

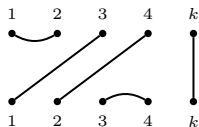
# Representation theory of the two-boundary Temperley-Lieb algebra

Zajj Daugherty  
(Joint work in progress with Arun Ram)

September 10, 2014

## Temperley-Lieb algebras

The *Temperley-Lieb algebra*  $TL_k(q)$  is the algebra of non-crossing pairings on  $2k$  vertices

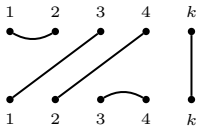


with multiplication given by stacking diagrams, subject to the relation

$$\bigcirc = q + q^{-1}$$

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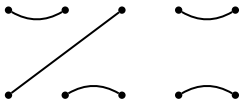
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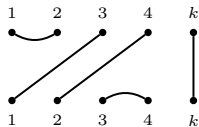
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**Multiplication:**



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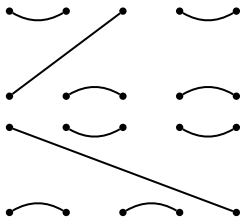
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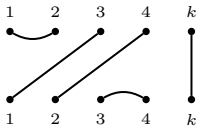
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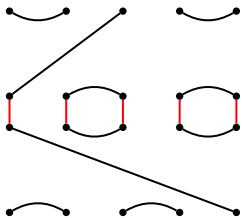
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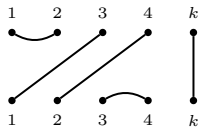
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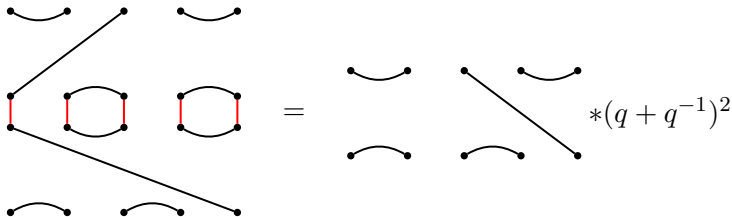
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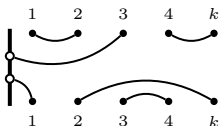
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**Multiplication:**



## Temperley-Lieb algebras

The *one-boundary Temperley-Lieb algebra*  $TL_k^{(1)}(q, z_0)$  is the algebra of one-walled non-crossing pairings on  $2k$  vertices



with multiplication given by stacking diagrams, subject to the relations

$$\bigcirc = q + q^{-1} \quad \text{and}$$

$$\begin{array}{c} \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \end{array} \begin{array}{c} \text{if even \#} \\ \text{connections} \\ \text{below} \end{array} \Big| \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} = 1 \quad \text{or} \quad \begin{array}{c} \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \\ \circ \\ \vdots \end{array} \begin{array}{c} \text{if odd \#} \\ \text{connections} \\ \text{below} \end{array} \Big| \begin{array}{c} \circ \\ \vdots \\ \circ \end{array} = z_0.$$

## Odd/even relations

The algebra  $TL_k^{(1)}(q, z_0)$  is generated by

$$e_i = \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right| \begin{array}{c} \overset{i}{\curvearrowright} \\ \vdots \\ \underset{i}{\curvearrowleft} \\ \vdots \\ \vdots \end{array} \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right| \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right| \quad \text{and} \quad e_0 = \left| \begin{array}{c} \overset{1}{\curvearrowright} \\ \vdots \\ \underset{1}{\curvearrowleft} \\ \vdots \\ \vdots \end{array} \right| \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right| \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right| \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right|$$

for  $i = 1, \dots, k - 1$



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for  $i = 1, \dots, k - 1$ , with relations

$$e_i e_{i \pm 1} e_i = e_i \text{ for } i \geq 1$$

The diagrammatic relation  $e_i e_{i \pm 1} e_i = e_i$  is shown in a box. On the left, a vertical strand has a crossing with another strand. On the right, a vertical strand has a crossing with another strand.

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$$\boxed{\begin{array}{c} \curvearrowright \\ \vdots \\ \curvearrowleft \end{array} = \begin{array}{c} \curvearrowright \\ \vdots \\ \curvearrowleft \end{array} \left| \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right|$$

or

$$\boxed{\begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \begin{array}{c} \curvearrowright \\ \vdots \\ \curvearrowleft \end{array} = \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \begin{array}{c} \curvearrowright \\ \vdots \\ \curvearrowleft \end{array}$$

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$$e_i^2 = a e_i$$

$$\boxed{\begin{array}{c} \curvearrowright \\ \circ \\ \curvearrowleft \end{array} = (q + q^{-1}) \begin{array}{c} \curvearrowright \\ \vdots \\ \curvearrowleft \end{array}$$

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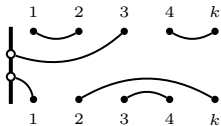
$$e_i^2 = a e_i$$

or

Side loops are resolved with a 1 or a  $z_0$  depending on whether there are an even or odd number of connections below their lowest point.

## Temperley-Lieb algebras

The *one-boundary Temperley-Lieb algebra*  $TL_k^{(1)}(q, z_0)$  is the algebra of one-walled non-crossing pairings on  $2k$  vertices



with multiplication given by stacking diagrams, subject to the relations

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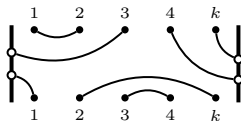
$$\begin{array}{c} \text{if even \#} \\ \text{connections} \\ \text{below} \end{array} \left[ \begin{array}{c} \vdots \\ \circlearrowright \\ \vdots \\ \circlearrowleft \\ \vdots \\ \circlearrowright \\ \vdots \\ \circlearrowleft \\ \vdots \end{array} \right] = 1 \quad \text{or}$$

$$\begin{array}{c} \text{if odd \#} \\ \text{connections} \\ \text{below} \end{array} \left[ \begin{array}{c} \vdots \\ \circlearrowright \\ \vdots \\ \circlearrowleft \\ \vdots \\ \circlearrowright \\ \vdots \\ \circlearrowleft \\ \vdots \end{array} \right] = z_0.$$

# Our main object: two-boundary Temperley-Lieb algebra

**Nienhuis, De Gier, Batchelor (2004):**

The *two-boundary Temperley-Lieb algebra*  $TL_k^{(2)}(q, z_0, z_k) = \mathcal{T}_k$  is the algebra of two-walled non-crossing pairings on  $2k$  vertices



so that each wall always has an even number of connections, with multiplication given by stacking diagrams, subject to the relations

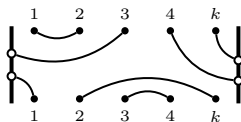
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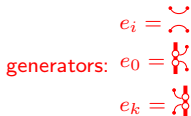
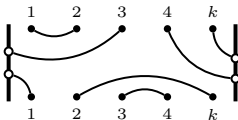
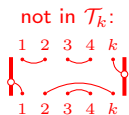
if even # connections below  $\bigcirc = 1$  or if odd # connections below  $\bigcirc = z_0$ ,  $\bigcirc = z_k$ .



