Combinatorics of the two-boundary Hecke algebra.

Zajj Daugherty
(joint with Arun Ram)

April 30, 2013

The two-boundary Hecke algebra H_k is generated by invertible elements $T_0, T_1, \ldots, T_k$ with relations



$$(T_0 - t_0)(T_0 - t_0^{-1}) = 0 = (T_k - t_k)(T_k - t_k^{-1})$$

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Goal today:

Tell you 3 descriptions of calibrated irreducible reps of H_k, where "calibrated" means $\mathbb{C}[Y_1^{\pm 1}, \ldots, Y_k^{\pm 1}]$ is simultaneously diagonalized.

Central characters

The center of H_k is symmetric Laurent polynomials

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w.r.t. the Weyl group W_0 of type C.

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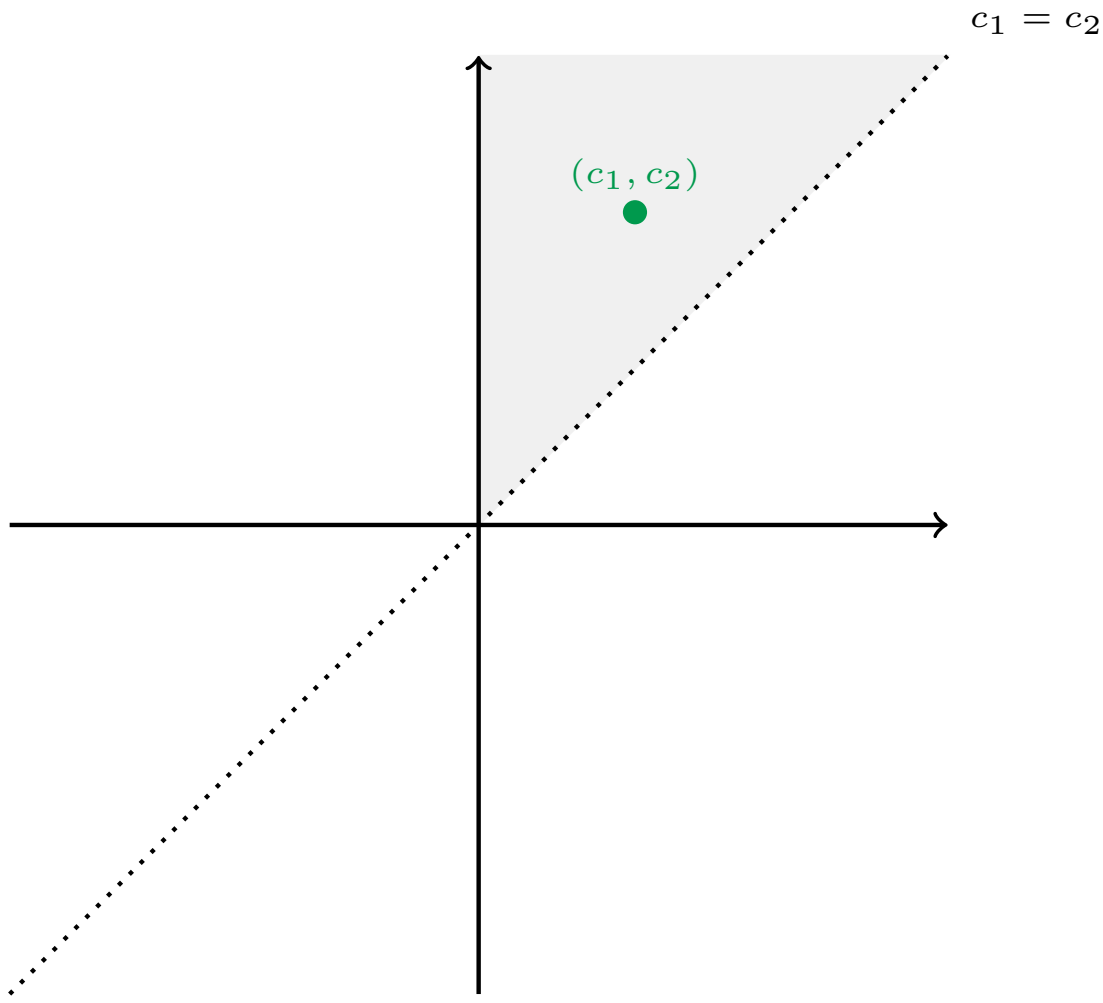
Central characters as points

Restrict to real points.

Fav equivalence class reps: $0 \leq c_1 \leq \cdots \leq c_k$. ($W_0$ acts by signed permutations)
When $k = 2$:



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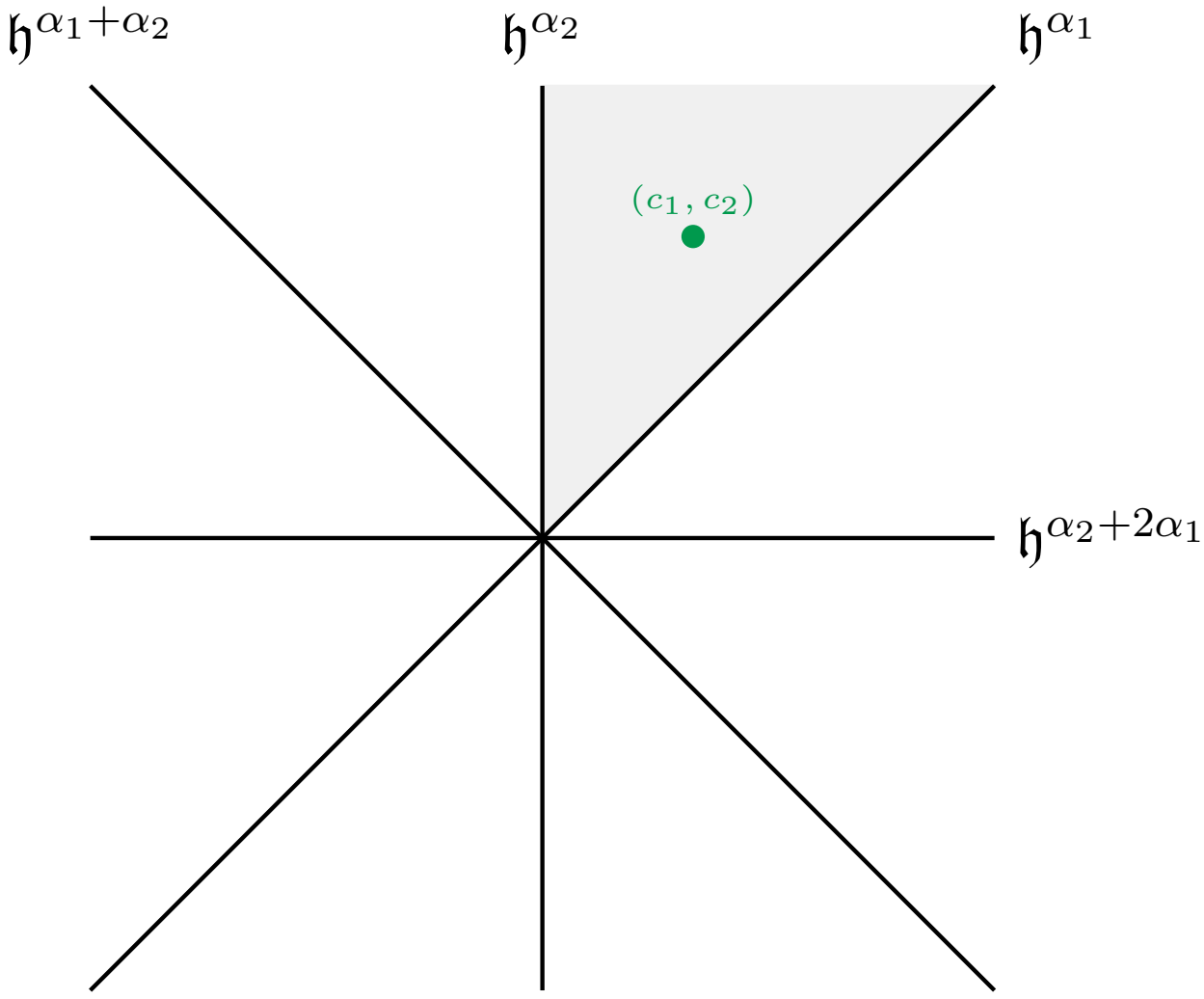

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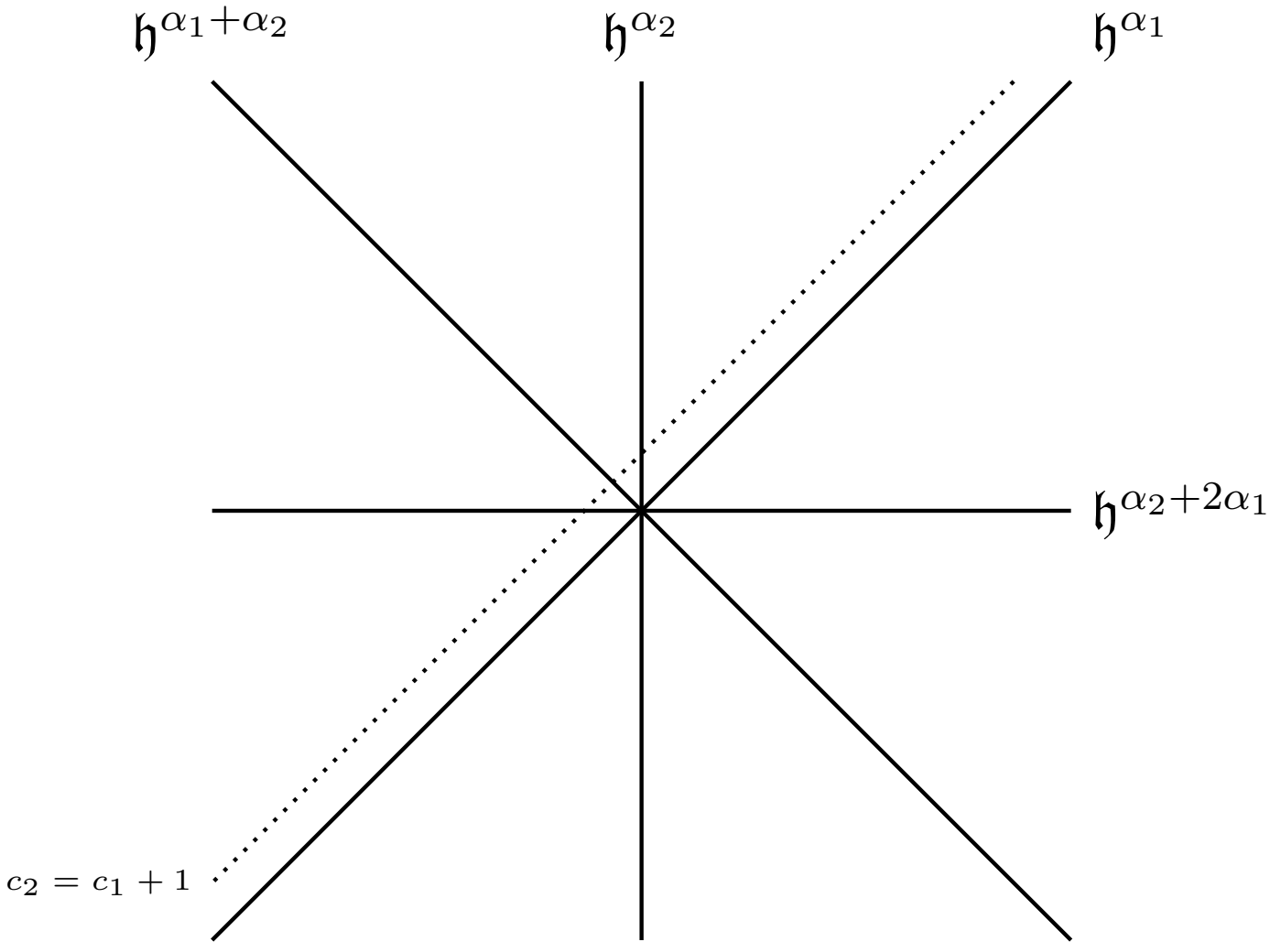

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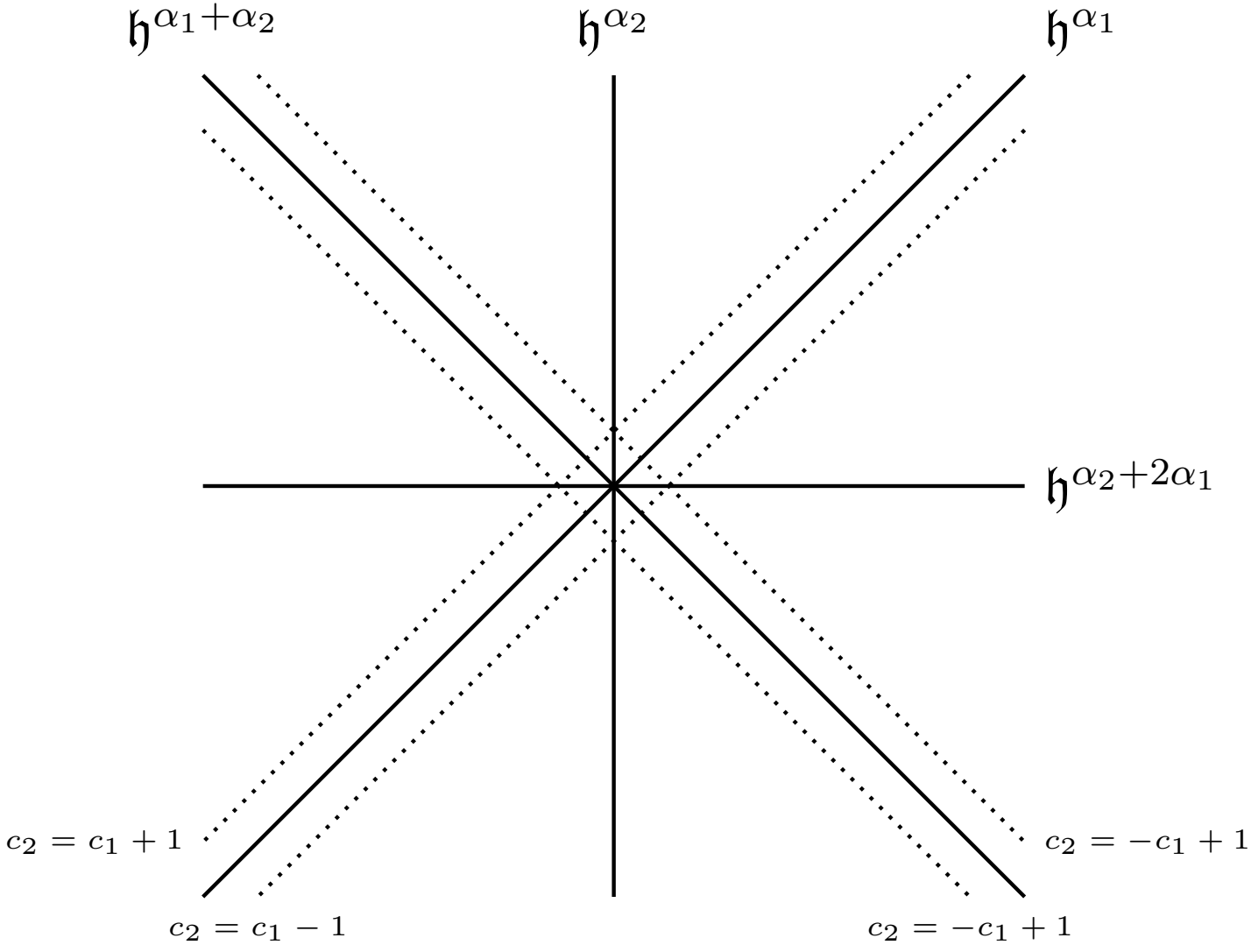

