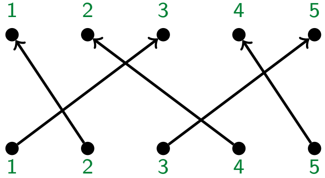
Tensor spaces, braid groups, and some remarkable quotients.

Zajj Daugherty

April 10, 2014

Motivating example: Schur-Weyl Duality

The **symmetric group** S_k (permutations) as diagrams:



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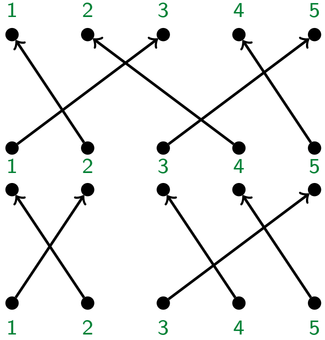

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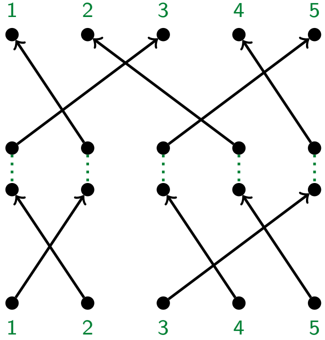

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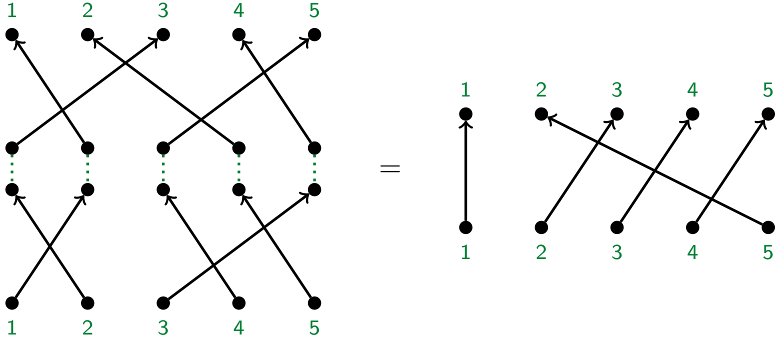

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$$g \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_k) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_k.$$

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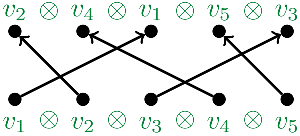

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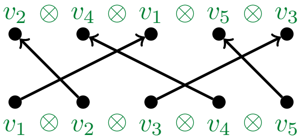
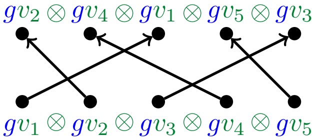
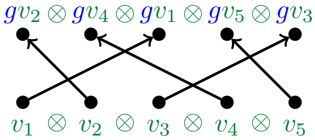


These actions commute!



vs.



Motivating example: Schur-Weyl Duality

Schur (1901): S_k and GL_n have commuting actions on $(\mathbb{C}^n)^{\otimes k}$.

Even better,

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Why this is exciting:

The double-centralizer relationship produces

$$(\mathbb{C}^n)^{\otimes k} \cong \bigoplus_{\lambda \vdash k} G^\lambda \otimes S^\lambda \quad \text{as a } \mathrm{GL}_n\text{-}S_k \text{ bimodule,}$$

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For example,

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More centralizer algebras

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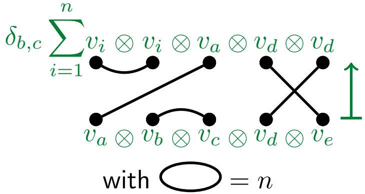
Orthogonal and symplectic groups acting on $(\mathbb{C}^n)^{\otimes k}$ diagonally centralize the **Brauer algebra**:



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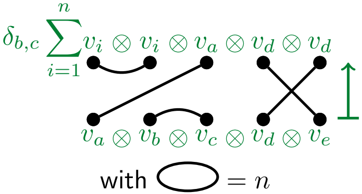

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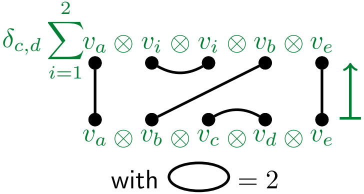