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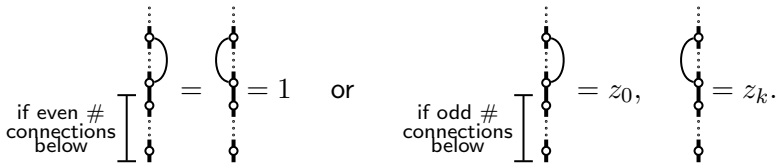
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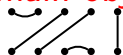


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$TL_k^{(2)} = \mathcal{T}_k$  is infinite-dimensional!



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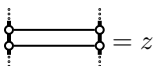
$TL_k^{(1)}$  is finite-dimensional



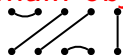
$TL_k^{(2)} = \mathcal{T}_k$  is infinite-dimensional!



**de Gier, Nichols (2008):** Explored representation theory of  $\mathcal{T}_k$ .

- 1 Take quotients giving   $= z$   
to get finite-dimensional algebras.
- 2 Establish connection to the affine Hecke algebras of type A and C to facilitate calculations.
- 3 Use diagrammatics and an action on  $(\mathbb{C}^2)^{\otimes k}$  to help classify representations in quotient (most modules are  $2^k$  dim'l; some split).

## Our main object: two-boundary Temperley-Lieb algebra



$TL_k$  is finite-dimensional ( $n$ th Catalan number) SWD✓



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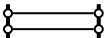
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## Quantum groups and braids

Fix  $q \in \mathbb{C}^*$ . Let  $U = U_q \mathfrak{g}$  be the Drinfel'd-Jimbo quantum group associated to a reductive Lie algebra  $\mathfrak{g}$ . Let  $V, M$  be  $U$ -modules. Then  $U \otimes U$  has invertible  $R = \sum_R R_1 \otimes R_2$  that yields a map


$$\check{R}_{VM}: \begin{array}{ccc} V \otimes M & \longrightarrow & M \otimes V \\ v \otimes m & \longmapsto & \sum_R R_1 m \otimes R_2 v \end{array} \quad \begin{array}{c} M \otimes V \\ \swarrow \quad \searrow \\ \bullet \quad \bullet \\ \searrow \quad \swarrow \\ V \otimes M \end{array}$$

that (1) satisfies braid relations, and  
(2) commutes with the action of  $U_q \mathfrak{g}$ .

## Quantum groups and braids

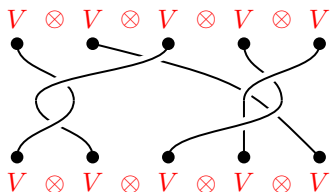
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The braid group shares a commuting action with  $U_q \mathfrak{g}$  on  $V^{\otimes k}$ :






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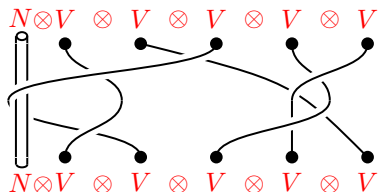
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The **one-boundary/affine** braid group shares a commuting action with  $U_q\mathfrak{g}$  on  $N \otimes V^{\otimes k}$ :



Around the pole:




$$= \check{R}_{NV} \check{R}_{VN}$$

## Quantum groups and braids

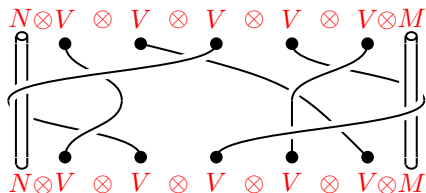
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The **two-boundary** braid group shares a commuting action with  $U_q \mathfrak{g}$  on  $N \otimes V^{\otimes k} \otimes M$ :



Around the pole:



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# Affine type C Hecke algebra and two-boundary braids

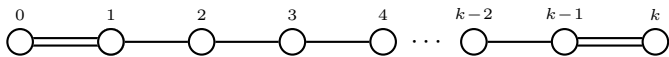


Fix constants  $t_0, t_k$ , and  $t = t_1 = \dots = t_{k-1}$ . The affine Hecke algebra of type C,  $\mathcal{H}_k$ , is generated by  $T_0, T_1, \dots, T_k$  with relations

$$\underbrace{T_i T_j \dots}_{m_{i,j} \text{ factors}} = \underbrace{T_j T_i \dots}_{m_{i,j} \text{ factors}} \quad \text{where} \quad m_{i,j} = \begin{array}{ll} 2 & \text{if } \begin{array}{c} i \quad j \\ \circ \quad \circ \end{array} \\ 3 & \text{if } \begin{array}{c} i \quad j \\ \circ \text{---} \circ \end{array} \\ 4 & \text{if } \begin{array}{c} i \quad j \\ \circ \text{=}= \circ \end{array} \end{array}$$

and  $T_i^2 = (t_i^{1/2} - t_i^{-1/2})T_i + 1$ .

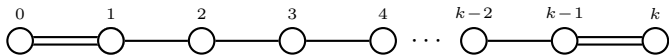
## Affine type C Hecke algebra and two-boundary braids



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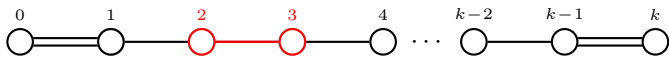
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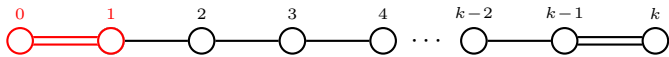
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Relations:

$$T_i T_{i+1} T_i = \begin{array}{c} \text{---} \quad \text{---} \\ \diagdown \quad \diagup \\ \diagup \quad \diagdown \\ \text{---} \quad \text{---} \end{array} = \begin{array}{c} \text{---} \quad \text{---} \\ \diagup \quad \diagdown \\ \diagdown \quad \diagup \\ \text{---} \quad \text{---} \end{array} = T_{i+1} T_i T_{i+1}$$

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Theorem (D.-Ram, degenerate versions of 1&2 in [D. 10])

- (1) Let  $U = U_q \mathfrak{g}$  for any complex reductive Lie algebras  $\mathfrak{g}$ .  
Let  $M$ ,  $N$ , and  $V$  be finite-dimensional modules.

The two-boundary braid group  $\mathcal{B}_k$  acts on  $N \otimes (V)^{\otimes k} \otimes M$  and this action commutes with the action of  $U$ .

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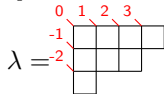
$$e_i e_{i \pm 1} e_i \quad \text{for } 1 \leq i \leq k-1 : \quad \begin{array}{c} \diagup \\ \diagdown \end{array} = \begin{array}{c} \diagdown \\ \diagup \end{array} \quad | \quad \text{or} \quad \begin{array}{c} \diagup \\ \diagdown \end{array} = \begin{array}{c} \diagdown \\ \diagup \end{array} \quad \left( \begin{array}{c} \text{and} \\ \text{reverses} \end{array} \right)$$

- (3) When  $\mathfrak{g} = \mathfrak{gl}_2$ ,  $\mathcal{T}_k$  acts on  $N \otimes (V)^{\otimes k} \otimes M$  (for good choices).