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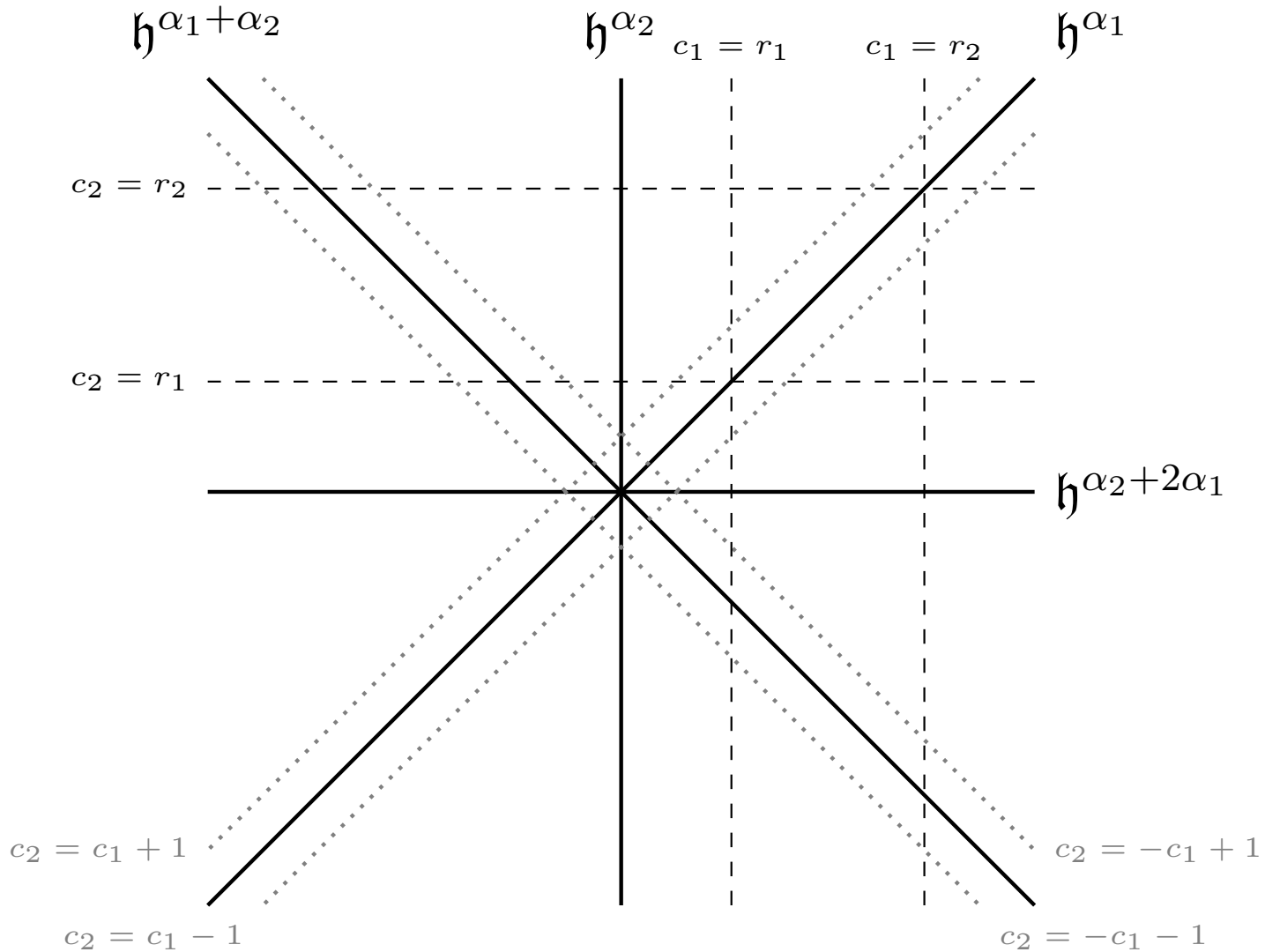

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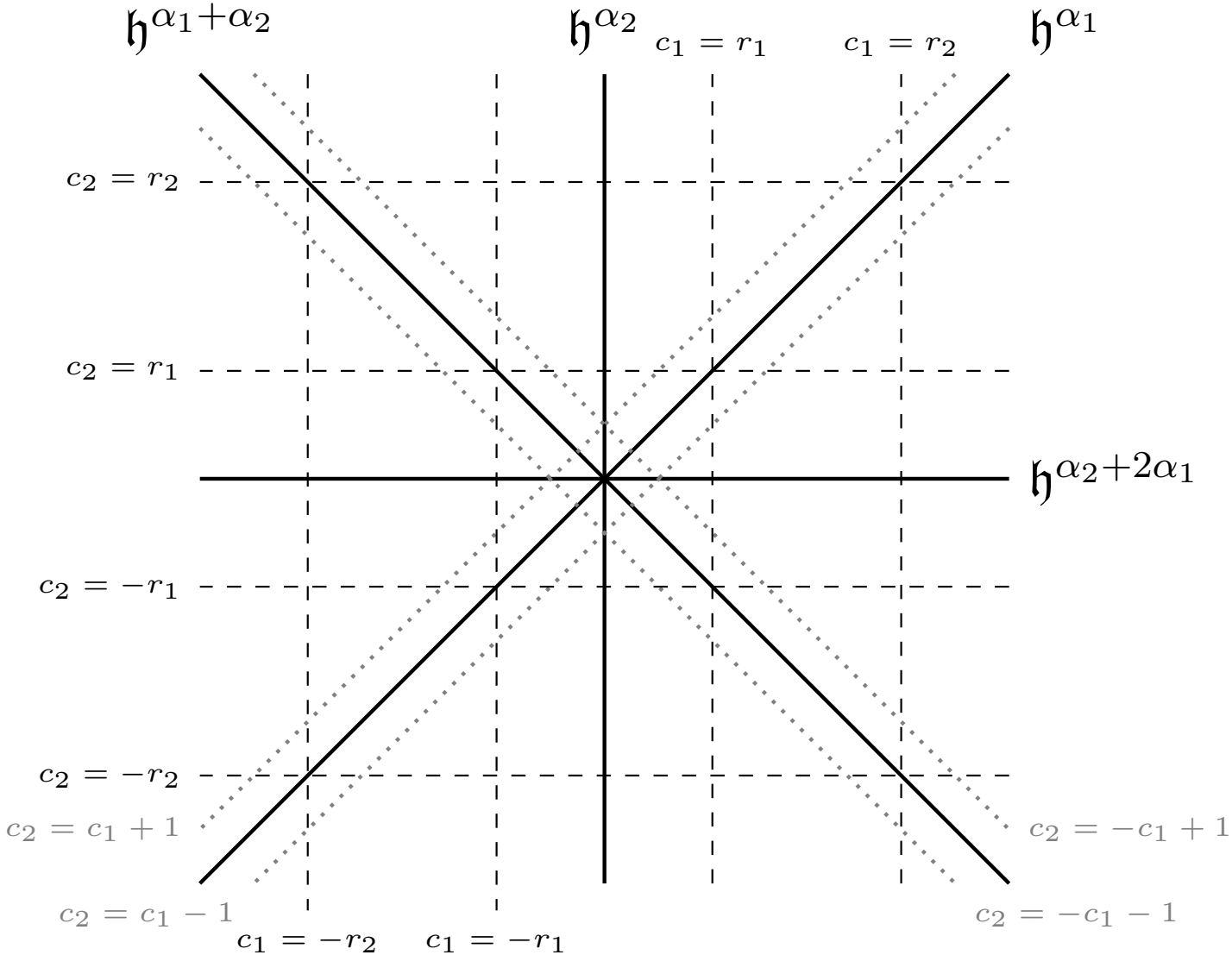

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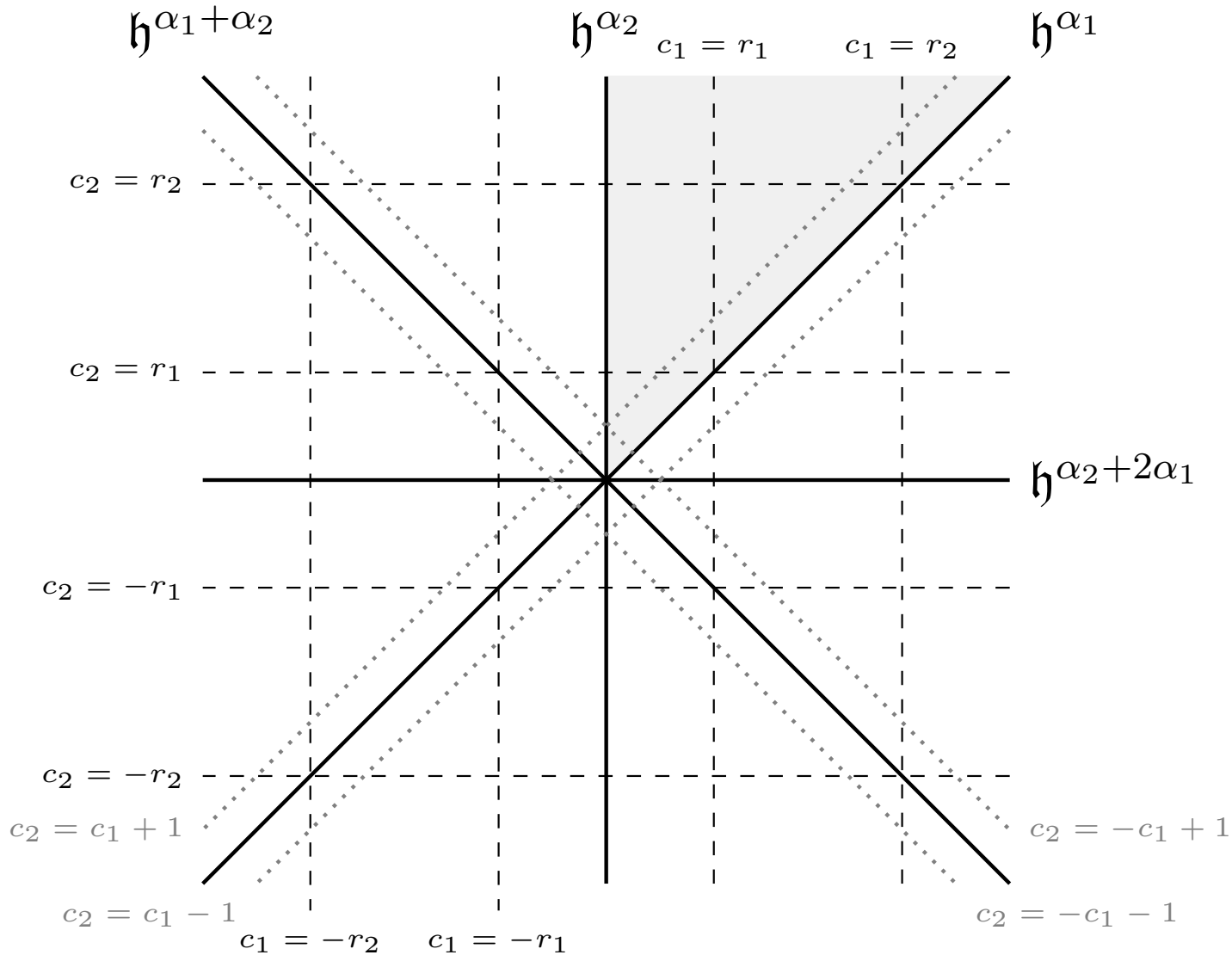

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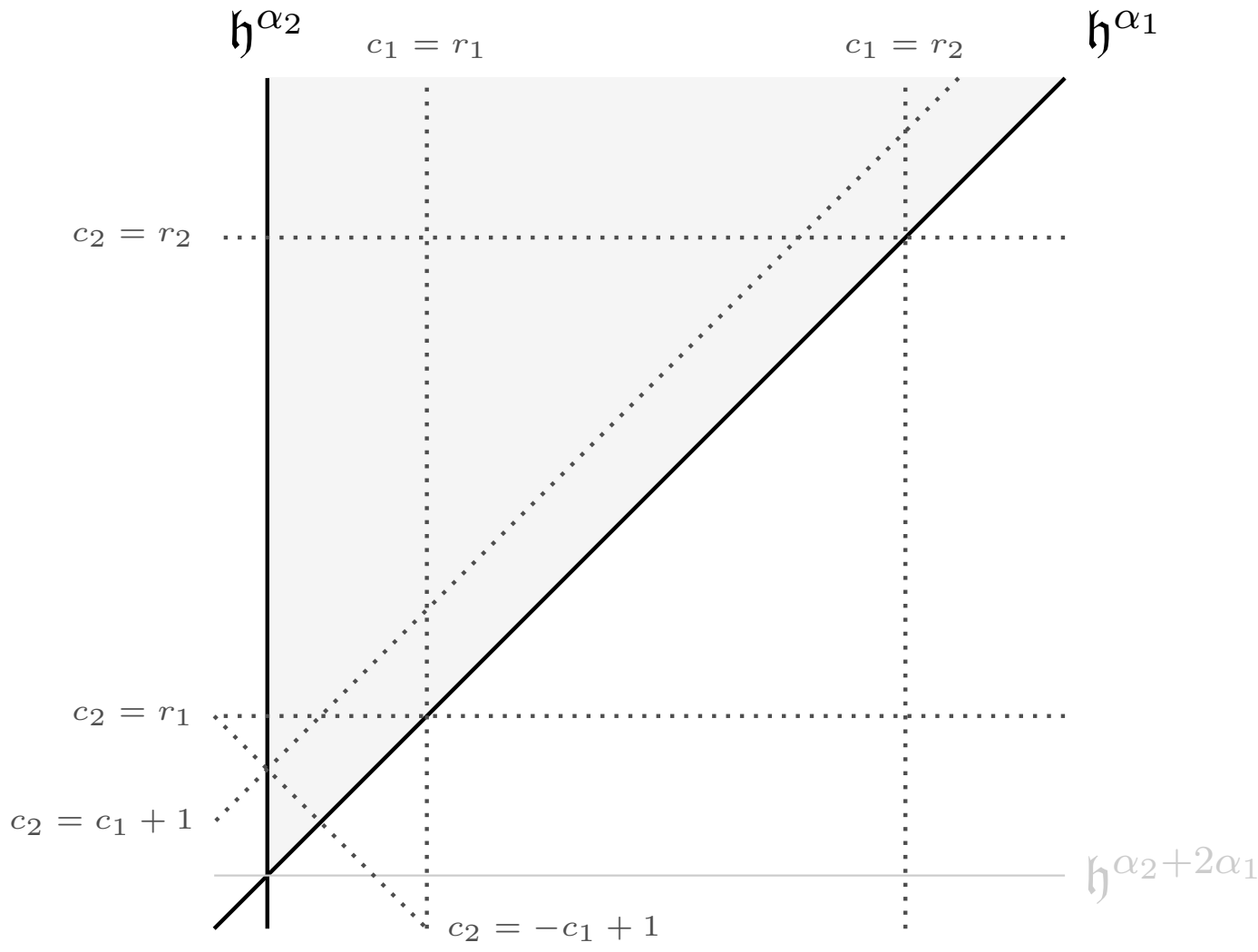

