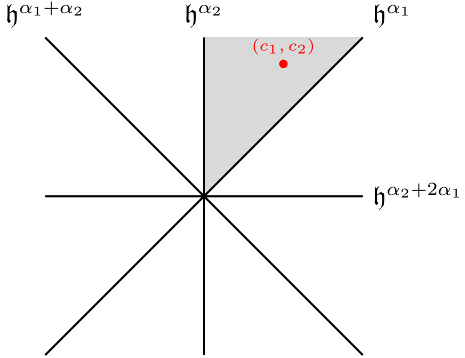

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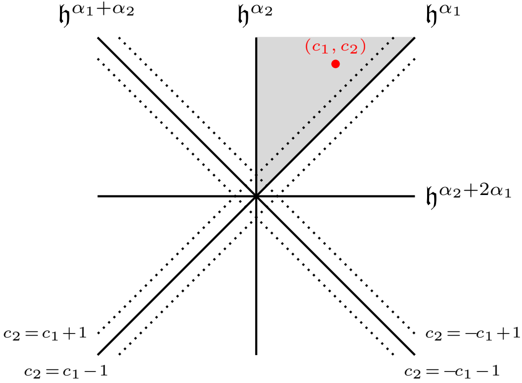

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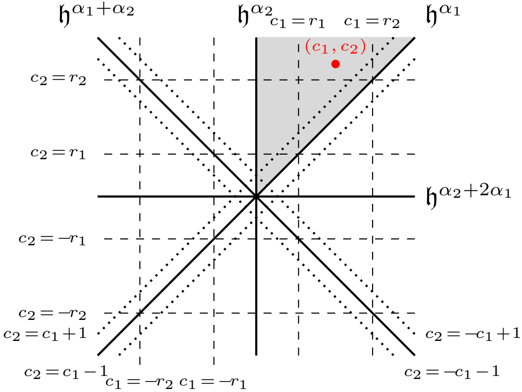

The r_is depend on $\mathcal{H}_k$'s parameters t_0 and t_k: $r_1 = \log_t(t_0/t_k)$, $r_2 = \log_t(t_0 t_k)$

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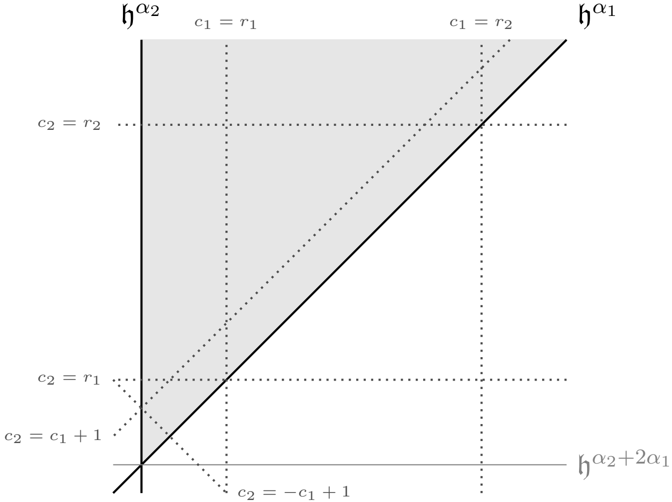

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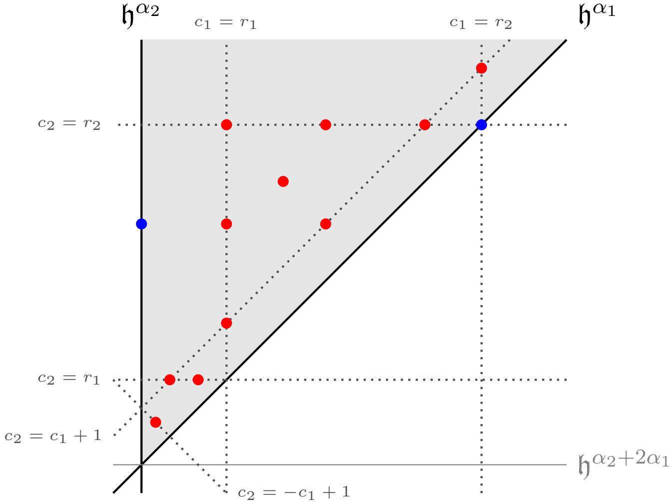