Consider the fin-dim'l simple  $U_q\mathfrak{gl}_n$ -modules  $L(\lambda)$  indexed by **partitions**:

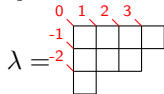
$$\lambda = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \\ \hline \square & & & \\ \hline \end{array}$$

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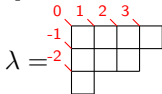
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The eigenvalues of these operators (of which there should be two, since  $(T_k - t_k^{1/2})(T_k + t_k^{-1/2}) = (T_0 - t_0^{1/2})(T_0 + t_0^{-1/2}) = (T_i - t^{1/2})(T_i + t^{-1/2}) = 0$ ) are controlled by contents of addable boxes.

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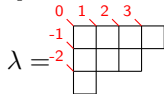
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$$(a^c) = c \begin{array}{c} a \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ c \end{array}$$

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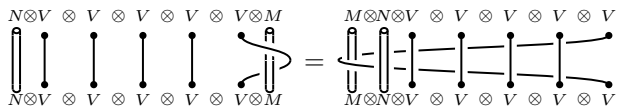
$\mathcal{H}_k$  has a commuting action with  $U_q \mathfrak{gl}_n$  on the space

$$L((b^d)) \otimes (L(\square))^{\otimes k} \otimes L((a^c)) \quad \text{with } c, d < n$$



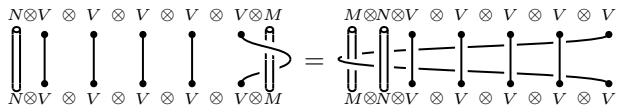
## Exploring tensor space structure

Move the right pole to the left:



# Exploring tensor space structure

Move the right pole to the left:



New favorite generators:

$$T_0 = \begin{array}{|c|} \hline \text{[diagram of two vertical lines, one with a loop]} \\ \hline \end{array}, \quad T_i = \begin{array}{|c|} \hline \text{[diagram of two lines crossing]} \\ \hline \end{array} \quad \text{and} \quad Y_j = \begin{array}{|c|} \hline \text{[diagram of two lines with a crossing and a loop]} \\ \hline \end{array}$$

# Exploring tensor space structure

Move the right pole to the left:

The diagram shows an equality between two tensor network expressions. On the left, a pole from the space  $N \otimes V$  passes through a series of tensor products  $\otimes V \otimes V \otimes V \otimes V \otimes V \otimes M$ . On the right, the pole from the space  $M \otimes N \otimes V$  passes through the same series of tensor products  $\otimes V \otimes V \otimes V \otimes V$ . The equality indicates that the pole can be moved from the right side to the left side of the tensor product.

New favorite generators:

The diagram defines three generators:
 

- $T_0$ : A vertical line from  $N \otimes V$  splits into two lines going to  $V$ .
- $T_i$ : Two lines from  $V$  merge into one line going to  $V$ .
- $Y_j$ : A line from  $M \otimes N \otimes V$  splits into two lines going to  $V$ , with a dot on the right line.

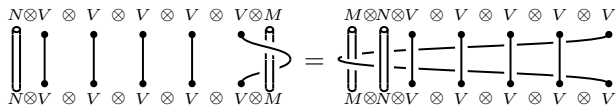
Then

$$M \otimes N = L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda), \quad (\text{multiplicity one!})$$

where  $\Lambda$  is the following set of partitions:

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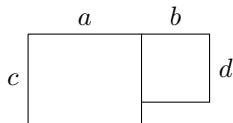
New favorite generators:

$$T_0 = \begin{array}{|c|} \hline \text{pole} \\ \hline \end{array}, \quad T_i = \begin{array}{c} i \quad i+1 \\ \diagdown \quad \diagup \\ i \quad i+1 \end{array} \quad \text{and} \quad Y_j = \begin{array}{c} \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \end{array}$$

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The diagram shows an equality between two tensor network expressions. On the left, there are five vertical lines representing vector spaces  $N \otimes V \otimes V \otimes V \otimes V \otimes M$  at the top and  $N \otimes V \otimes V \otimes V \otimes V \otimes M$  at the bottom. A pole (a pair of lines) is shown on the right side, with a curved arrow indicating its movement to the left. On the right, the pole has moved to the left side, and the tensor spaces are now  $M \otimes N \otimes V \otimes V \otimes V \otimes V$  at both the top and bottom.

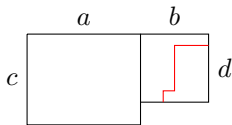
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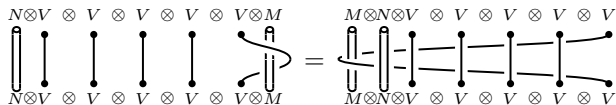
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Move the right pole to the left:



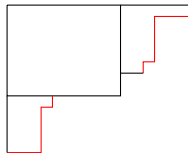
New favorite generators:

$$T_0 = \begin{array}{|c|} \hline \text{pole} \\ \hline \end{array}, \quad T_i = \begin{array}{c} i \quad i+1 \\ \diagdown \quad \diagup \\ i \quad i+1 \end{array} \quad \text{and} \quad Y_j = \begin{array}{c} \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \quad \text{pole} \end{array}$$

Then

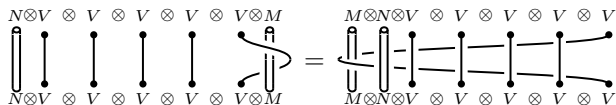
$$M \otimes N = L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda), \quad (\text{multiplicity one!})$$

where  $\Lambda$  is the following set of partitions:



# Exploring tensor space structure

Move the right pole to the left:



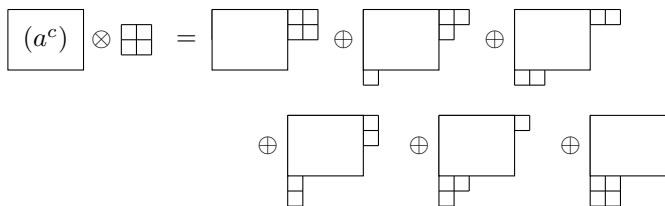
New favorite generators:

$$T_0 = \left[ \begin{array}{c} \text{pole from } N \otimes V \text{ to } N \otimes V \\ \text{pole from } V \otimes M \text{ to } V \otimes M \end{array} \right], \quad T_i = \begin{array}{c} i \quad i+1 \\ \text{pole from } V \otimes V \text{ to } V \otimes V \\ i \quad i+1 \end{array} \quad \text{and} \quad Y_j = \begin{array}{c} \text{pole from } M \otimes N \otimes V \text{ to } M \otimes N \otimes V \\ \text{pole from } V \otimes V \text{ to } V \otimes V \\ j \end{array}$$

Then

$$M \otimes N = L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda), \quad (\text{multiplicity one!})$$

where  $\Lambda$  is the following set of partitions...