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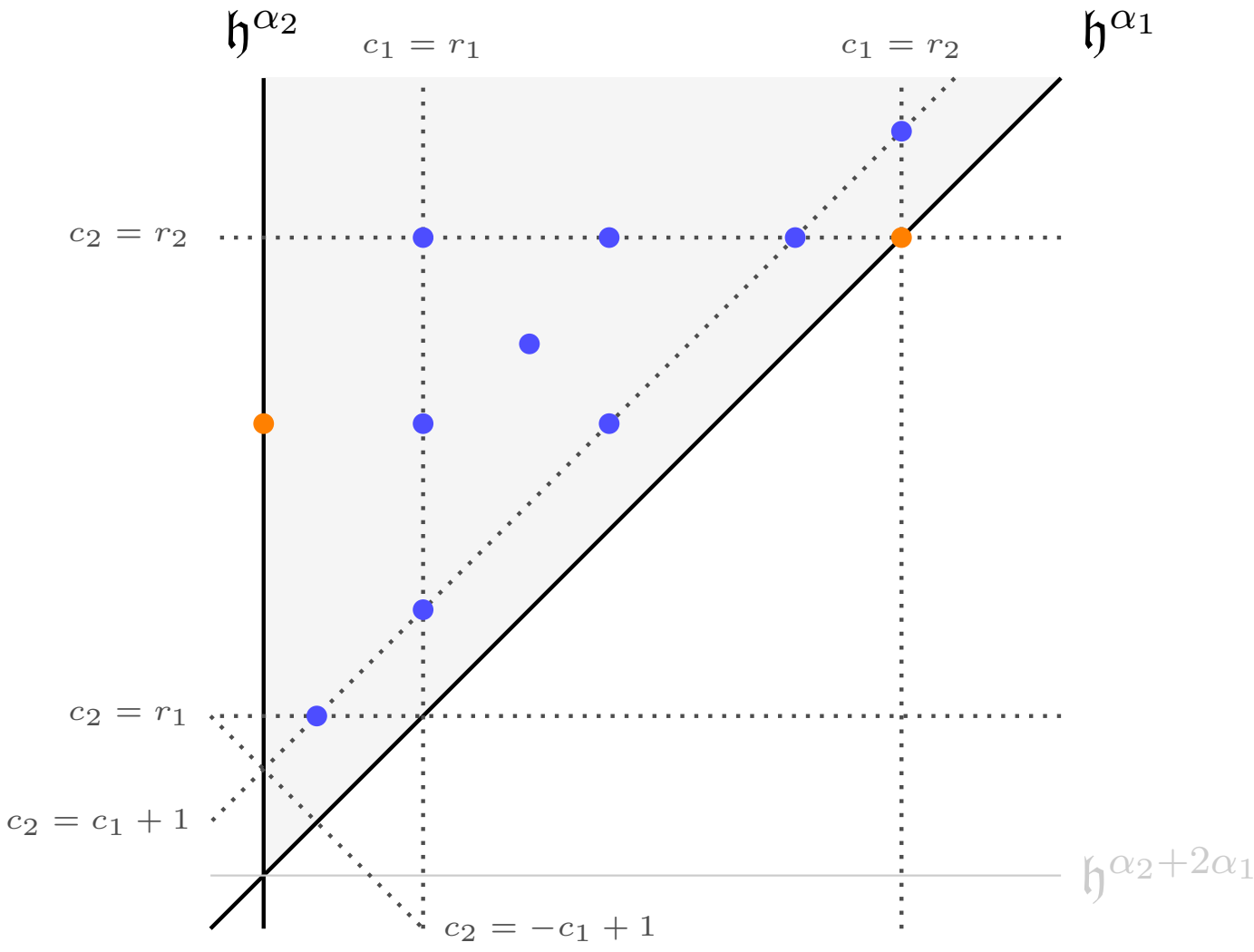

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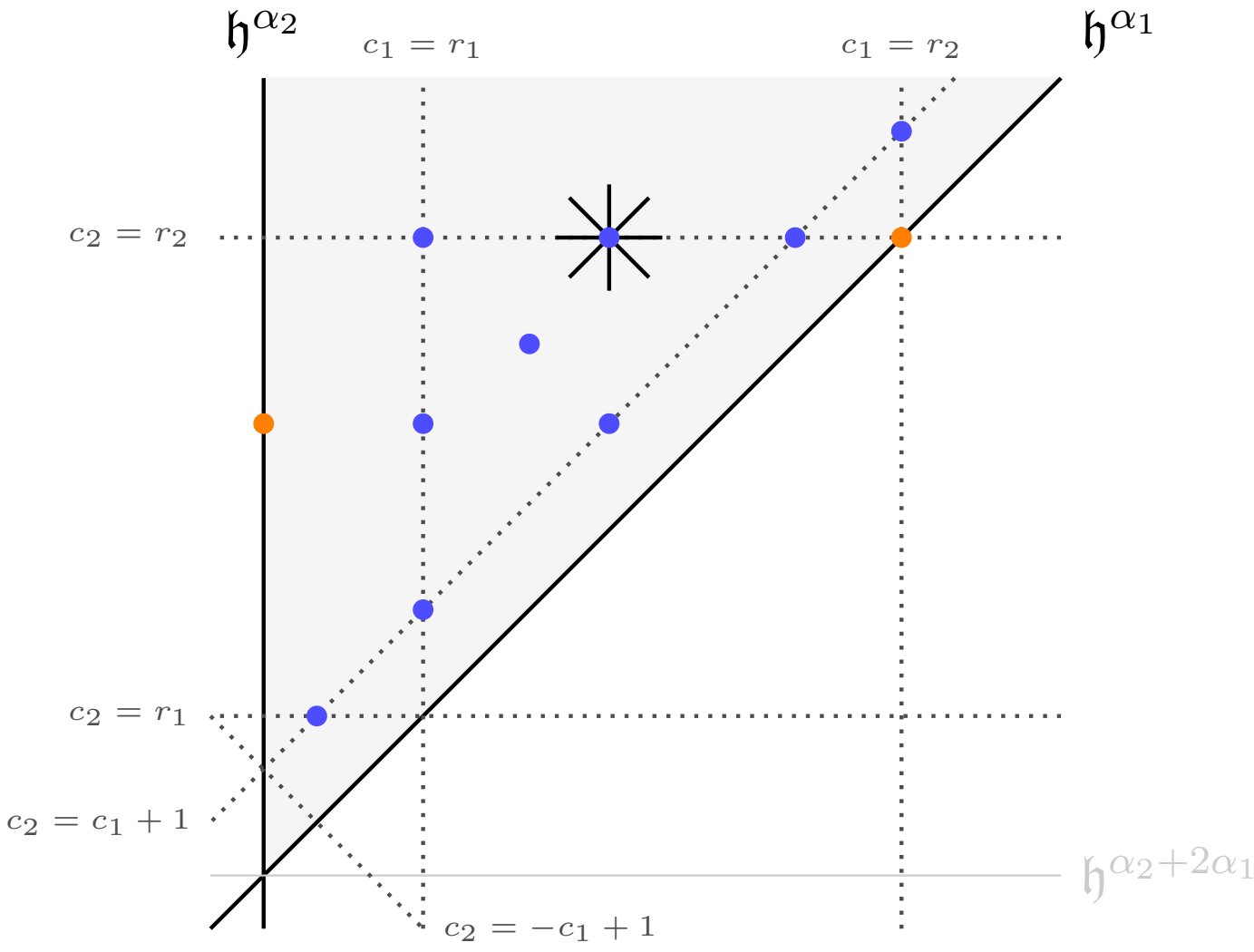

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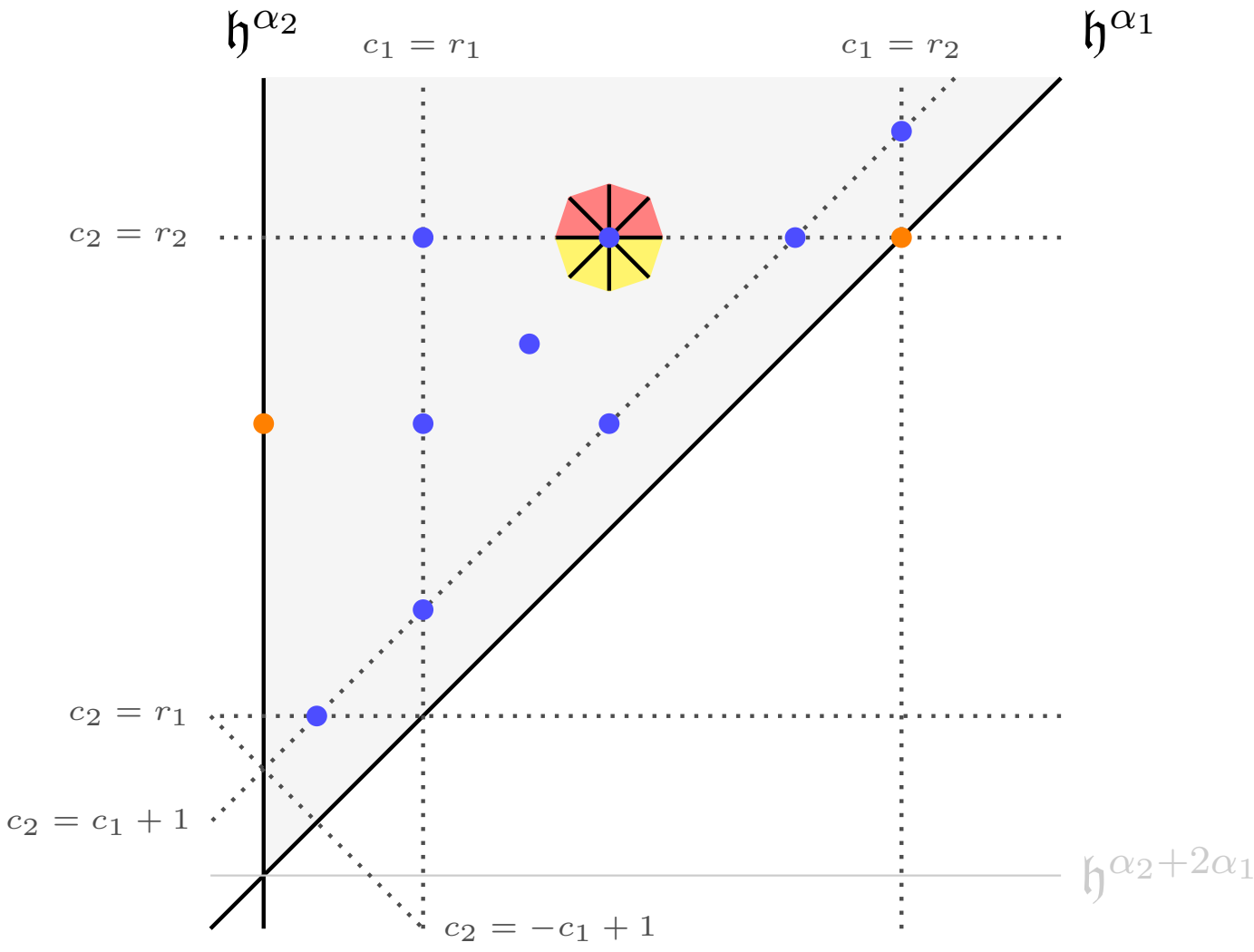

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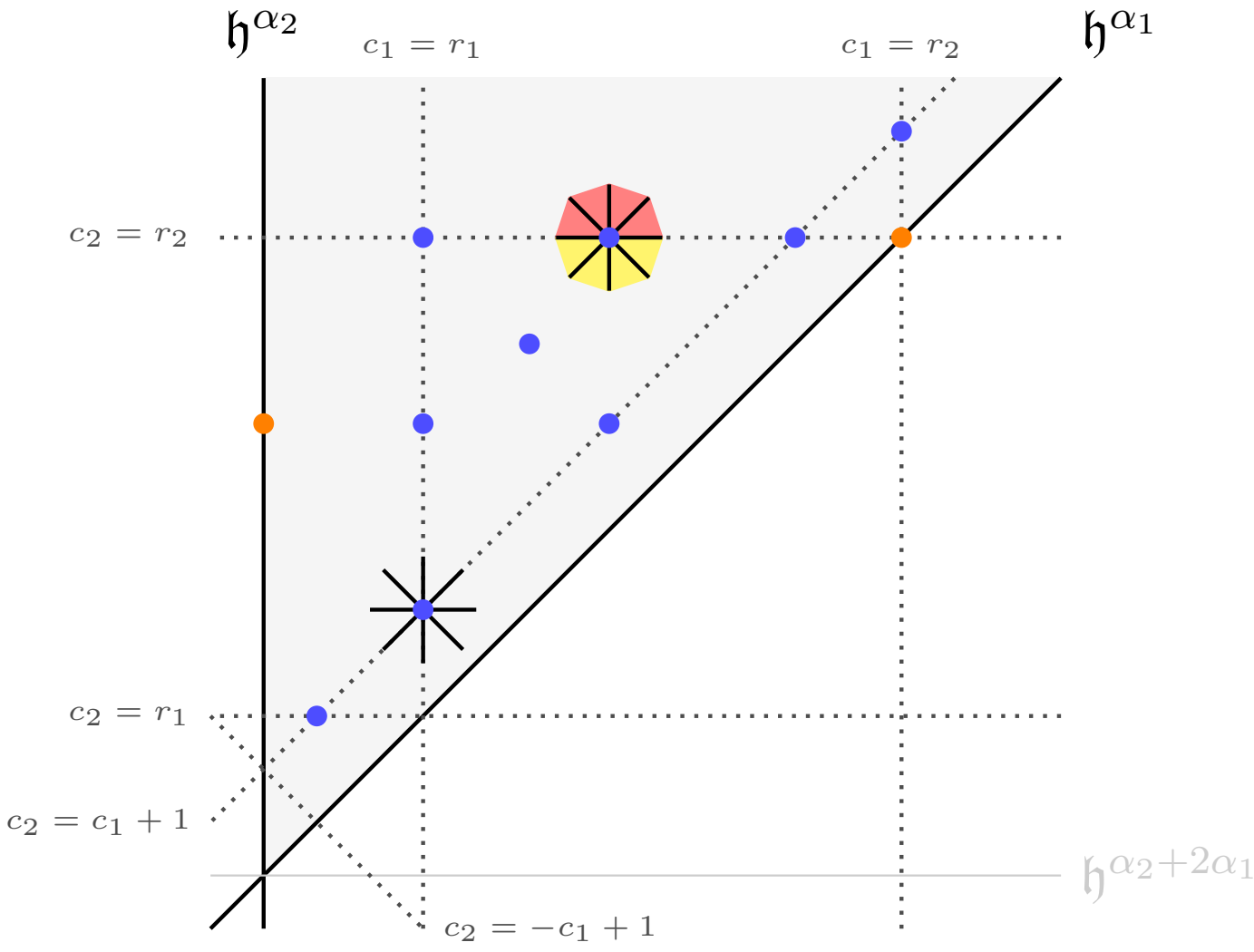

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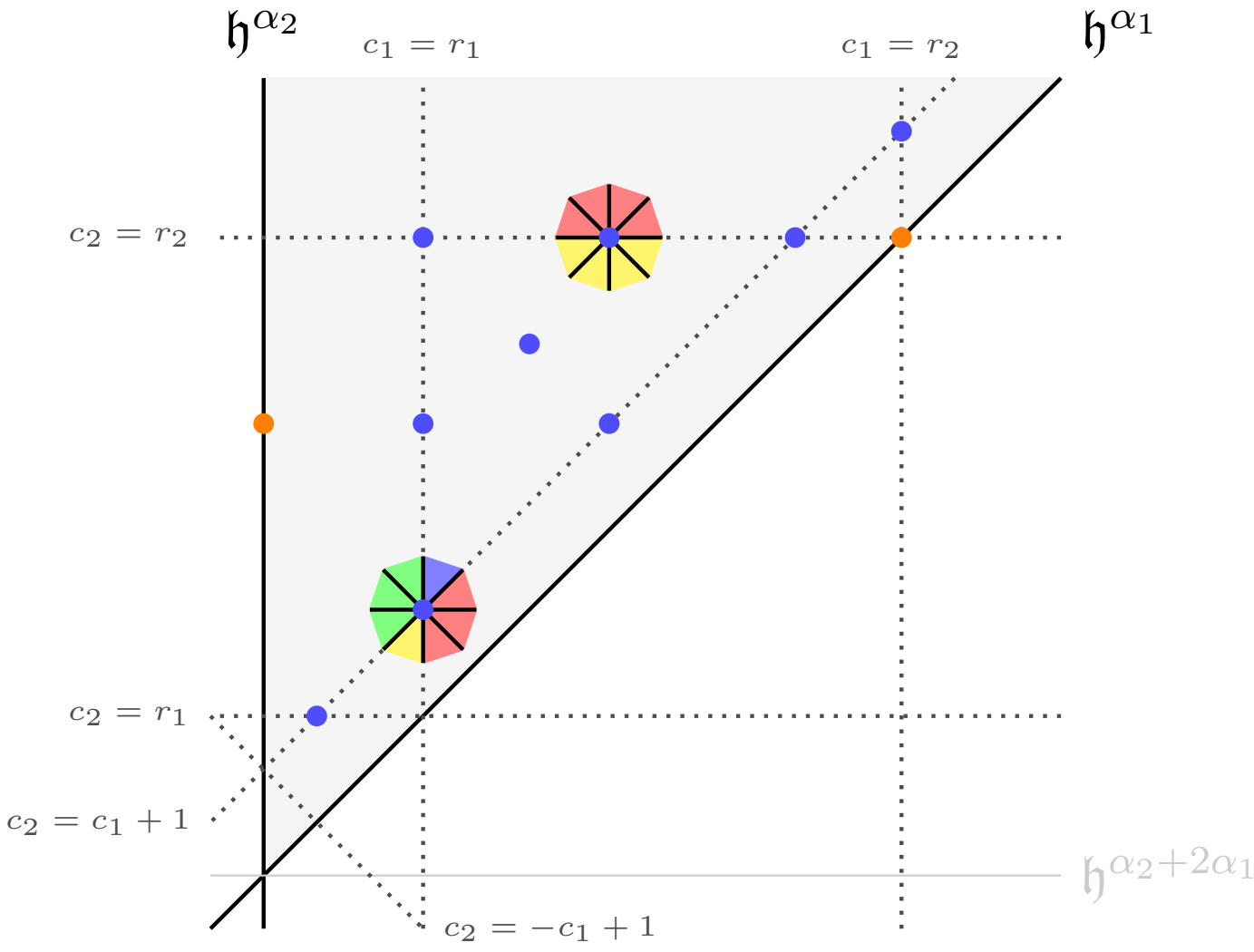