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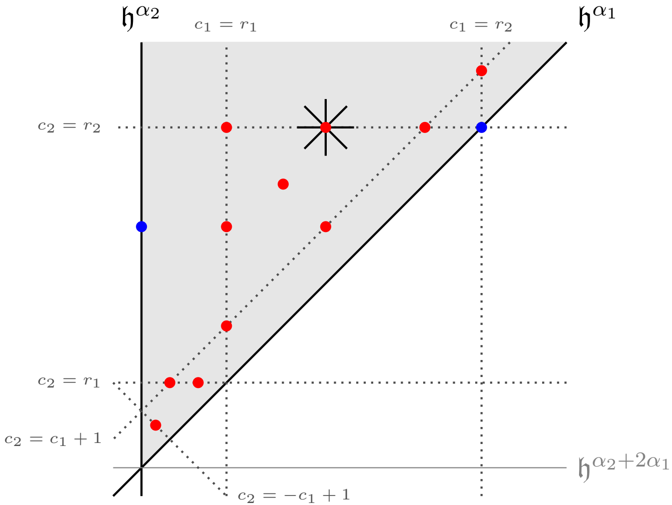

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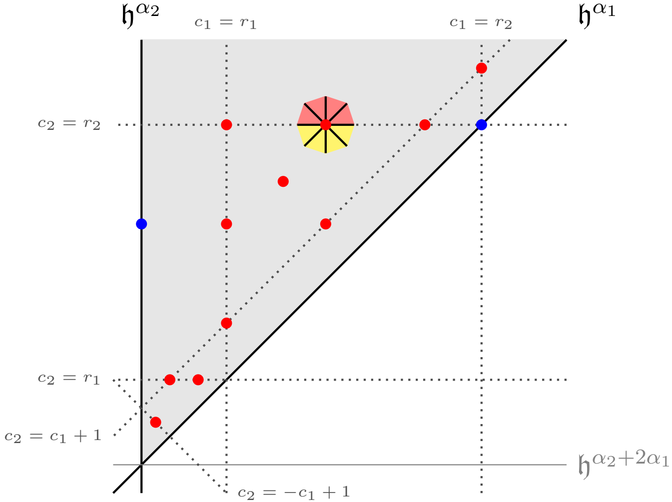

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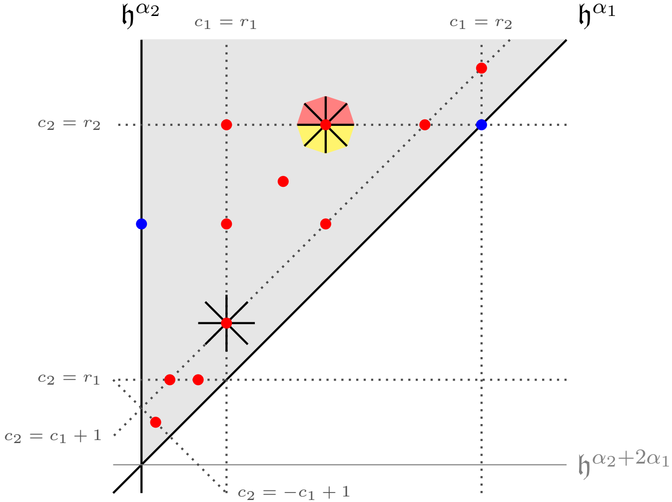