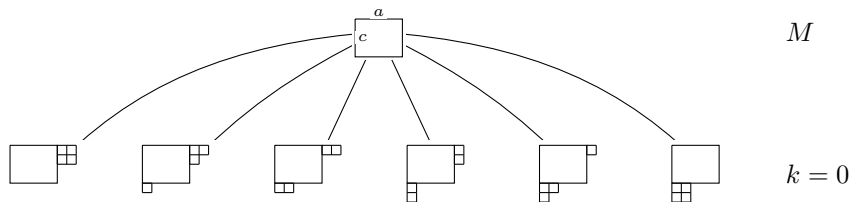
## Exploring tensor space structure



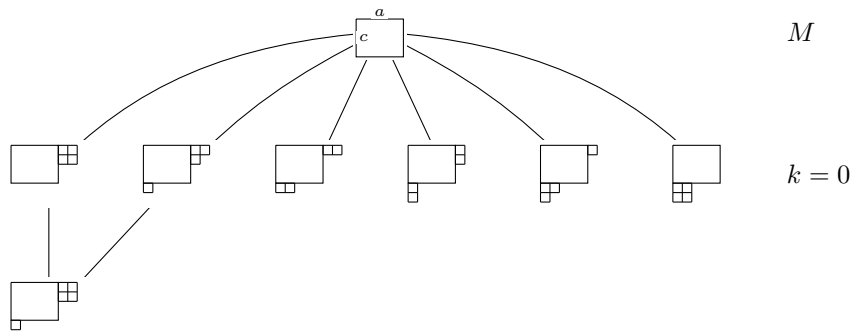
$M$



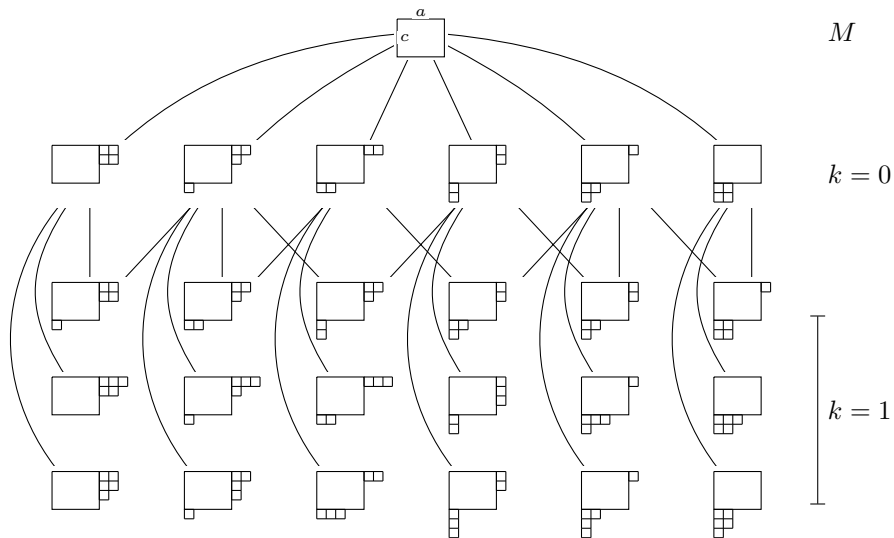
## Exploring tensor space structure



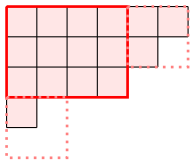
## Exploring tensor space structure



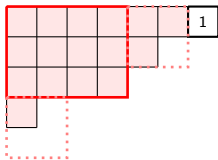
# Exploring tensor space structure



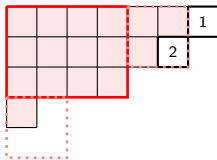
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right)$$



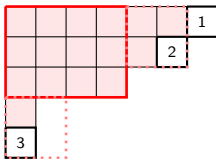
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square)$$



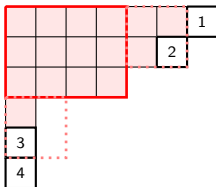
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square)$$



$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

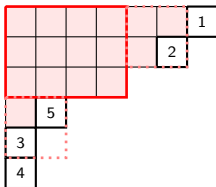


$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

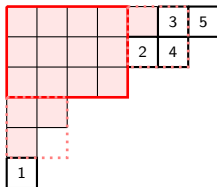
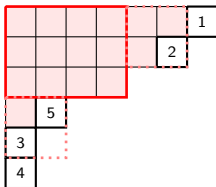




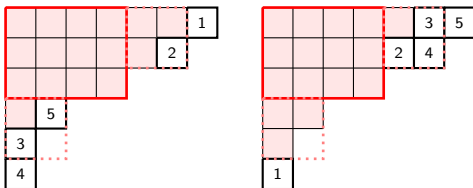
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

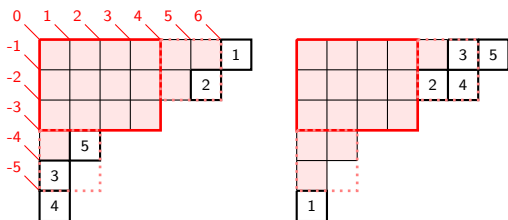


$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



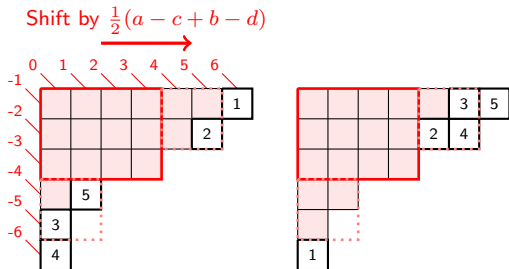
$\mathcal{H}_k$  representations in tensor space are labeled by certain partitions  $\lambda$ , with basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ .

$$L\left(\begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



$\mathcal{H}_k$  representations in tensor space are labeled by certain partitions  $\lambda$ , with basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ . Rep are calibrated, i.e.  $Y_j$ 's act by constants controlled by content.

$$L\left(\begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

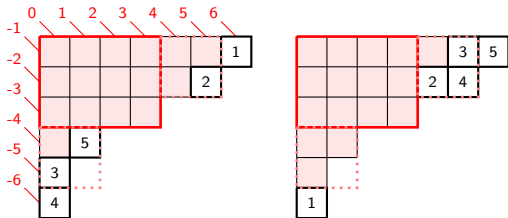


$\mathcal{H}_k$  representations in tensor space are labeled by certain partitions  $\lambda$ , with basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ . Rep are calibrated, i.e.  $Y_j$ 's act by constants controlled by content.

$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

Shift by  $\frac{1}{2}(a - c + b - d)$

$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



$$\begin{aligned} Y_1 &\mapsto t^{-5.5} \\ Y_2 &\mapsto t^{2.5} \\ Y_3 &\mapsto t^{4.5} \\ Y_4 &\mapsto t^{3.5} \\ Y_5 &\mapsto t^{5.5} \end{aligned}$$

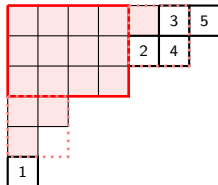
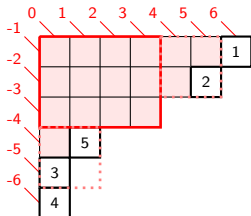
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$$L \left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

Shift by  $\frac{1}{2}(a - c + b - d)$

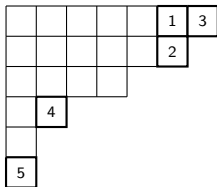


$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



$$\begin{aligned} Y_1 &\mapsto t^{-5.5} \\ Y_2 &\mapsto t^{2.5} \\ Y_3 &\mapsto t^{4.5} \\ Y_4 &\mapsto t^{3.5} \\ Y_5 &\mapsto t^{5.5} \end{aligned}$$

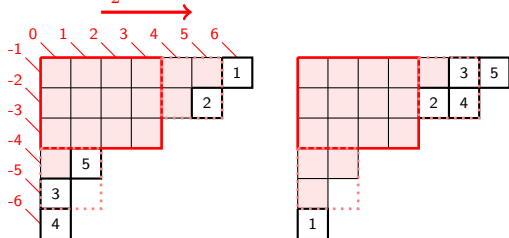
$\mathcal{H}_k$  representations in tensor space are labeled by certain partitions  $\lambda$ , with basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ . Rep are calibrated, i.e.  $Y_j$ 's act by constants controlled by content.