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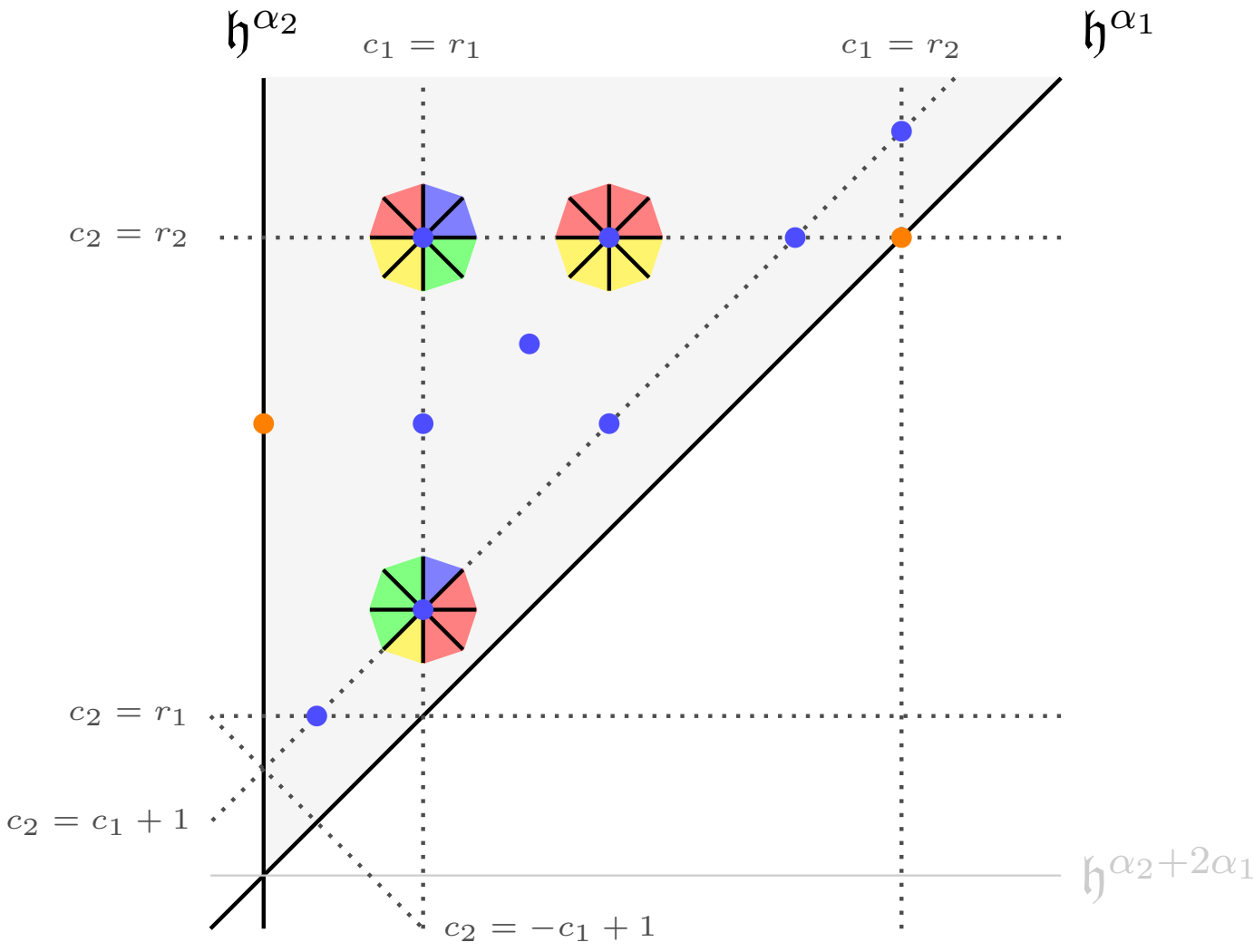

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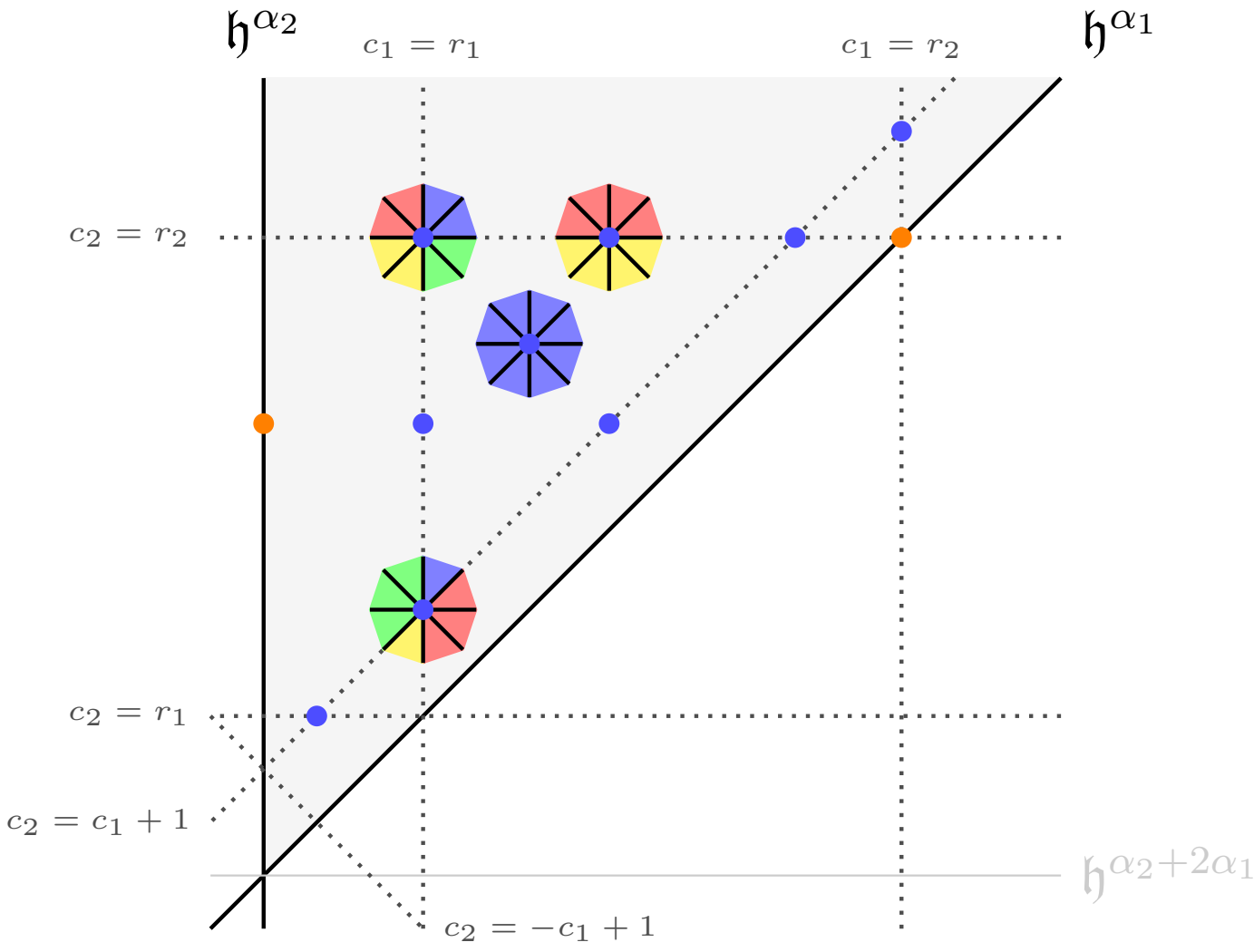

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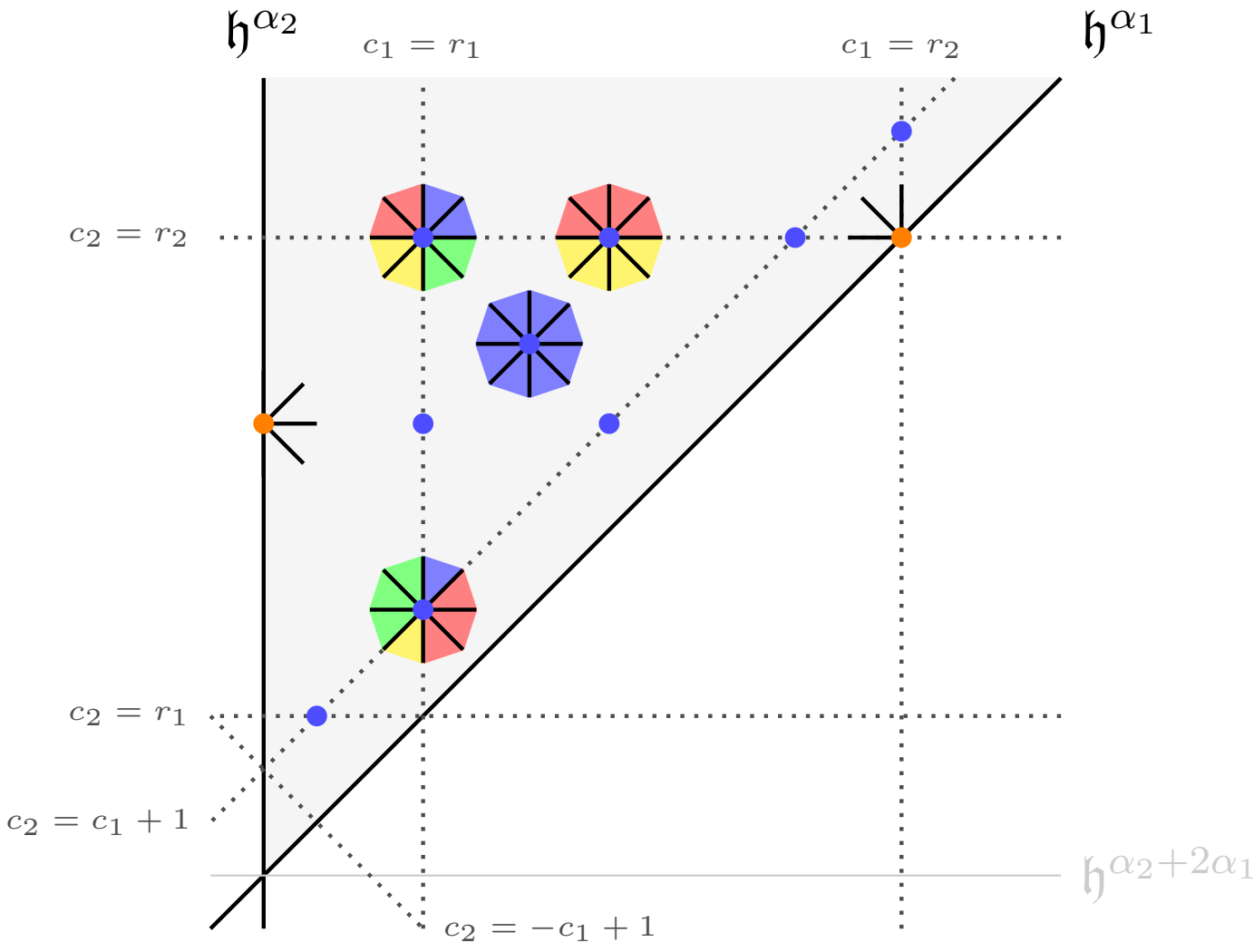

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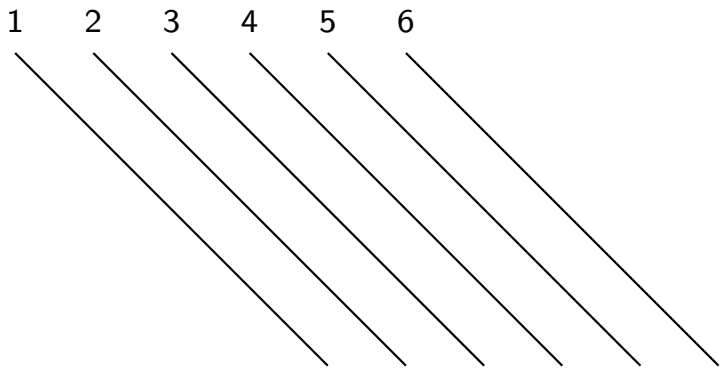

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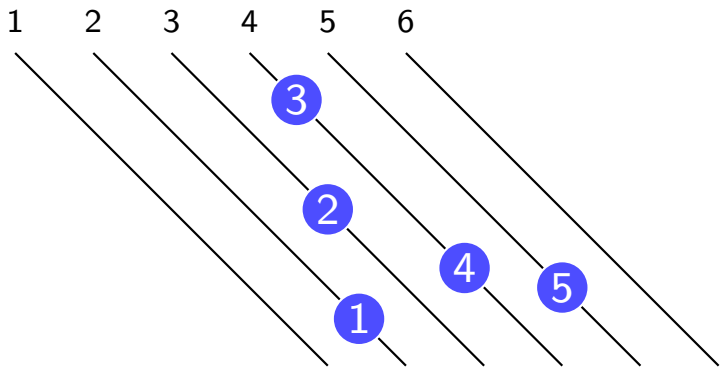

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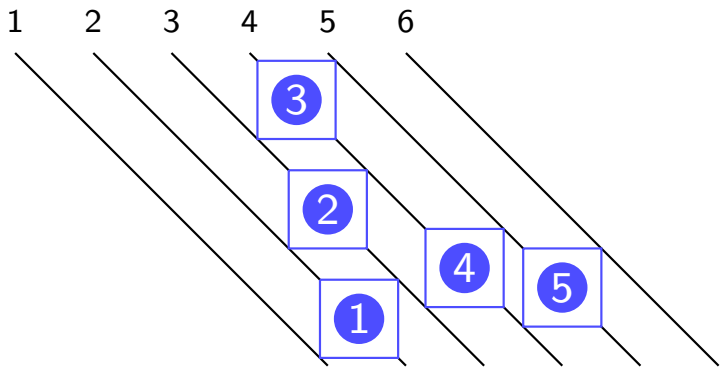

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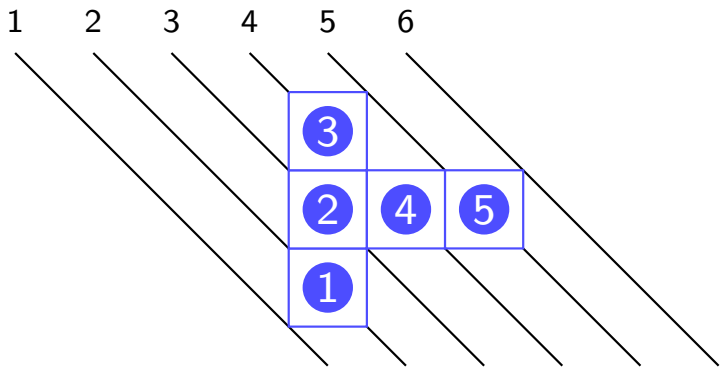