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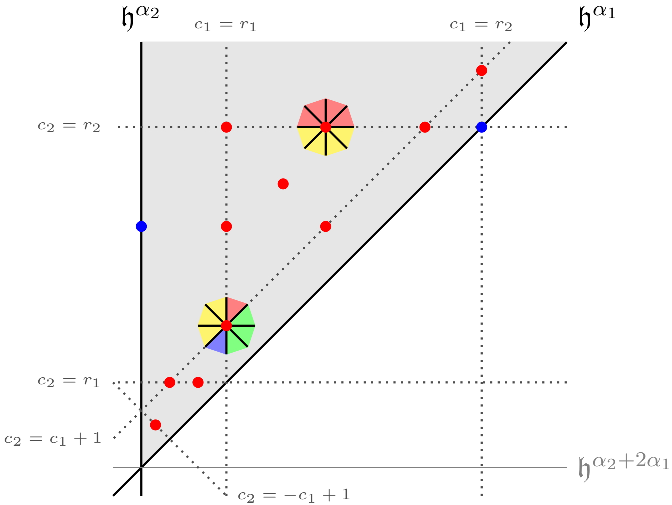

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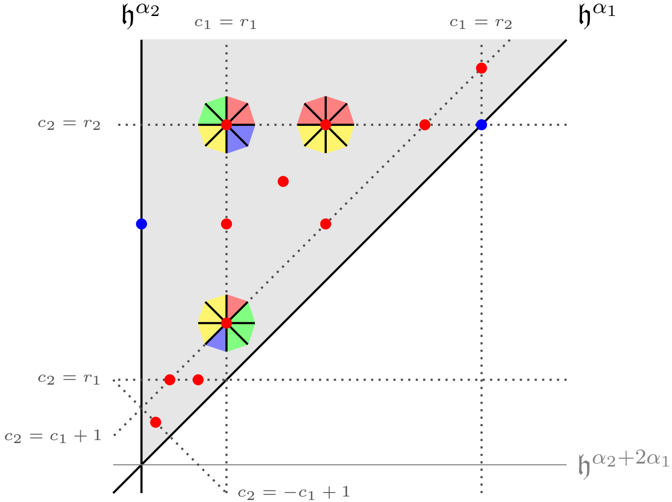

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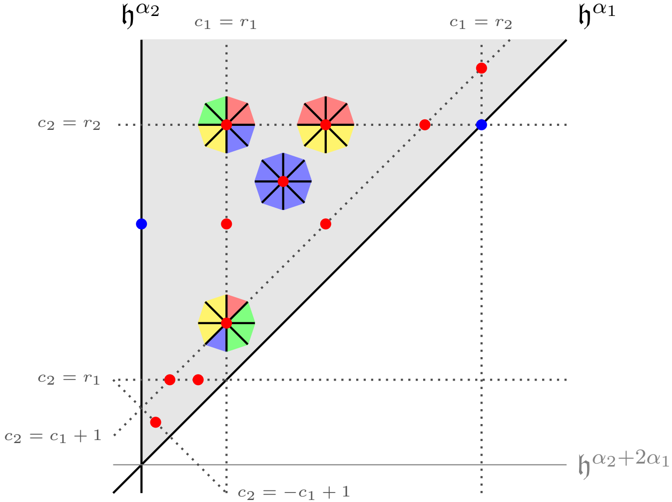

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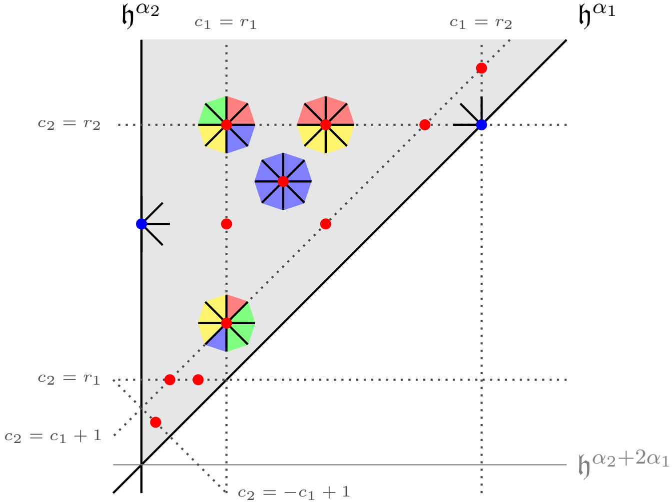