$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

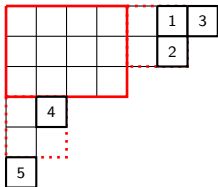
Shift by  $\frac{1}{2}(a - c + b - d)$

$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



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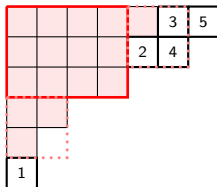
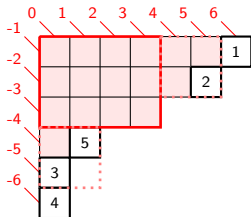


$$L \left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

Shift by  $\frac{1}{2}(a - c + b - d)$

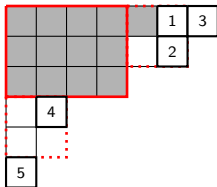


$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



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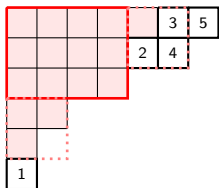
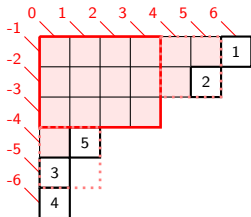


boxes that must  
 = appear in the partition  
 at level 0.

$$L \left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

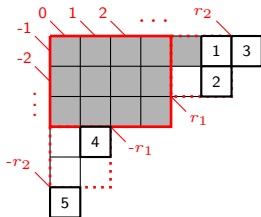
Shift by  $\frac{1}{2}(a - c + b - d)$


$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



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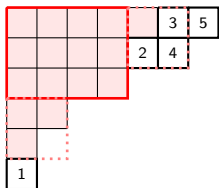
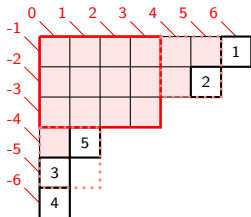


 = boxes that must appear in the partition at level 0.

$$L \left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

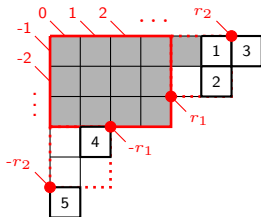
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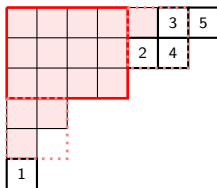
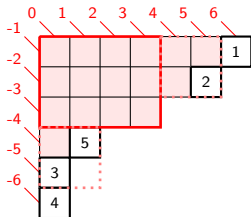


boxes that must  
 = appear in the partition  
 at level 0.

$$L \left( \begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array} \right) \otimes L \left( \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

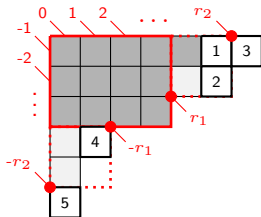
Shift by  $\frac{1}{2}(a - c + b - d)$

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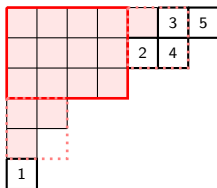
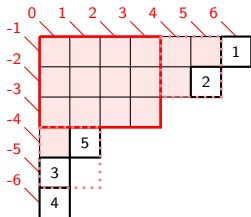


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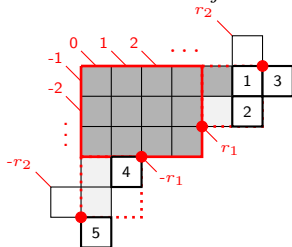
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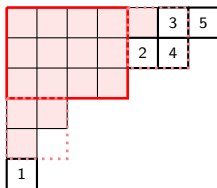
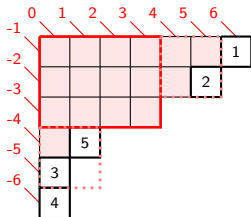


boxes that must  
 = appear in the partition  
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$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

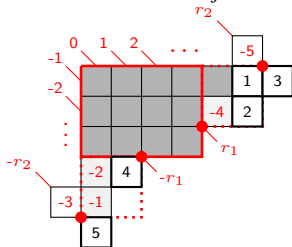
Shift by  $\frac{1}{2}(a - c + b - d)$


$$\begin{aligned} Y_1 &\mapsto t^{5.5} \\ Y_2 &\mapsto t^{3.5} \\ Y_3 &\mapsto t^{-4.5} \\ Y_4 &\mapsto t^{-5.5} \\ Y_5 &\mapsto t^{-2.5} \end{aligned}$$



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$\mathcal{H}_k$  representations in tensor space are labeled by certain partitions  $\lambda$ , with basis labeled by tableaux from some partition  $\mu$  in  $(a^c) \otimes (b^d)$  to  $\lambda$ . Rep are calibrated, i.e.  $Y_j$ 's act by constants controlled by content.

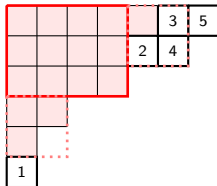
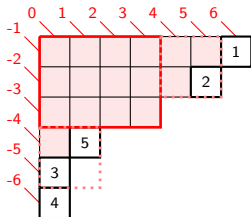


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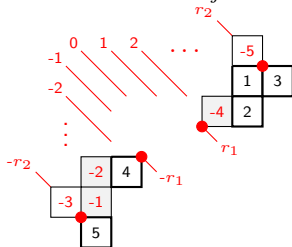
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## Central characters

The Hecke algebra  $\mathcal{H}_k$  features invertible, pairwise commuting elements  $Y_1, \dots, Y_k$  (weight lattice part).

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The Weyl group  $\mathcal{W}$  of type C (the group of signed permutations) acts on  $\mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]$  by permuting the subscripts, with  $Y_{-i} = Y_i^{-1}$ . Then the center of  $\mathcal{H}_k$  is symmetric Laurent polynomials

$$Z(\mathcal{H}_k) = \mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]^{\mathcal{W}}.$$

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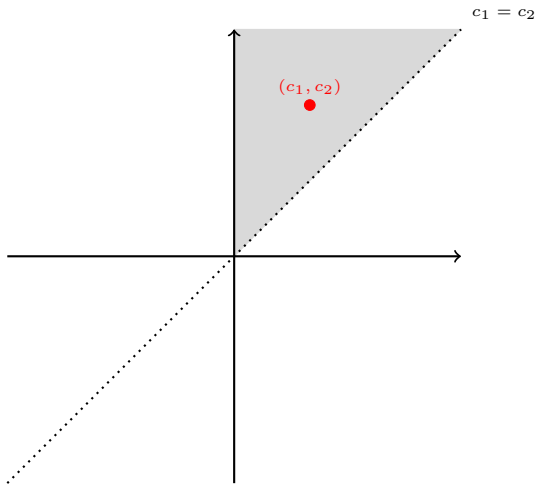
(when  $\mathbf{c}$  is real, favorite representatives satisfy  $0 \leq c_1 \leq \dots \leq c_k$ .)