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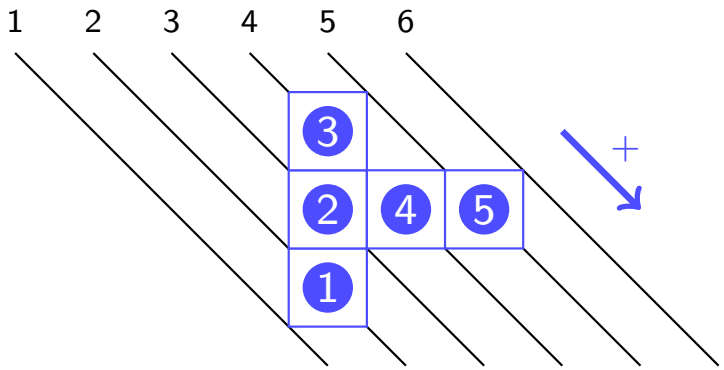

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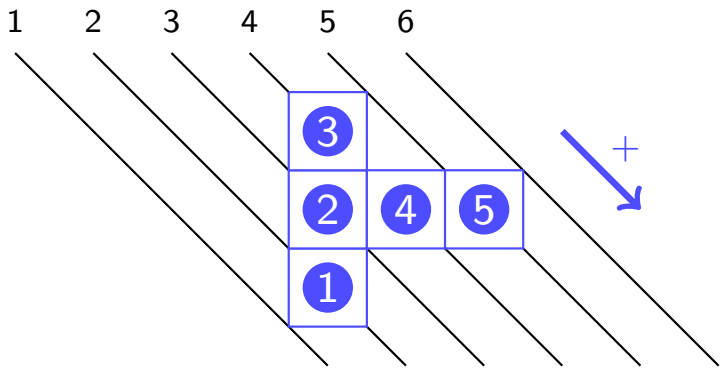

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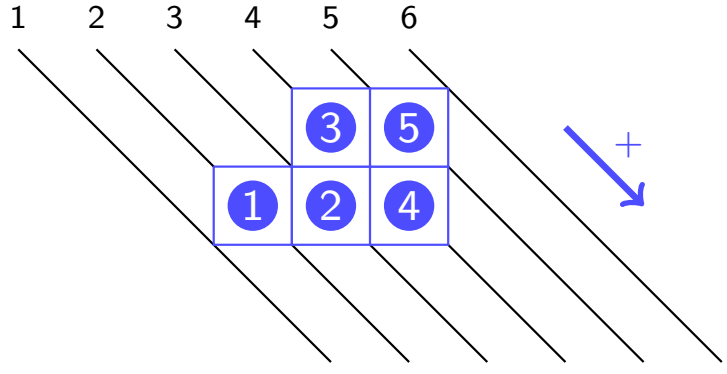

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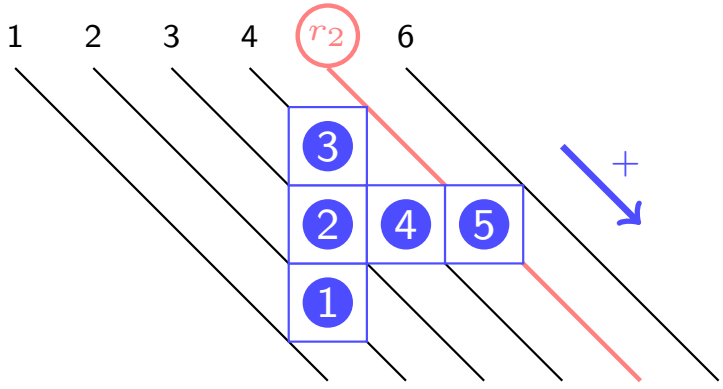

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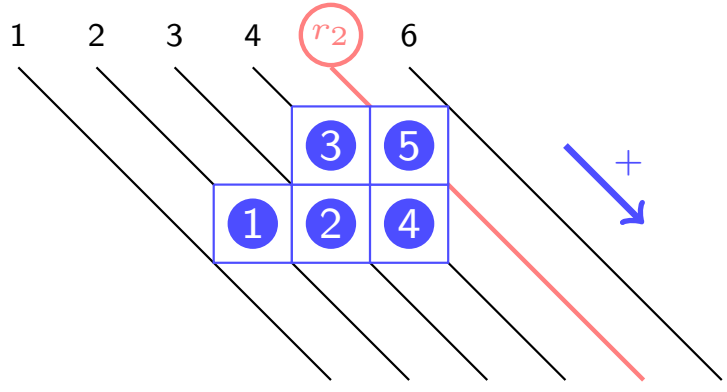

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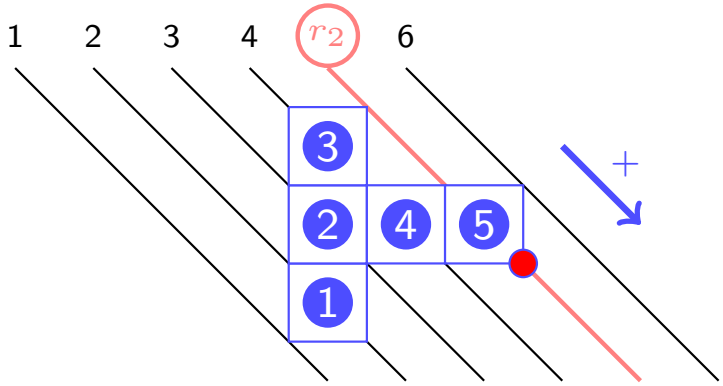

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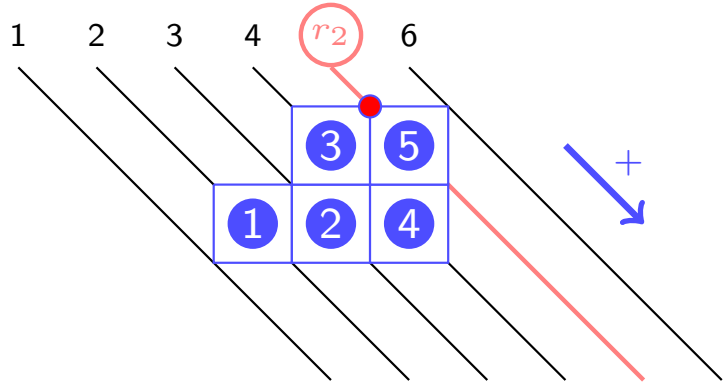

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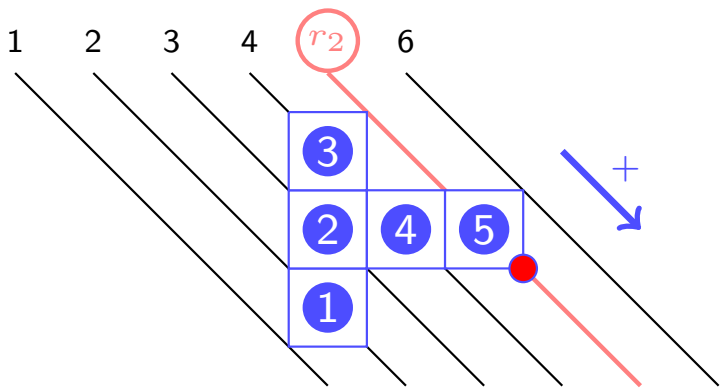

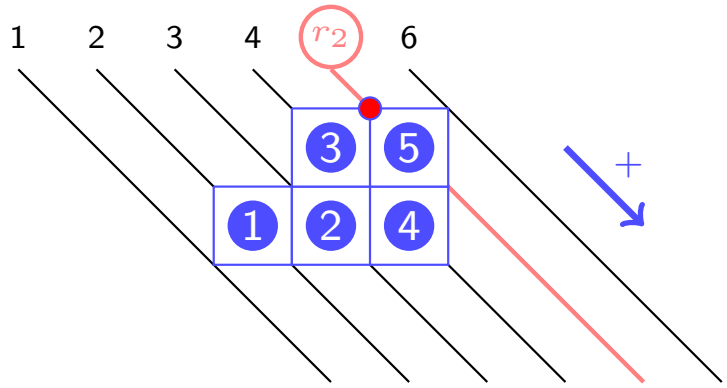

Basis indexed by standard fillings with $\{\pm 1, \ldots, \pm k\}$ with restrictions:

(1) Exactly one of i or $-i$ appears.

(2) If $\mathrm{box_i} <$ ● , then filling is negative. If $\mathrm{box_i} >$ ● , filling is positive.

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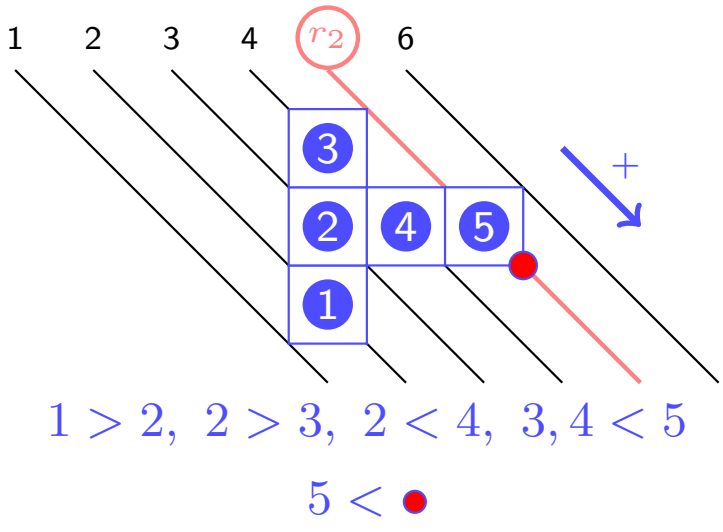
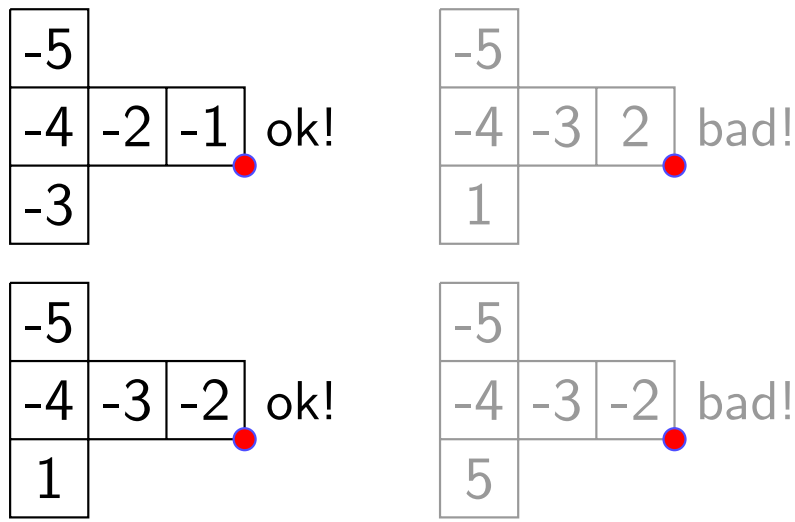
Restrict to $c_i \in \mathbb{Z} + \beta$ for some $\beta \in \mathbb{C}$.
A central character $\mathbf{c}$ gives a list of diagonal placements.
For example:

$$\mathbf{c} = (2, 3, 4, 4, 5)$$



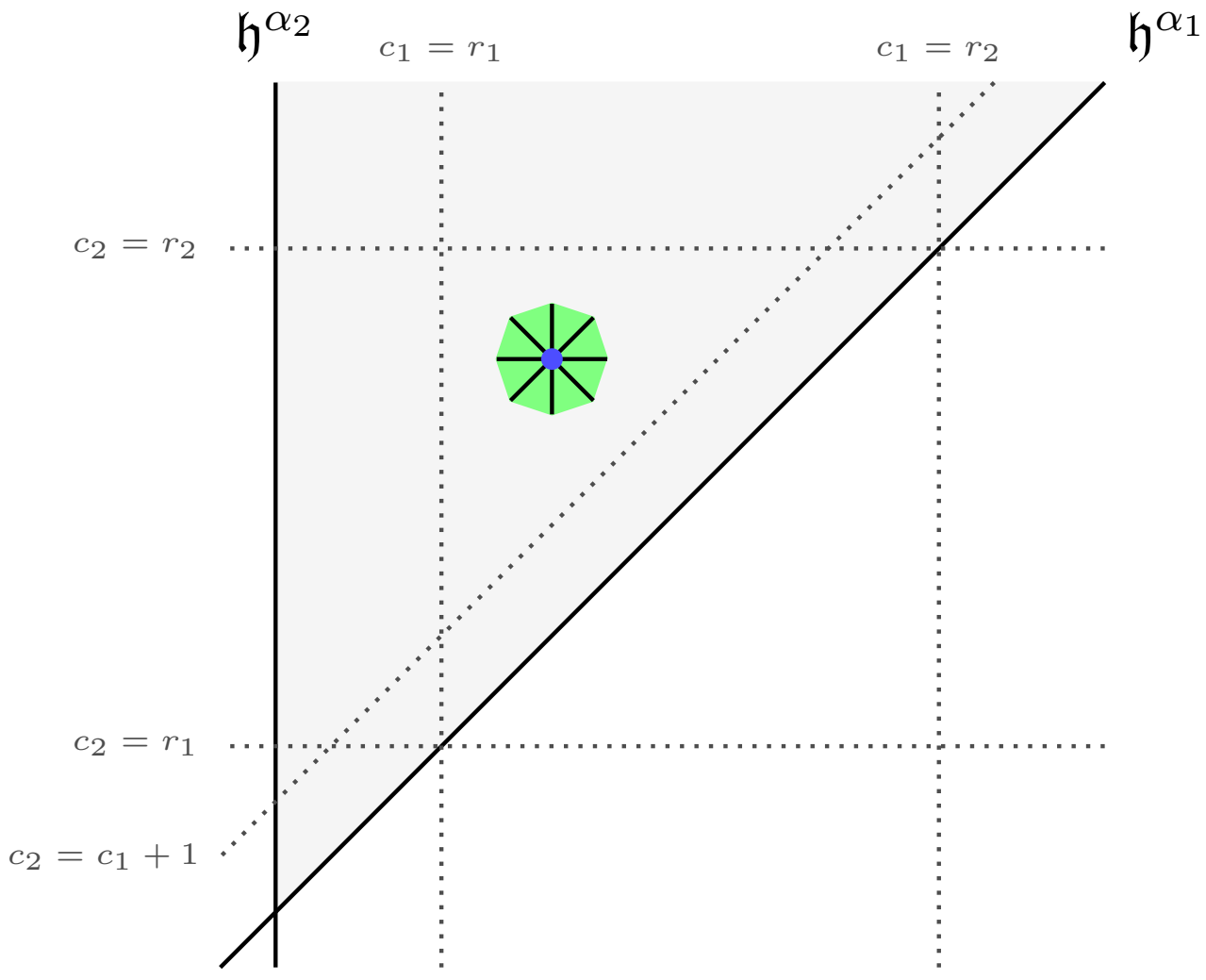


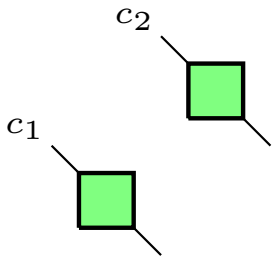
Basis indexed by standard fillings with $\{\pm 1, \ldots, \pm k\}$ with restrictions:

(1) Exactly one of i or $-i$ appears.

(2) If $\mathrm{box_i} <$ ● , then filling is negative. If $\mathrm{box_i} >$ ● , filling is positive.