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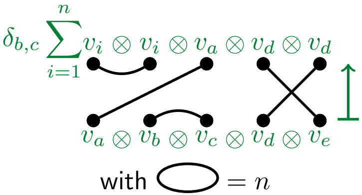

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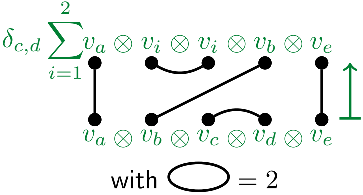

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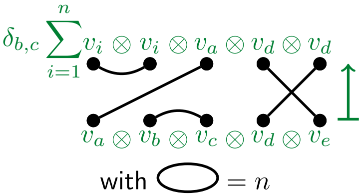

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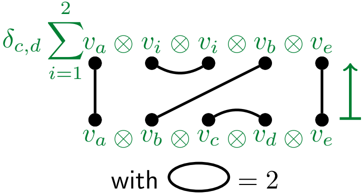

Either way:
Diagrams encoding maps $V^{\otimes k} \to V^{\otimes k}$ that commute with the action of some classical algebra.

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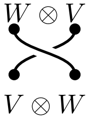
$$\check{\mathcal{R}}_{VW} \colon V \otimes W \longrightarrow W \otimes V$$



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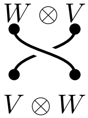

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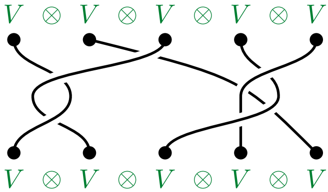

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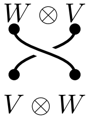

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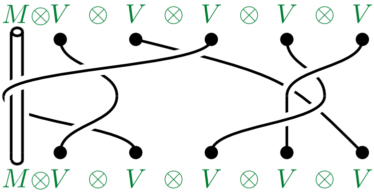
The one-pole/affine braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k}$:



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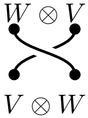

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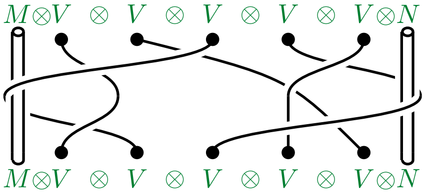
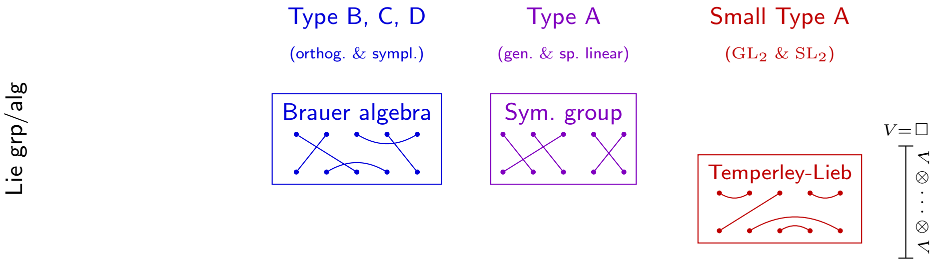
The two-pole braid group shares a commuting action with $\mathcal{U}$ on $M \otimes V^{\otimes k} \otimes N$:





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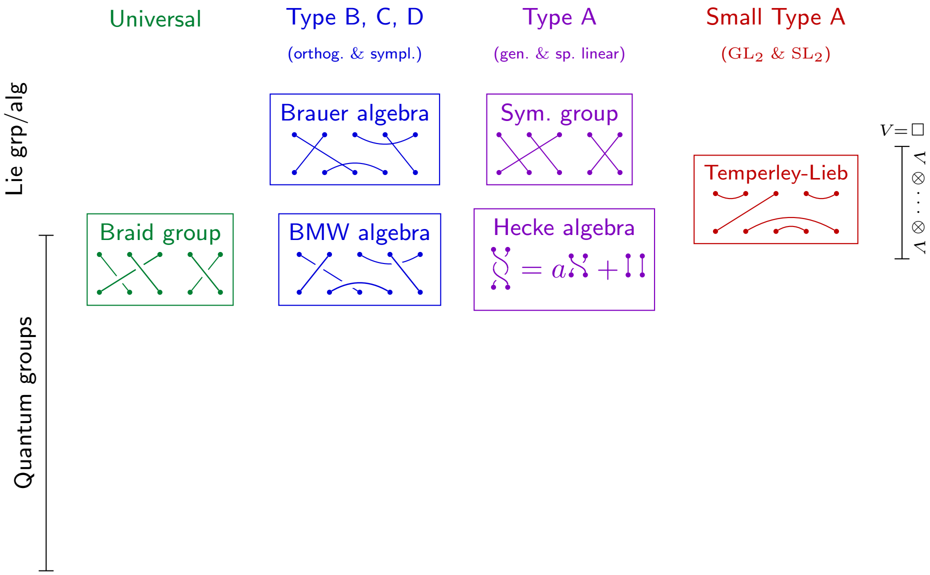
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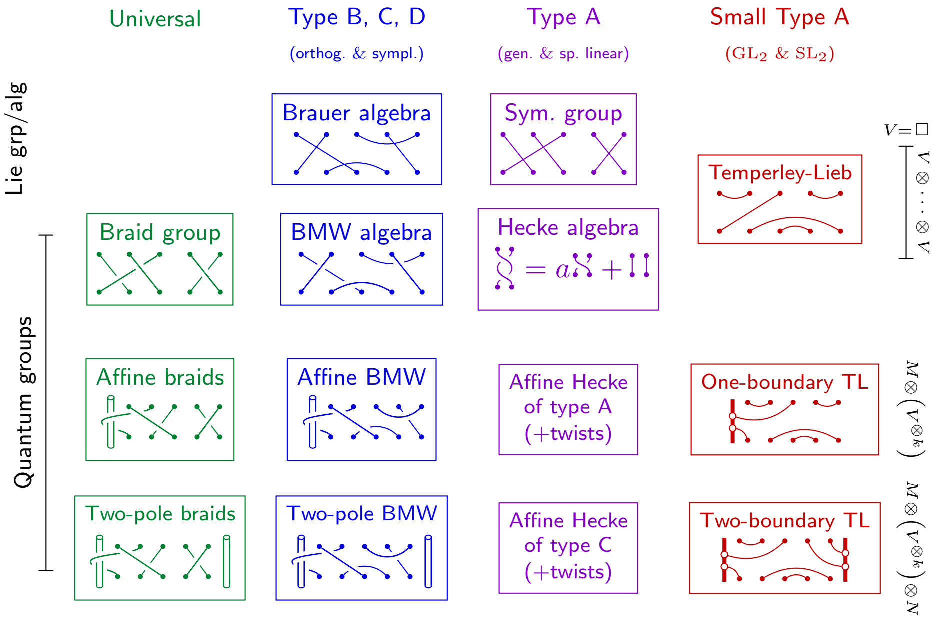

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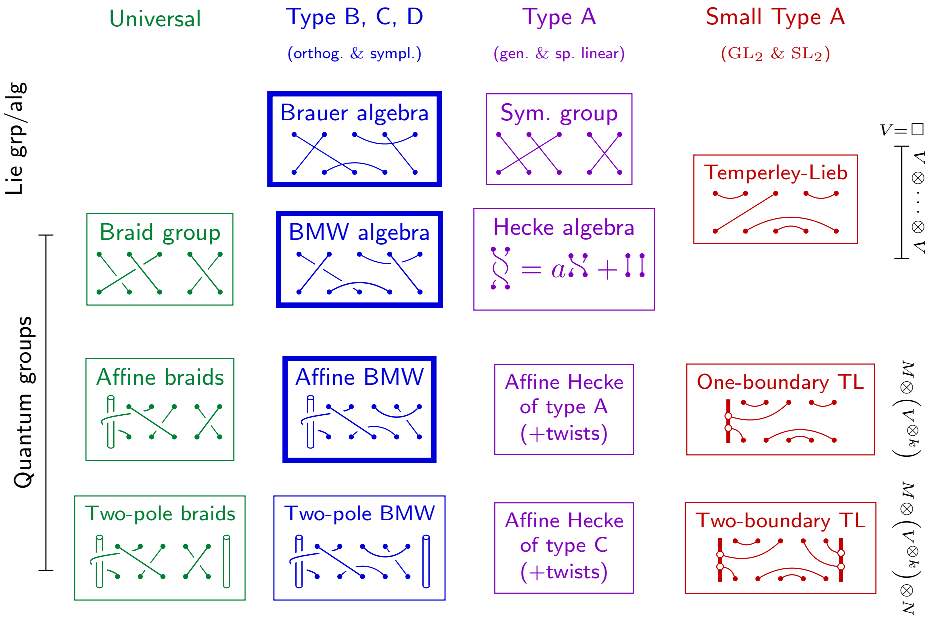
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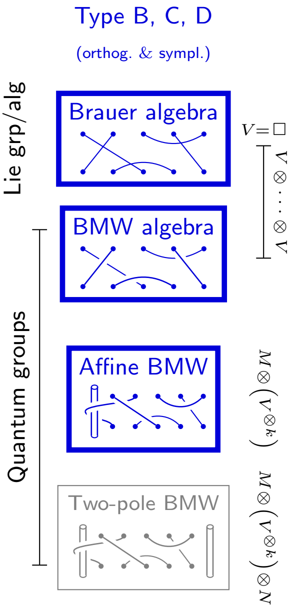