## Central characters as points

Restrict to real points.

Fav equivalence class reps:  $0 \leq c_1 \leq \cdots \leq c_k$ .

When  $k = 2$ :

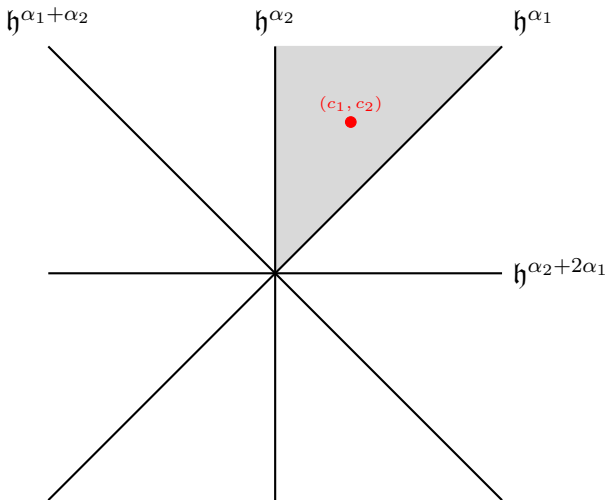


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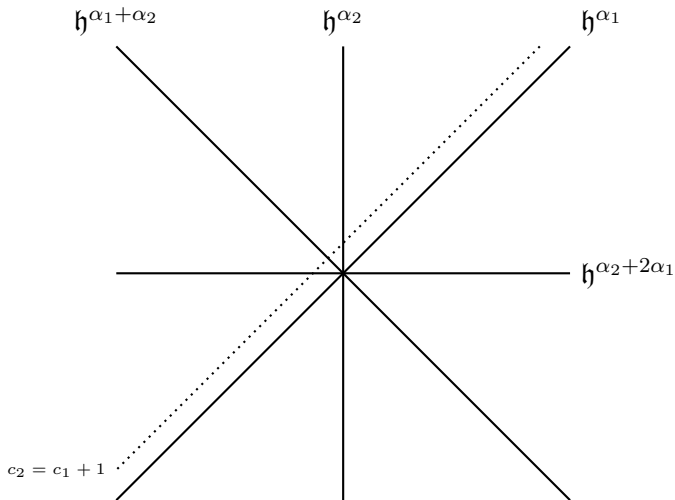


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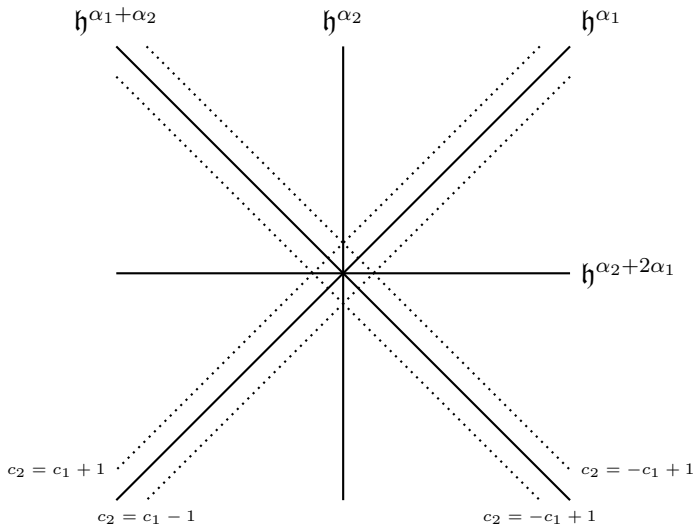


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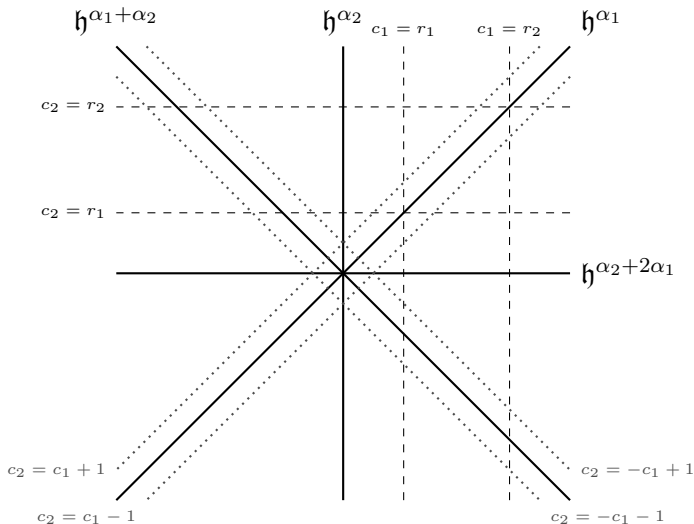