Points versus box arrangements





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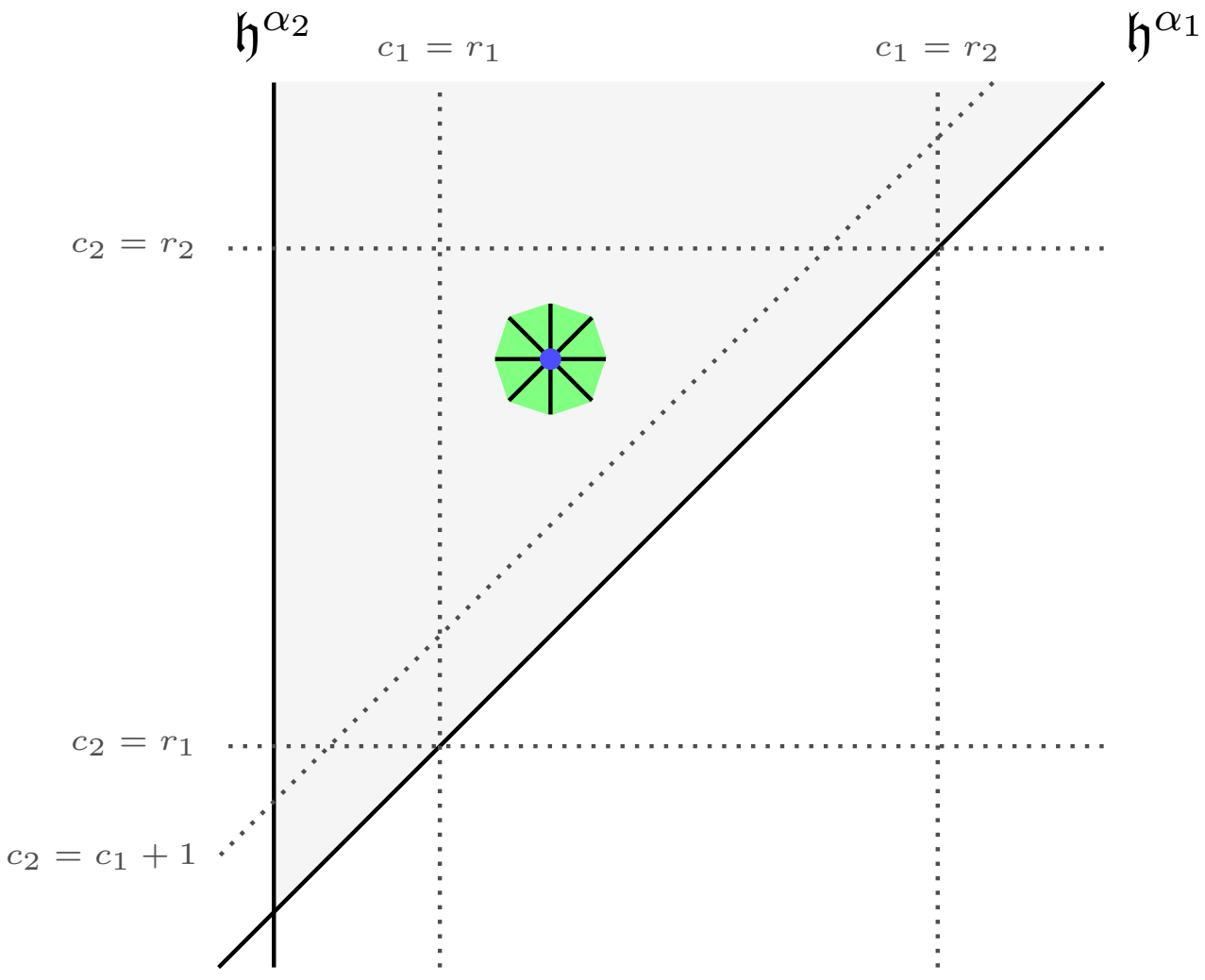
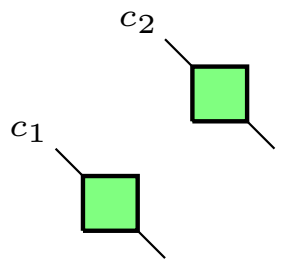
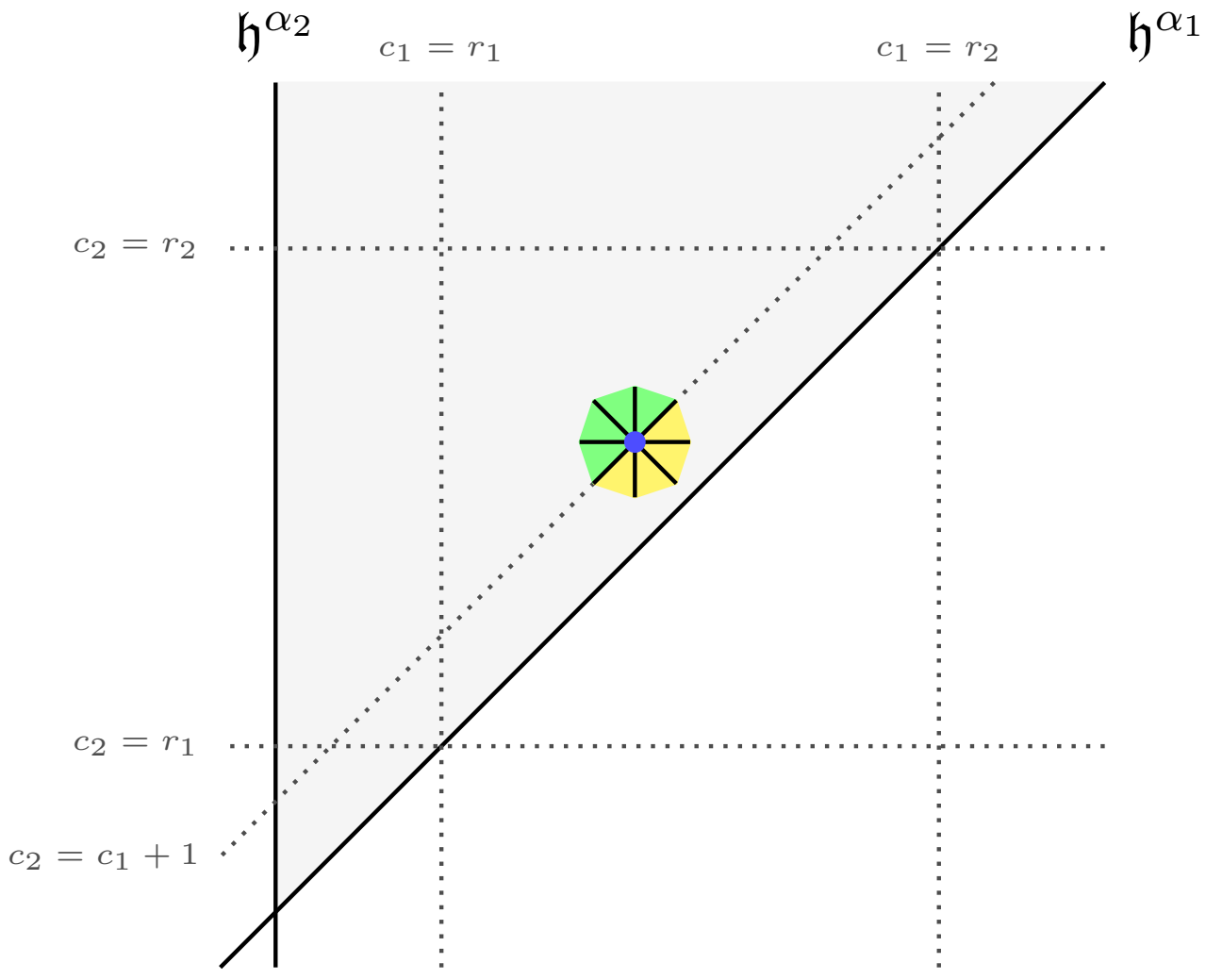
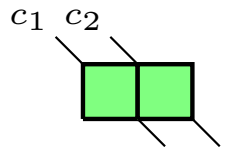


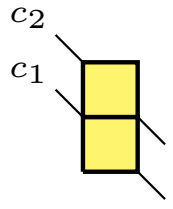


2 1	2 -1
1 2	-1 2
-2 -1	-2 1
-1 -2	1 -2

Points versus box arrangements







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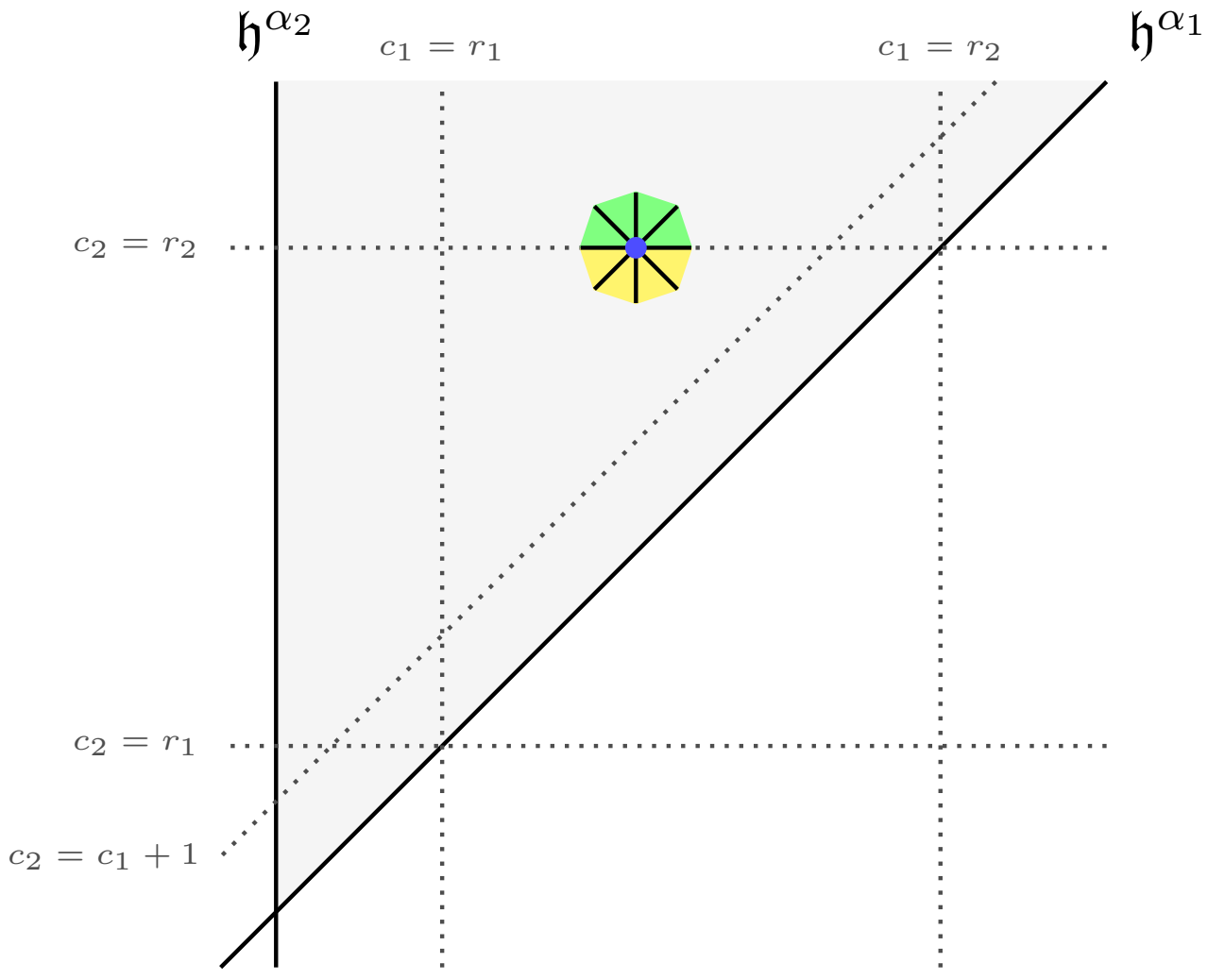

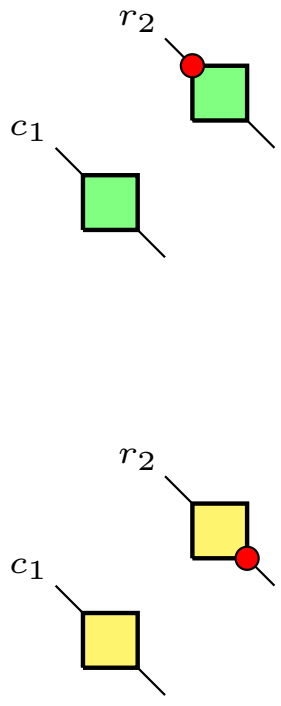

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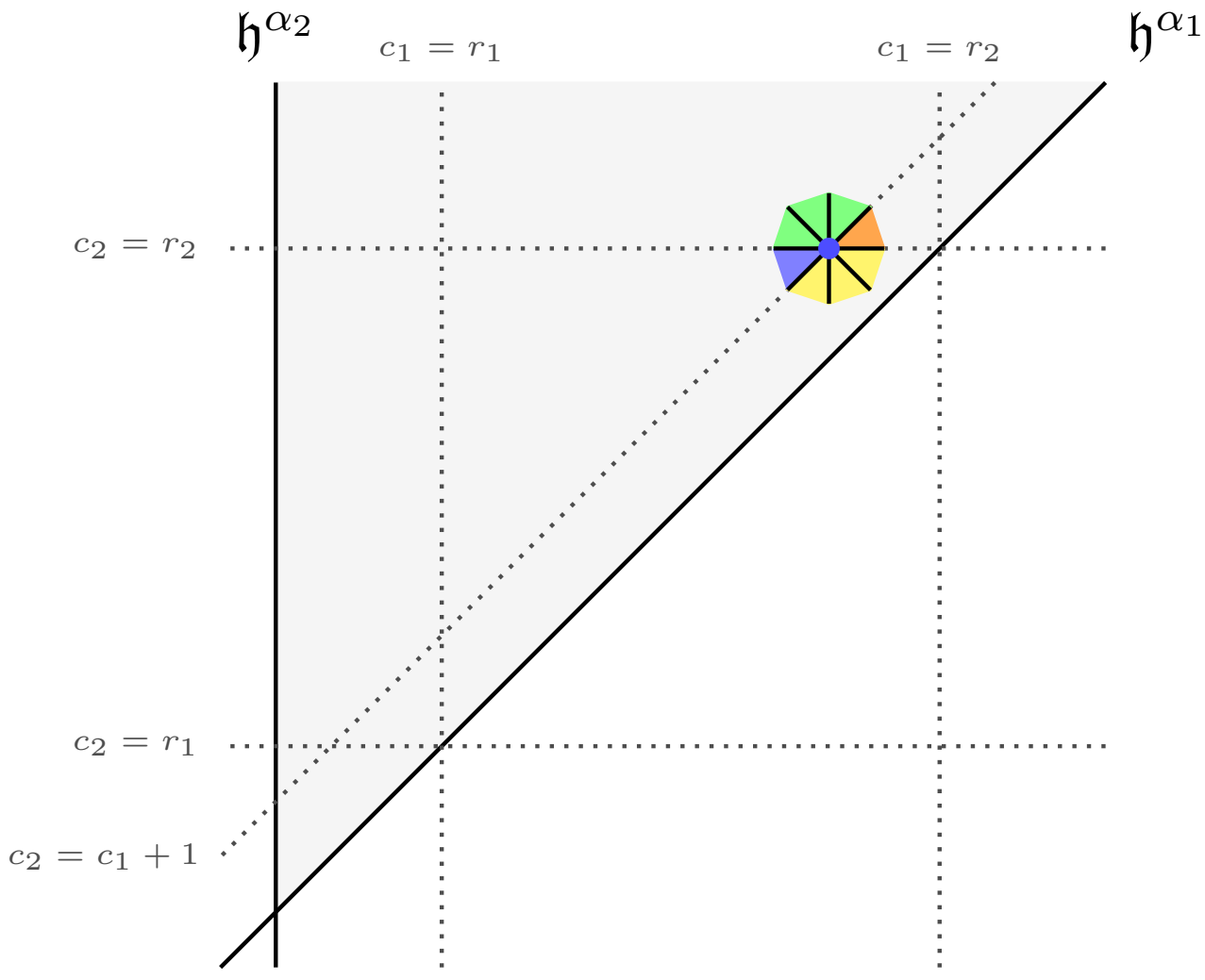

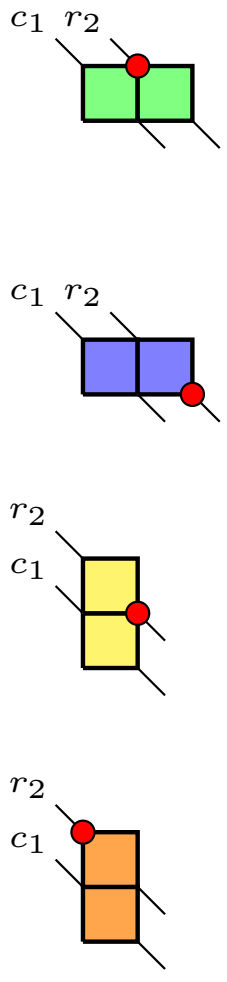

Description 1: Central characters are indexed by points $\mathbf{c}$ in $\mathbb{C}^k$.
Representations of H_k are indexed by skew local regions.
Basis indexed by chambers.