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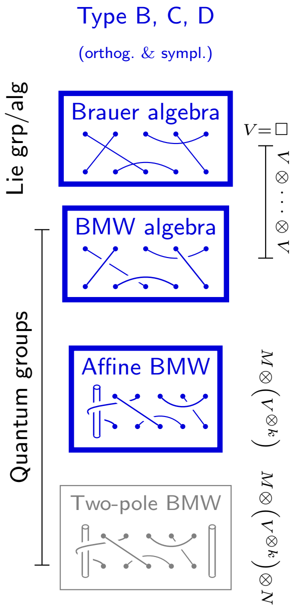

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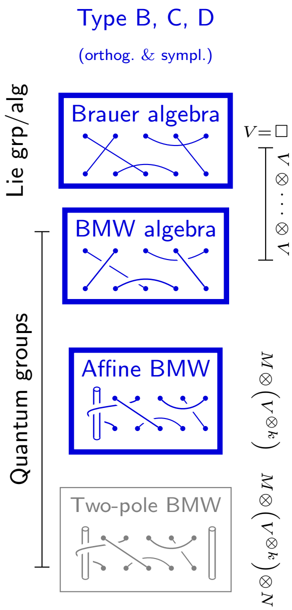

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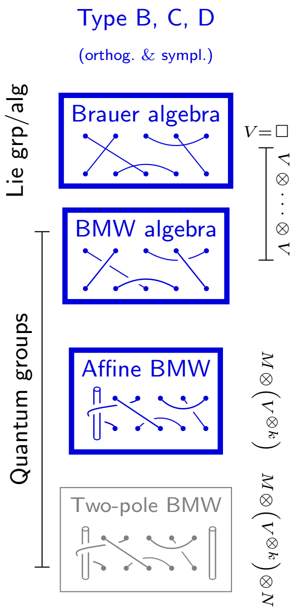

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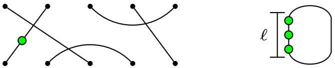

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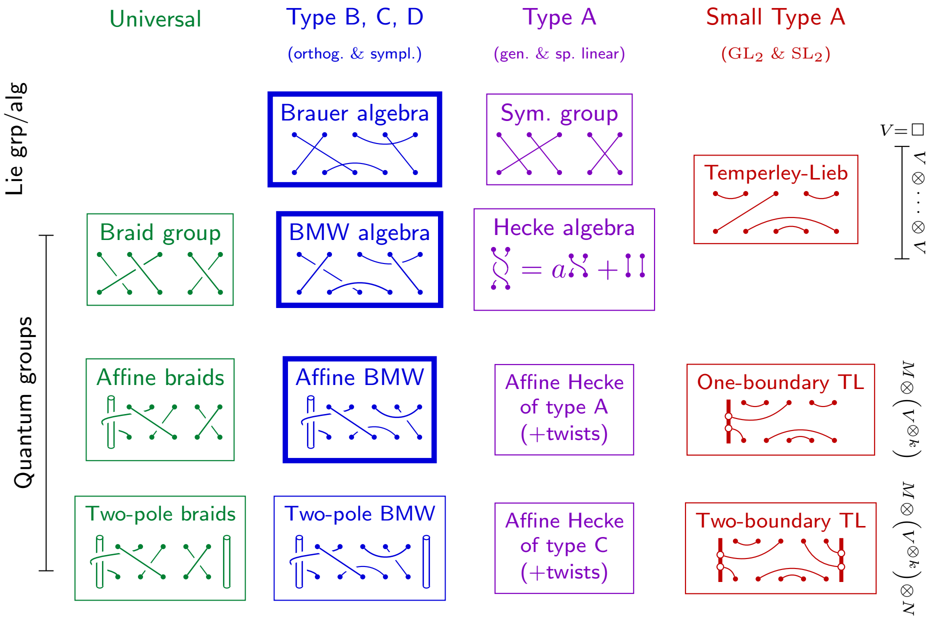
D.-Ram-Virk: Used these centralizer relationships to study these two algebras simultaneously. Some results:

(a) The center of each algebra.
(b) Difficult "admissibility conditions" handled.
(c) Powerful "intertwiner" operators.
(More to come)



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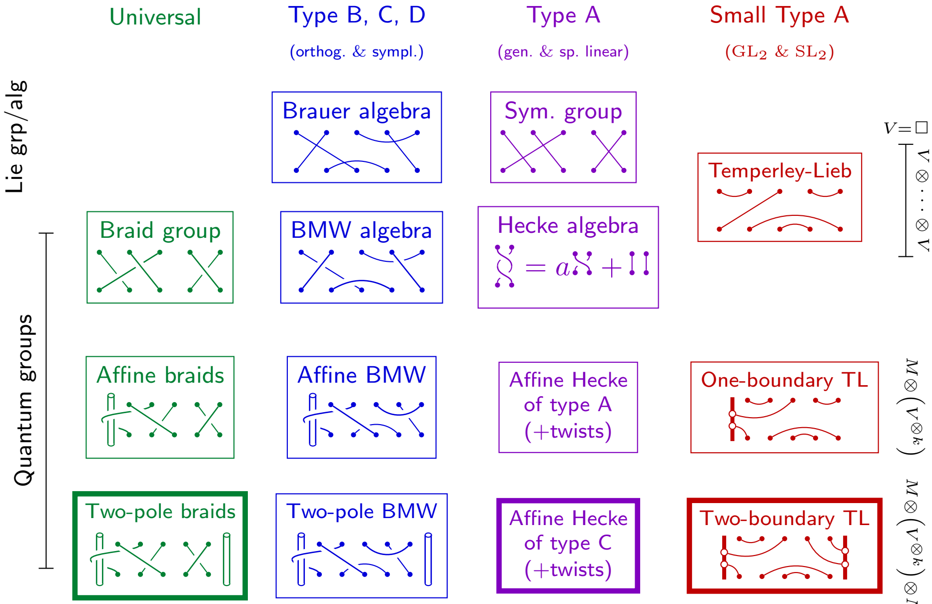

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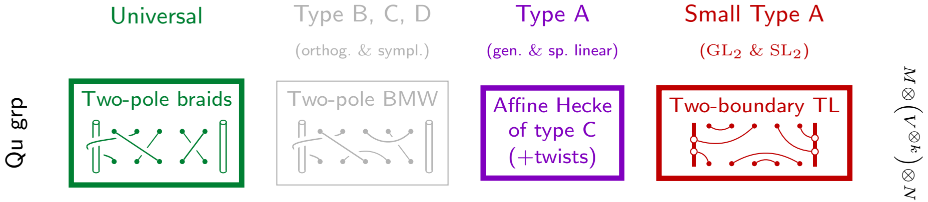
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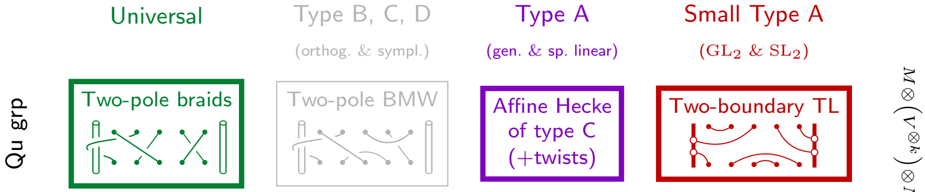
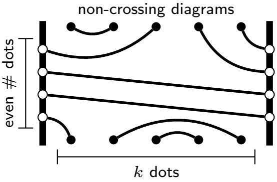
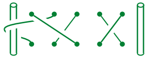


Two boundary algebras:

Nienhuis, de Gier, Batchelor (2004): Studying the six-vertex model with additional integrable boundary terms, introduced the two-boundary Temperley-Lieb algebra TL_k:



	Universal	Type B, C, D (orthog. & sympl.)	Type A (gen. & sp. linear)	Small Type A (GL_2 & SL_2)	
Qu grp	Two-pole braids 	Two-pole BMW 	Affine Hecke of type C (+twists)	Two-boundary TL 	$N \otimes (V \otimes^k) \otimes M$

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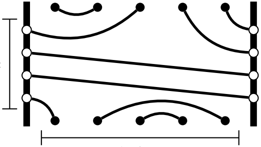

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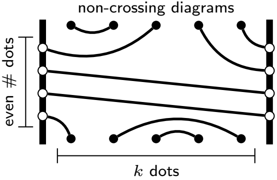

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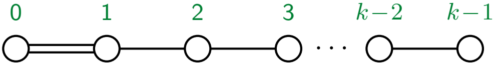
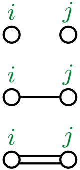
D. (2010): The centralizer of $\mathfrak{gl}_n$ acting on tensor space $M \otimes V^{\otimes k} \otimes N$ displays type C combinatorics for good choices of M, N, and V.

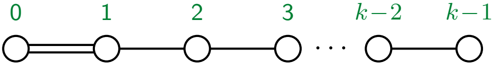
Type C Weyl group and affine Hecke algebra



$$m_{i,j} = \begin{cases} 2 & \text{if} \\ 3 & \text{if} \\ 4 & \text{if} \end{cases}$$

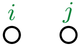


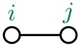
Type C Weyl group and affine Hecke algebra



The **Weyl group of type C** is generated by $s_0, \ldots, s_{k-1}$ with relations $s_i^2 = 1$ and

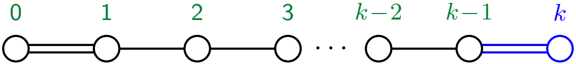
$$\underbrace{s_i s_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{s_j s_i \ldots}_{m_{i,j} \text{ factors}} \qquad \text{where} \qquad m_{i,j} = \begin{cases} 2 & \text{if} \\ 3 & \text{if} \\ 4 & \text{if} \end{cases}$$

2 if 

3 if 

4 if 

Type C Weyl group and affine Hecke algebra



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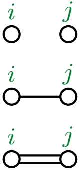
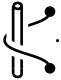
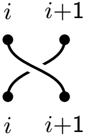


Fix constants a_0, a_k, and $a_1 = \cdots = a_{k-1}$. The **affine Hecke algebra of type C**, H_k, is generated by $T_0, T_1, \ldots, T_k$ with relations

$$T_i^2 = (a_i - a_i^{-1})T_i + 1, \qquad \underbrace{T_i T_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{T_j T_i \ldots}_{m_{i,j} \text{ factors}}.$$

Why the two-boundary braid group is type C

The **two-boundary (two-pole) braid group** B_k is generated by

$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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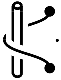
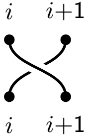
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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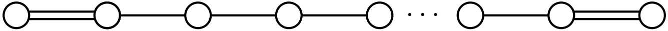

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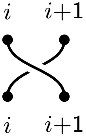
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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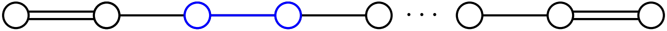
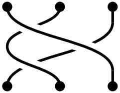
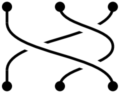


$T_i T_{i+1} T_i =$  $=$  $= T_{i+1} T_i T_{i+1}$

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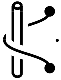
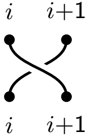
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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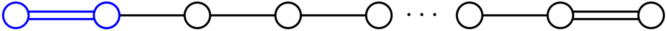
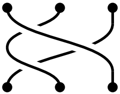
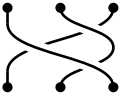
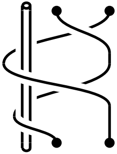
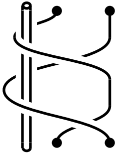


$T_iT_{i+1}T_i =$  $=$  $= T_{i+1}T_iT_{i+1}$

$T_0T_1T_0T_1 =$  $=$  $= T_1T_0T_1T_0$

(similar picture for $T_kT_{k-1}T_kT_{k-1} = T_{k-1}T_kT_{k-1}T_k$)

Theorem (D.-Ram, degenerate version in [Da10])

1. *Let $U = U_q\mathfrak{g}$ for any complex reductive Lie algebras $\mathfrak{g}$.*
   Let M, N, and V be finite-dimensional modules.