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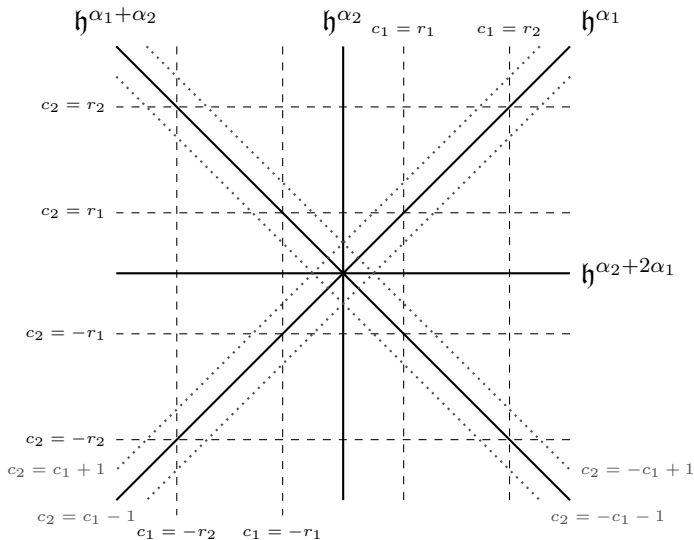
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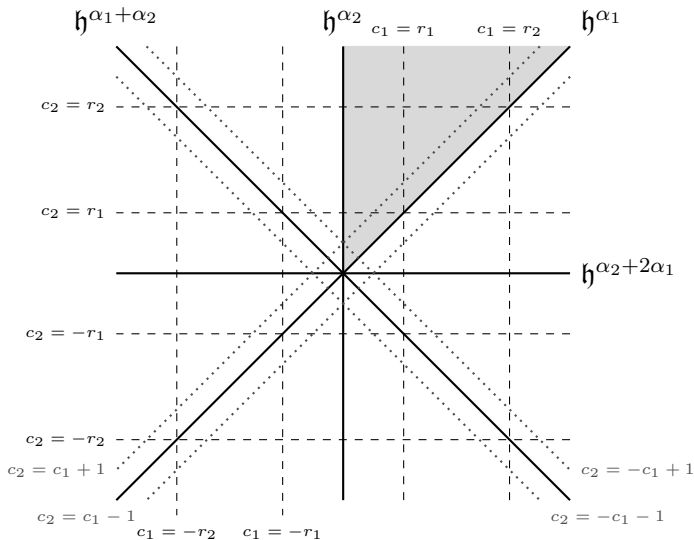
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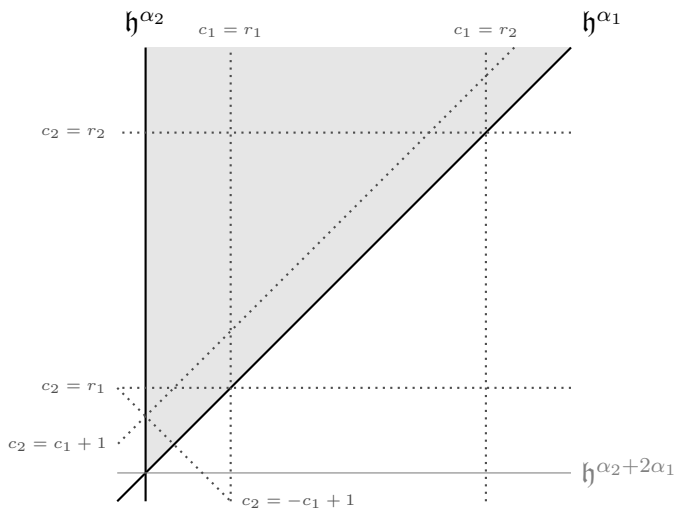
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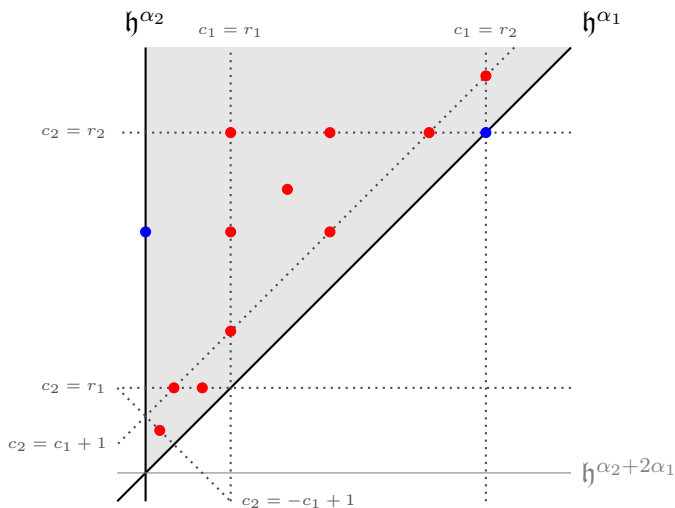
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 Calibrated reps as “skew local regions”



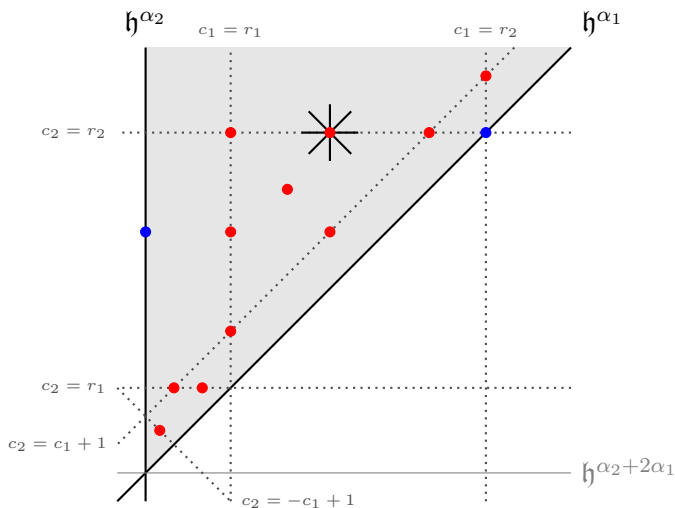
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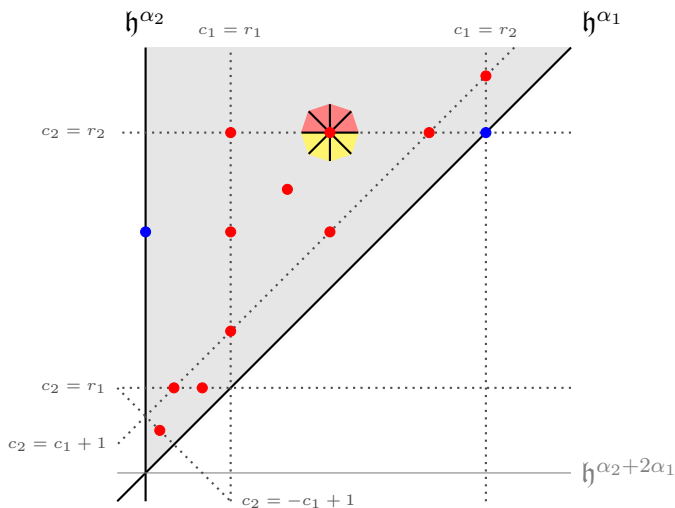
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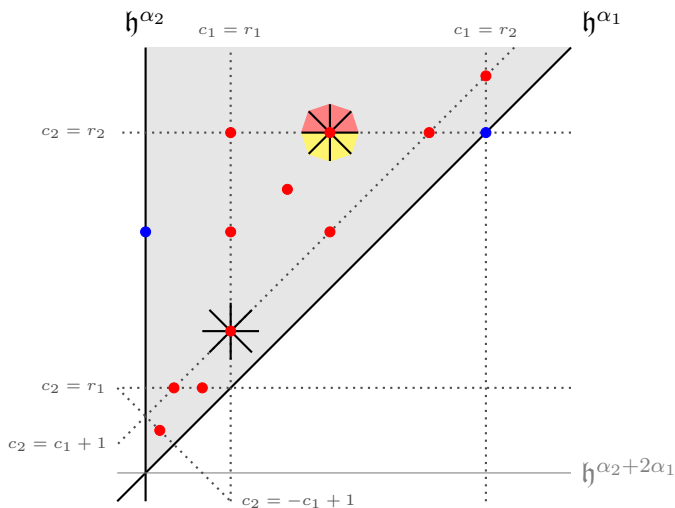
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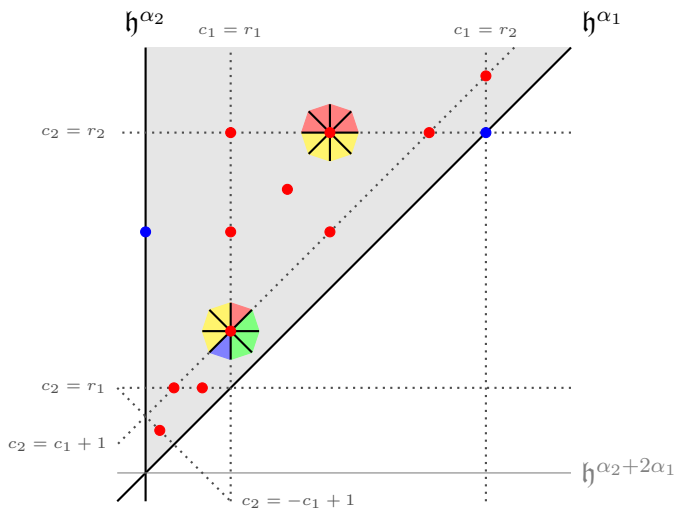
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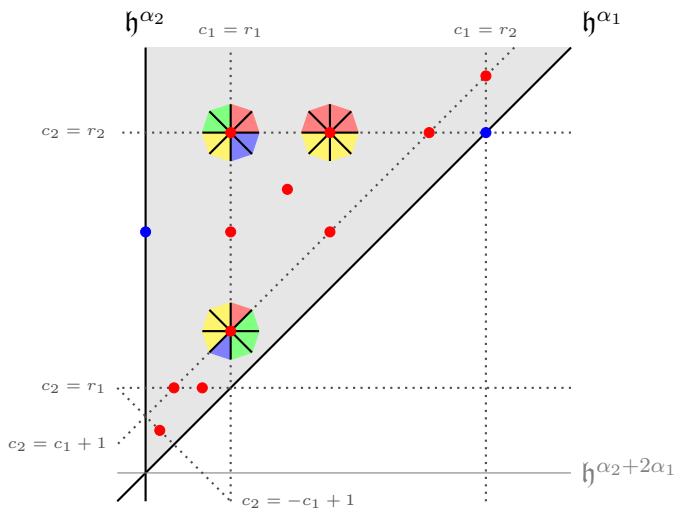