Description 2: Marked box arrangements.
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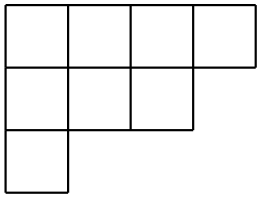
Description 3: Partitions.
Representation arise in Schur-Weyl duality with certain $U_q\mathfrak{gl}_n$ reps.

Centralizer properties

Let $U = U_q\mathfrak{gl}_n$ be the quantum group for $\mathfrak{gl}_n(\mathbb{C})$. We're interested in certain finite dimensional simple U-modules $L(\lambda)$ indexed by partitions:

$\lambda =$ 

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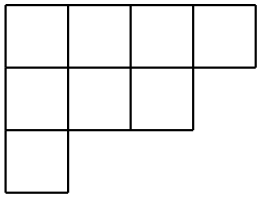
$\lambda =$ 

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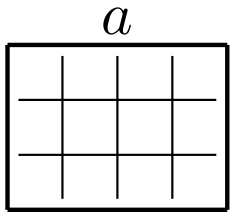
In particular, rectangular partitions:

$(a^c) = c$ 

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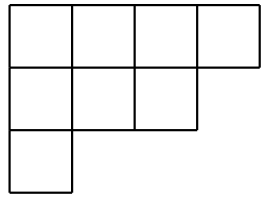
$\lambda =$ 

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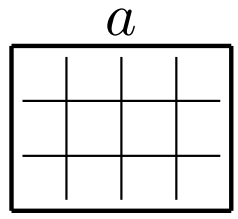
$(a^c) = c$ 

H_k has a commuting action with U on the space

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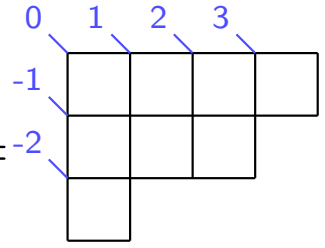
$\lambda =$ 

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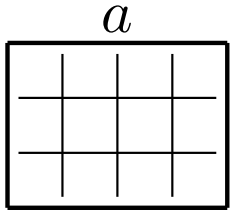
$(a^c) = c$ 

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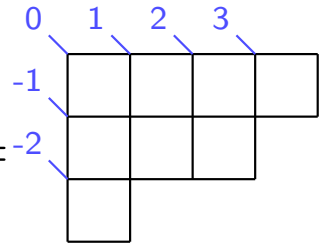
$\lambda =$ 

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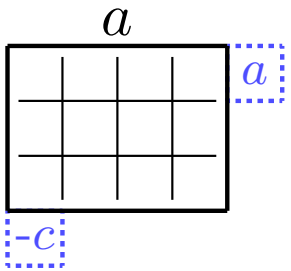
$(a^c) = c$ 

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The content of a box is its diagonal number.

The eigenvalues of T_0 and T_k are controlled by the contents of addable boxes to (a^c) and (b^d).

Exploring $L((a^c)) \otimes L((b^d)) \otimes \left(L(\square)\right)^{\otimes k}$

Products of rectangles:

$$L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda) \qquad \text{(multiplicity one!)}$$

where Λ is the following set of partitions:

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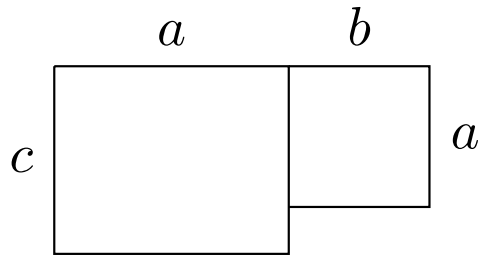

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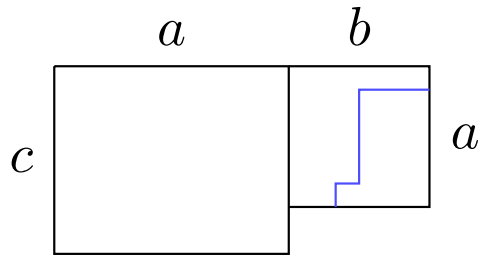

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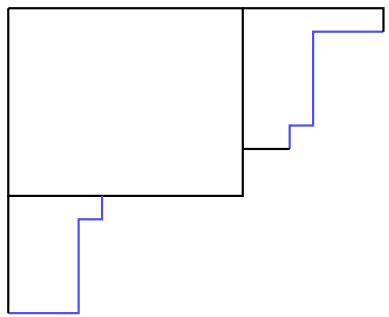

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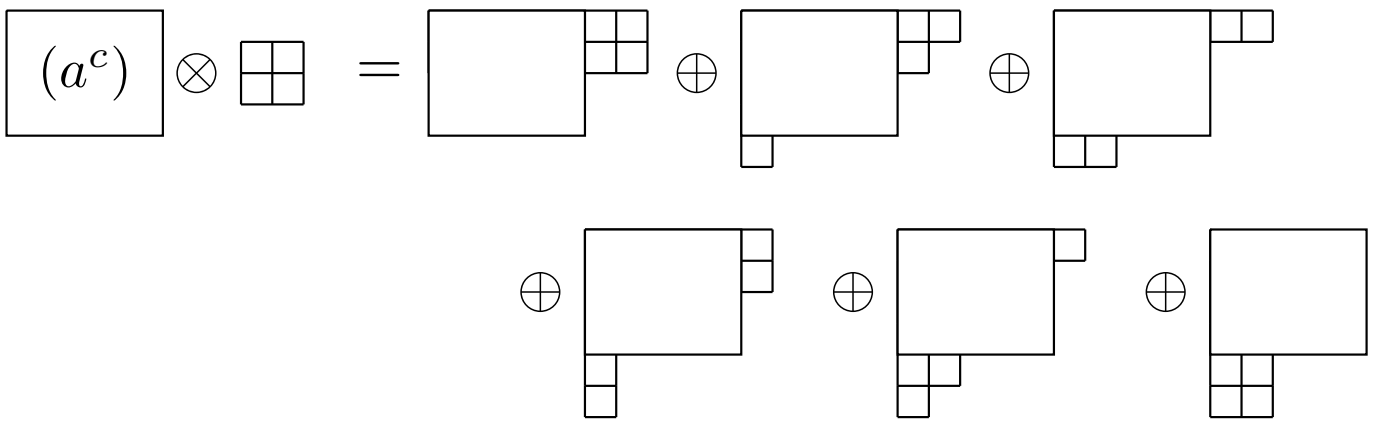
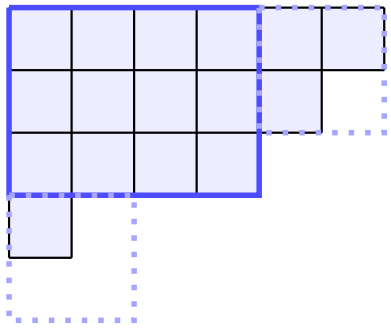
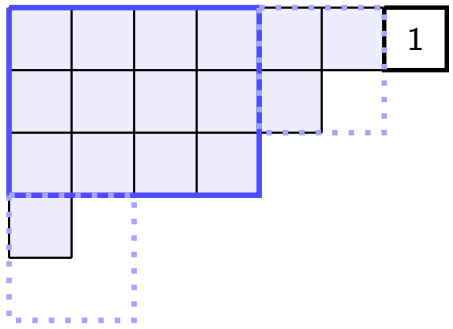


$$L\left(\begin{array}{|c|c|c|c|}\hline&&&\\\hline&&&\\\hline&&&\\\hline\end{array}\right)\otimes L\left(\begin{array}{|c|c|}\hline&\\\hline&\\\hline\end{array}\right)$$



$$L\left(\ydiagram{4,4,4}\right) \otimes L\left(\ydiagram{2,2}\right) \otimes L\left(\ydiagram{1}\right)$$



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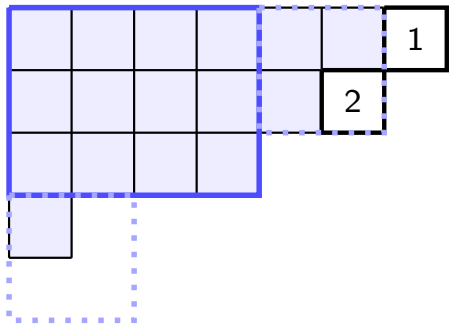

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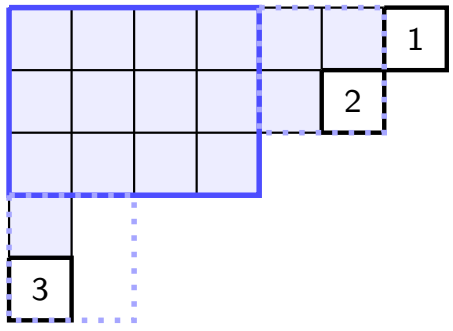

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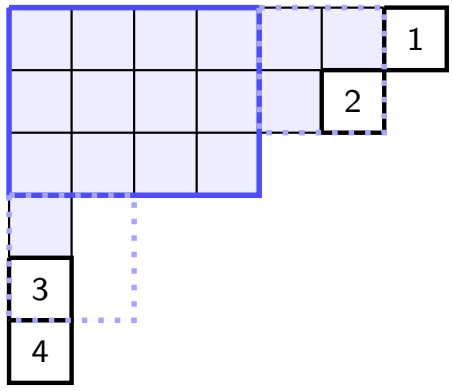

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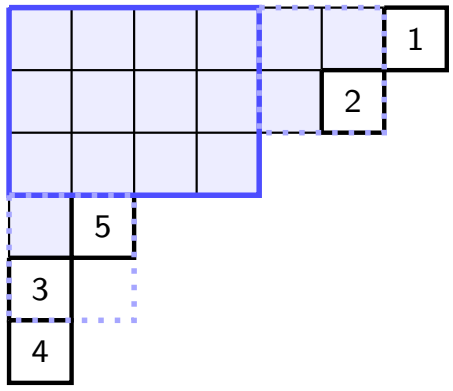

$(*)$ H_k representations in tensor space are labeled by certain partitions λ.

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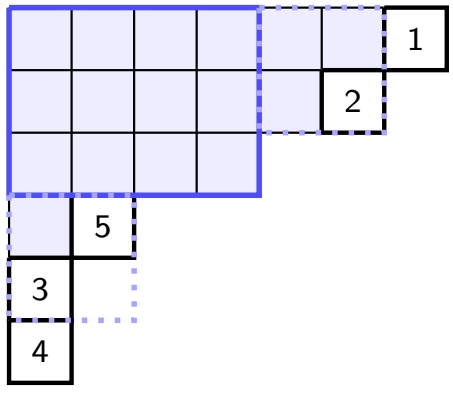

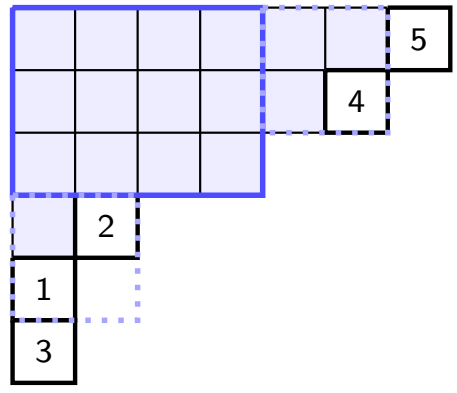

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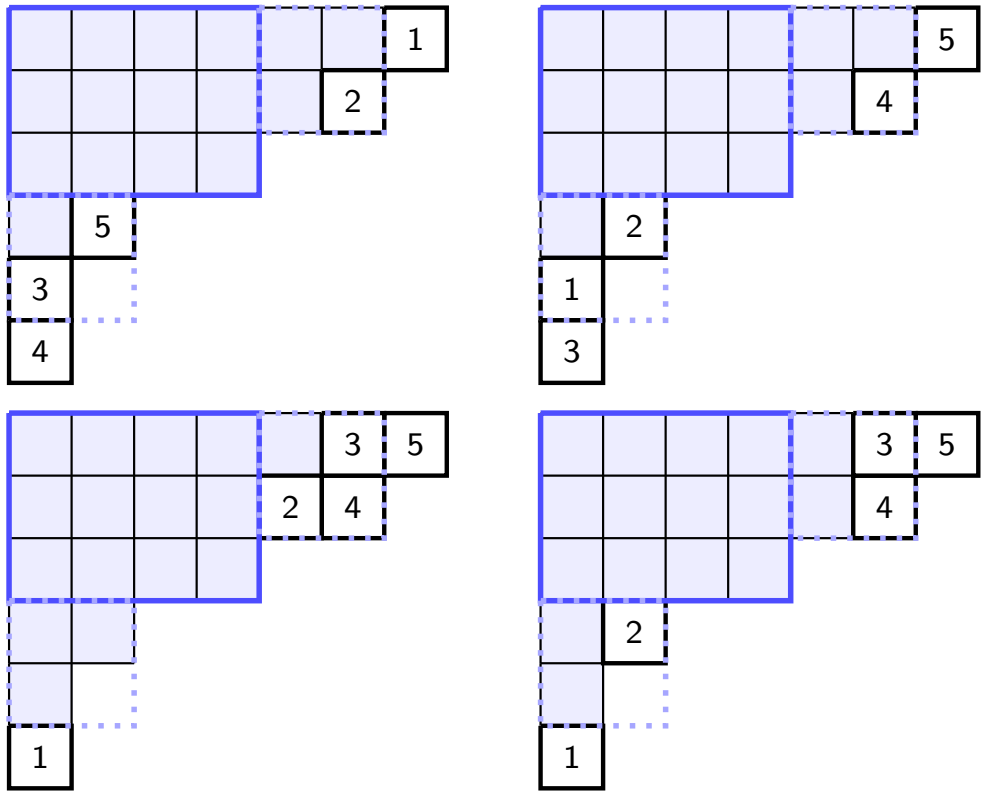

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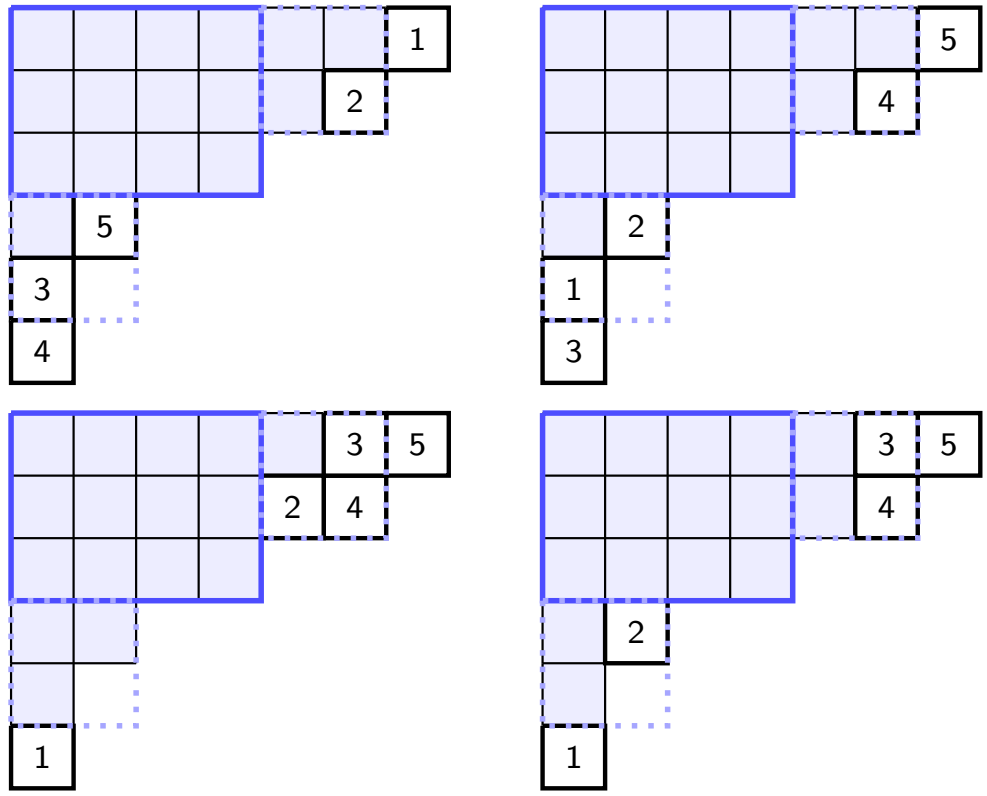

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(∗) Basis labeled by tableaux from *some* partition μ in $(a^c) \otimes (b^d)$ to λ.