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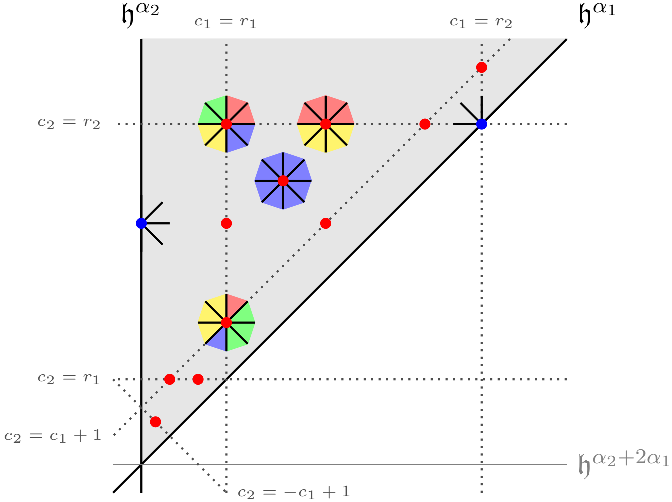

Thm. (D.-Ram)

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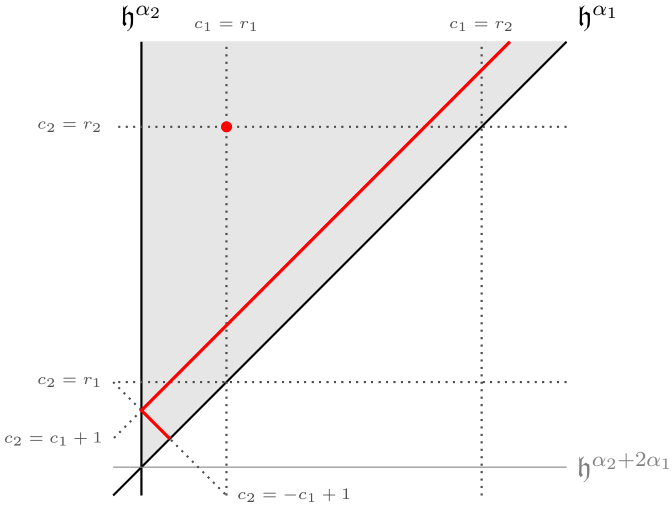

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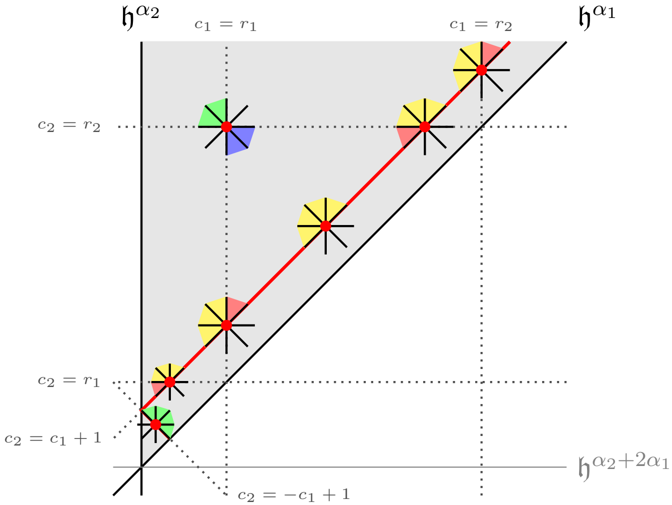

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