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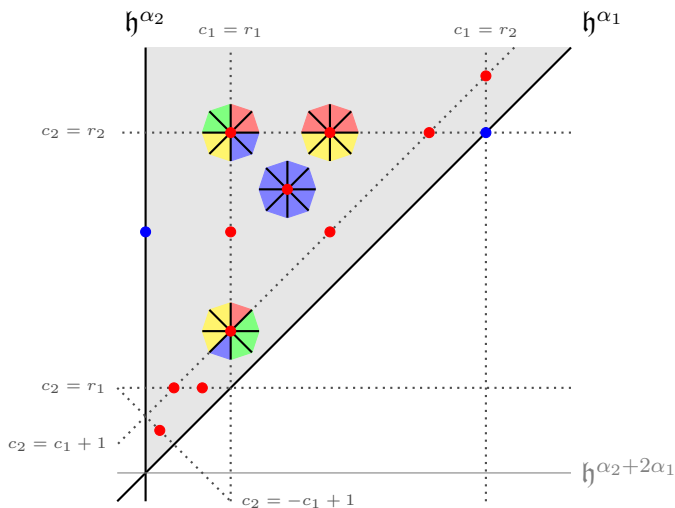
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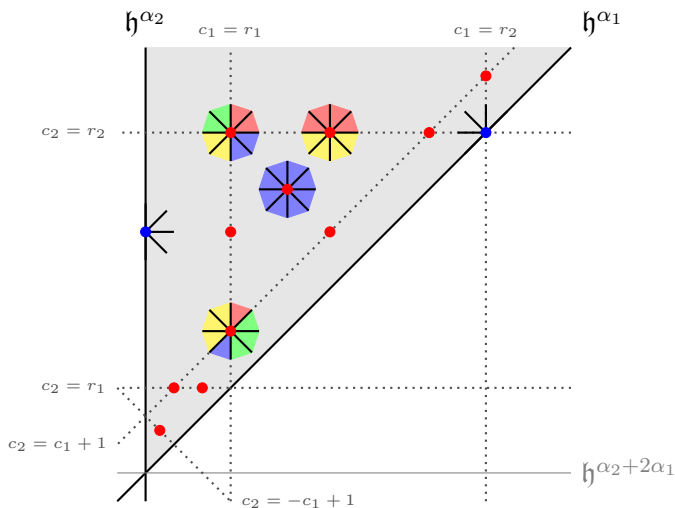
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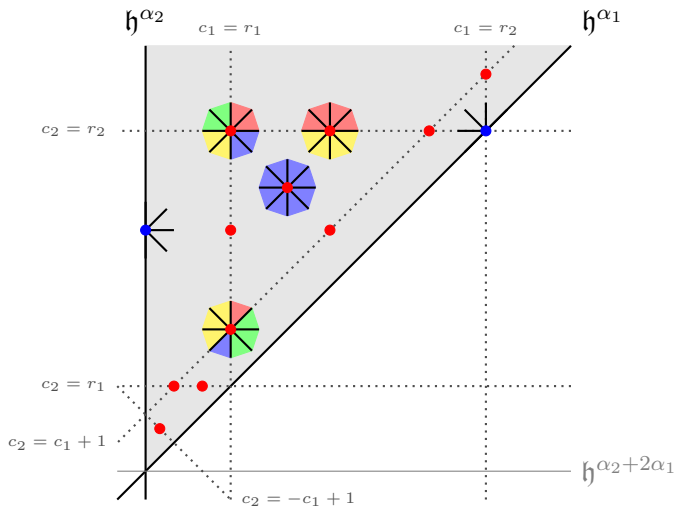


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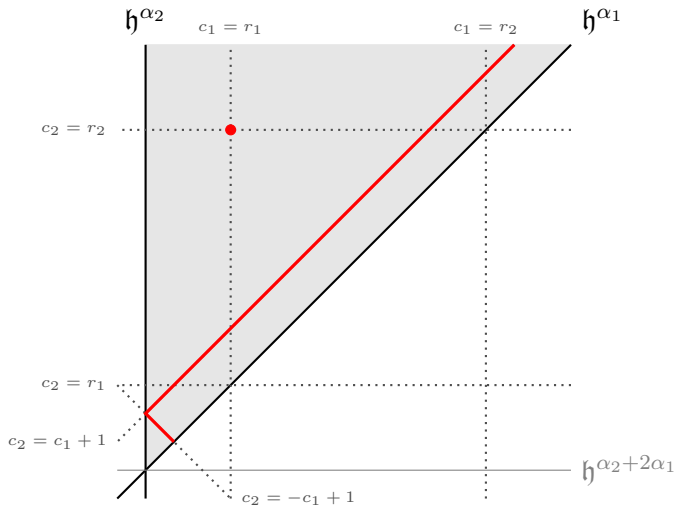


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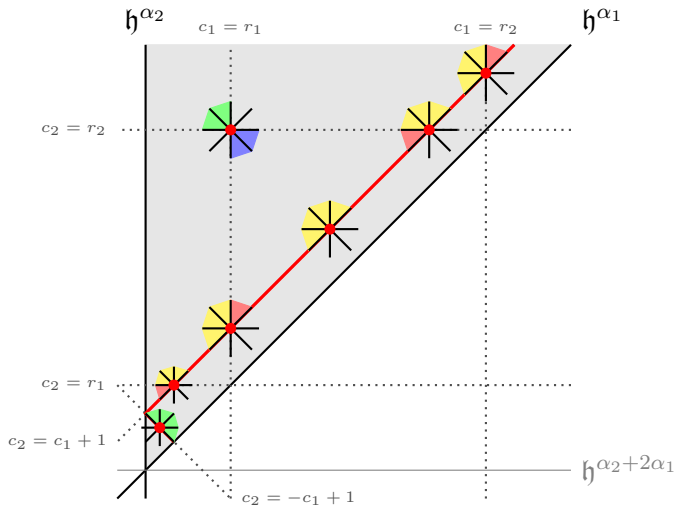
Thm. (D.-Ram)

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