   The two-boundary braid group $\mathcal{B}_k$ acts on $M \otimes (V)^{\otimes k} \otimes N$ and this action commutes with the action of U.

2. *If $\mathfrak{g} = \mathfrak{gl}_n$, then (for appropriate choices of M, N, and V), the affine Hecke algebra of type C, H_k, acts on $M \otimes (V)^{\otimes k} \otimes N$ and this action commutes with the action of U.*

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Some results:

(a) A combinatorial classification and construction of irreducible representations of H_k (type C with distinct parameters).

(b) A diagrammatic intuition behind otherwise unwieldy calculations in TL_k and H_k.

(c) A classification of the representations of TL_k in [dGN08] via central characters, including answers to open questions and conjectures regarding their irreducibility and isomorphism classes.

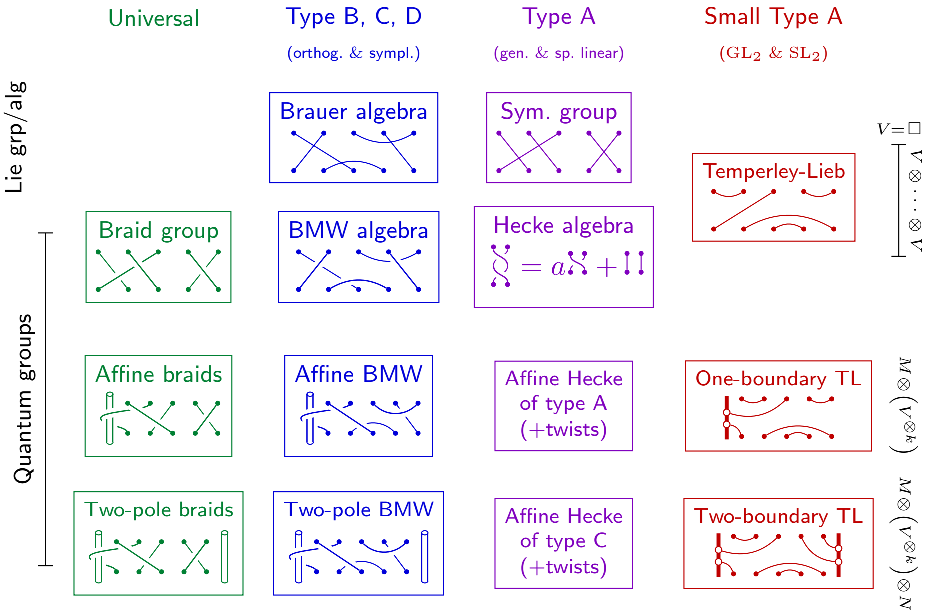
Thanks!

[Da10] *Degenerate two-boundary centralizer algebras*, Pacific J. Math., 258-1 (2012) 91–142.

[DRV14] *Affine and degenerate affine BMW algebras: the center*, with Arun Ram and Rahbar Virk, to appear in to appear in Osaka J. Math., 51-1 (2014).

[DRV13] *Affine and degenerate affine BMW algebras: actions on tensor space*, with Arun Ram and Rahbar Virk, Selecta Math., 19-2 (2013) 611–653.

[DR] *Two boundary Hecke Algebras and the combinatorics of type* $(C_n^\vee, C)$ *Hecke algebras*, with Arun Ram (in progress).


Universal
Type B, C, D
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Lie grp/alg
Quantum groups
Brauer algebra
Sym. group
$V = \square$
Temperley-Lieb
$V \otimes \cdots \otimes V$
Braid group
BMW algebra
Hecke algebra
$= a + $
Affine braids
Affine BMW
Affine Hecke of type A (+twists)
One-boundary TL
$(V^{\otimes k}) \otimes M$
Two-pole braids
Two-pole BMW
Affine Hecke of type C (+twists)
Two-boundary TL
$N \otimes (V^{\otimes k}) \otimes M$