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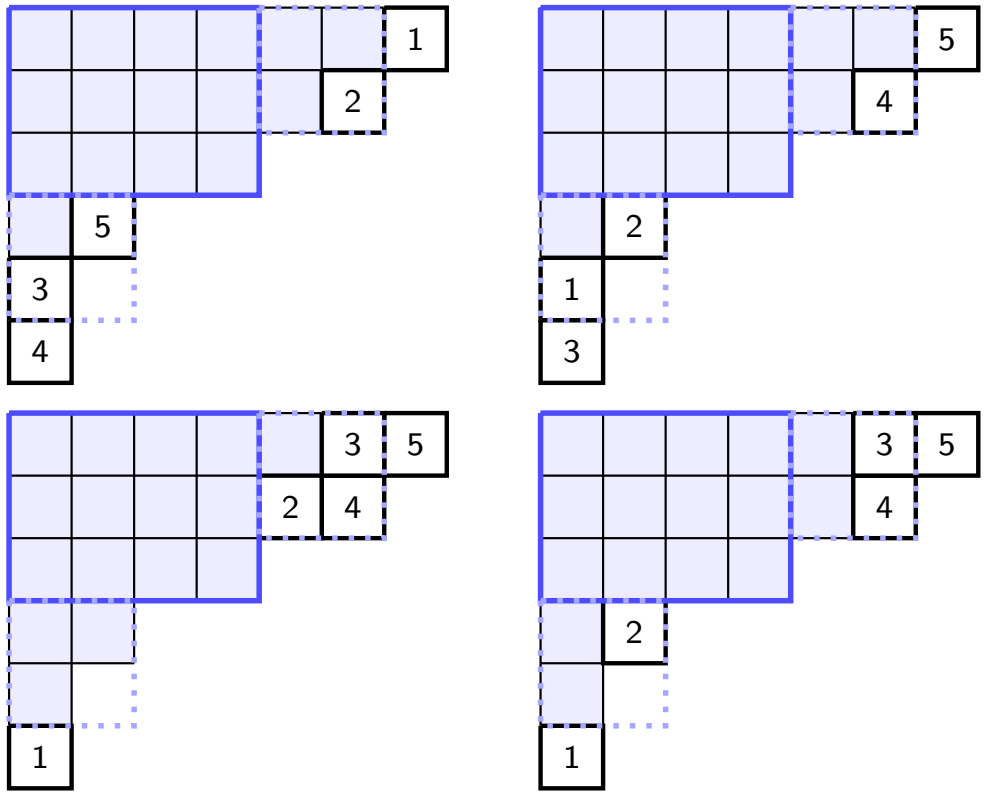

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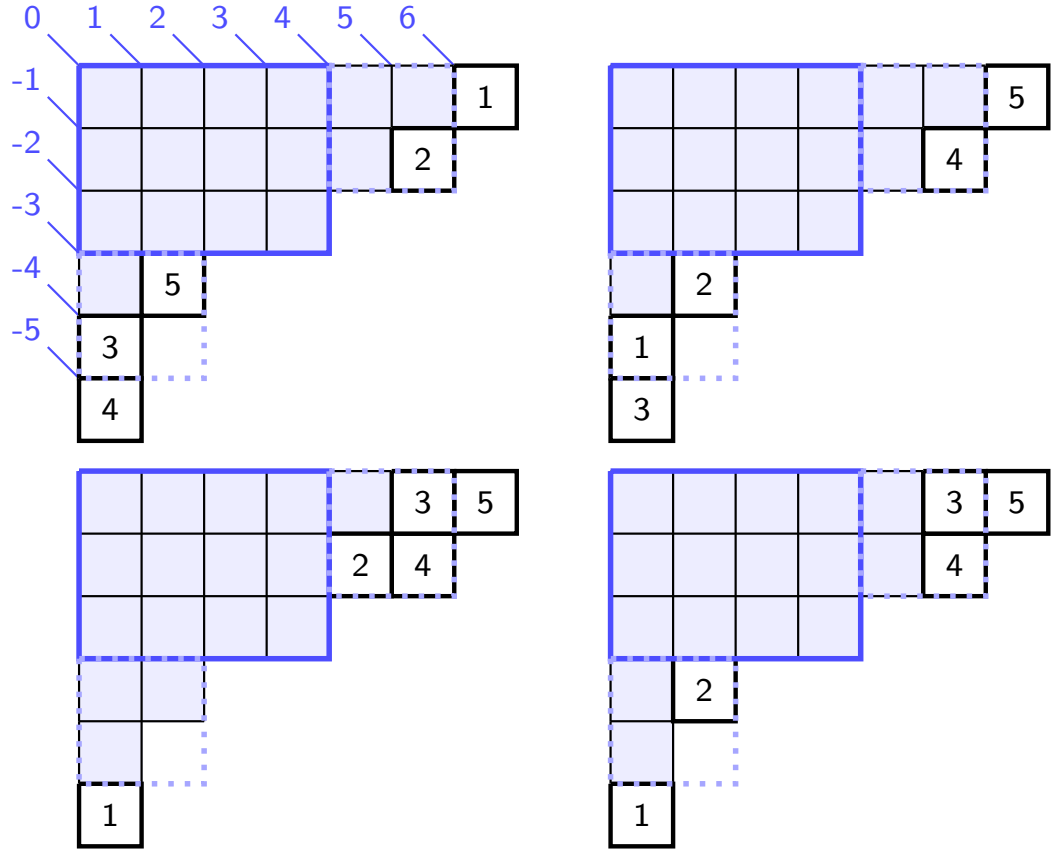

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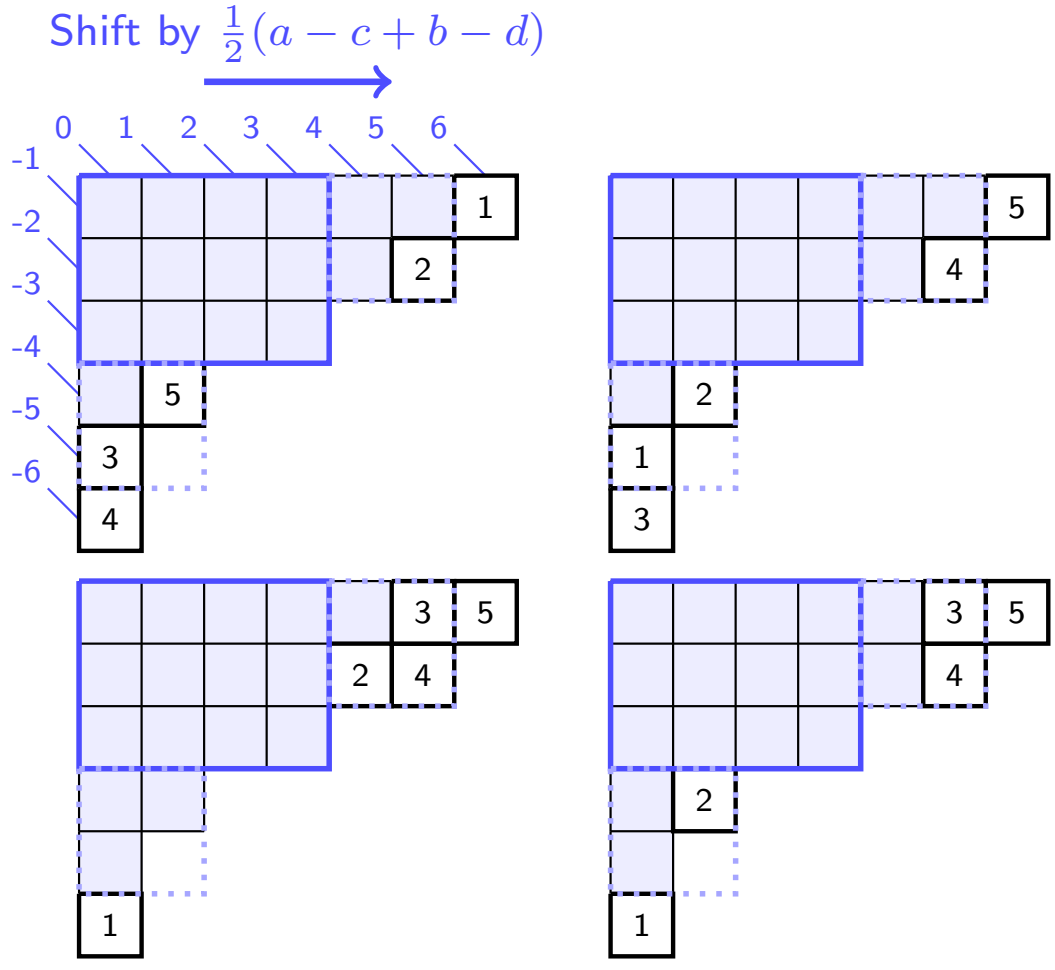

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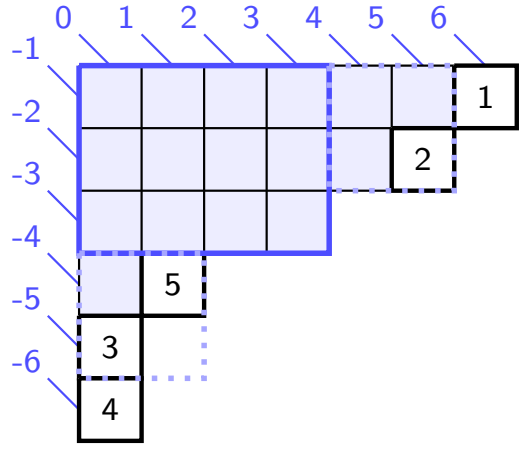
(∗) Basis labeled by tableaux from *some* partition μ in $(a^c) \otimes (b^d)$ to λ.

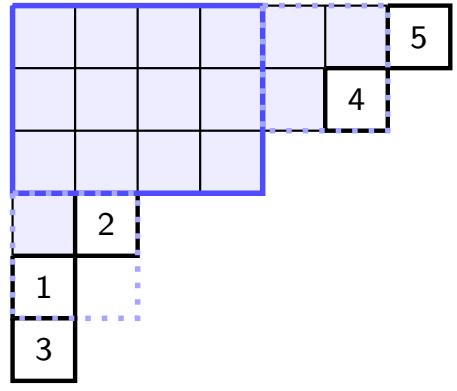
(∗) Calibrated

$$L\left(\begin{array}{|c|c|c|c|}\hline\ &\ &\ &\ \\\hline\ &\ &\ &\ \\\hline\ &\ &\ &\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|c|}\hline\ &\ \\\hline\ &\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|}\hline\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|}\hline\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|}\hline\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|}\hline\ \\\hline\end{array}\right) \otimes L\left(\begin{array}{|c|}\hline\ \\\hline\end{array}\right)$$

Shift by $\frac{1}{2}(a-c+b-d)$

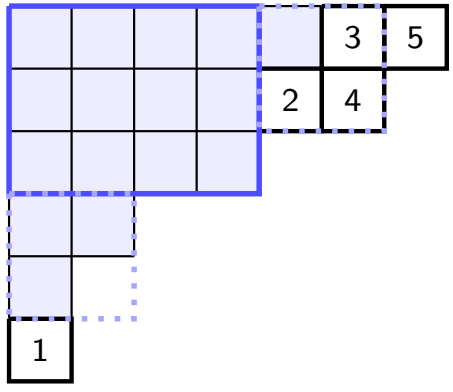
$Y_1 \mapsto t^{5.5}$
$Y_2 \mapsto t^{3.5}$
$Y_3 \mapsto t^{-4.5}$
$Y_4 \mapsto t^{-5.5}$
$Y_5 \mapsto t^{-2.5}$

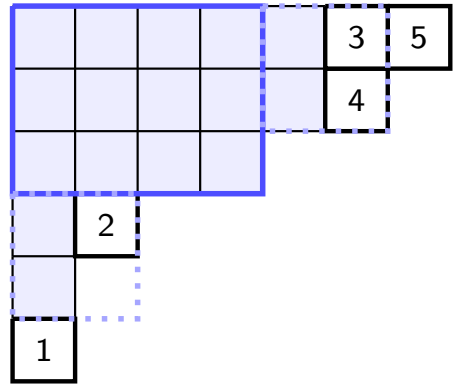




$Y_1 \mapsto t^{5.5}$
$Y_2 \mapsto t^{3.5}$
$Y_3 \mapsto t^{-4.5}$
$Y_4 \mapsto t^{-5.5}$
$Y_5 \mapsto t^{-2.5}$

$Y_1 \mapsto t^{-5.5}$
$Y_2 \mapsto t^{2.5}$
$Y_3 \mapsto t^{4.5}$
$Y_4 \mapsto t^{3.5}$
$Y_5 \mapsto t^{5.5}$





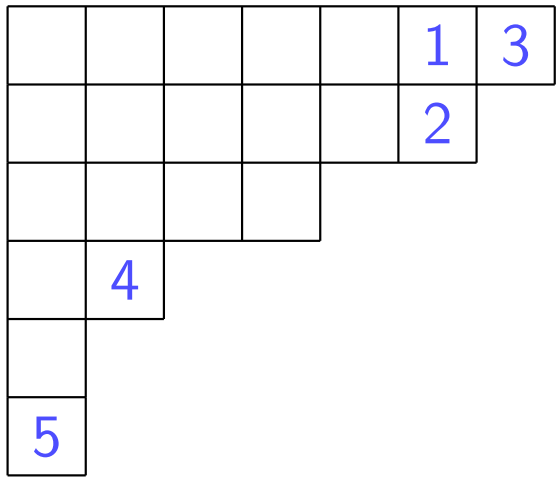
$Y_1 \mapsto t^{5.5}$
$Y_2 \mapsto t^{3.5}$
$Y_3 \mapsto t^{-4.5}$
$Y_4 \mapsto t^{-5.5}$
$Y_5 \mapsto t^{-2.5}$

(∗) H_k representations in tensor space are labeled by certain partitions λ.

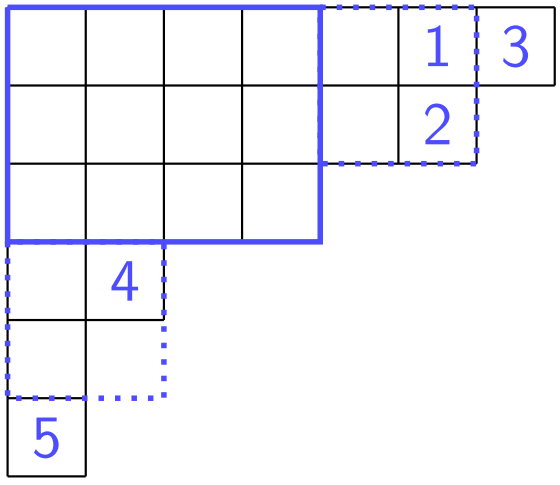
(∗) Basis labeled by tableaux from *some* partition μ in $(a^c) \otimes (b^d)$ to λ.

(∗) Calibrated: Y_i acts by t to the shifted content of box_i.

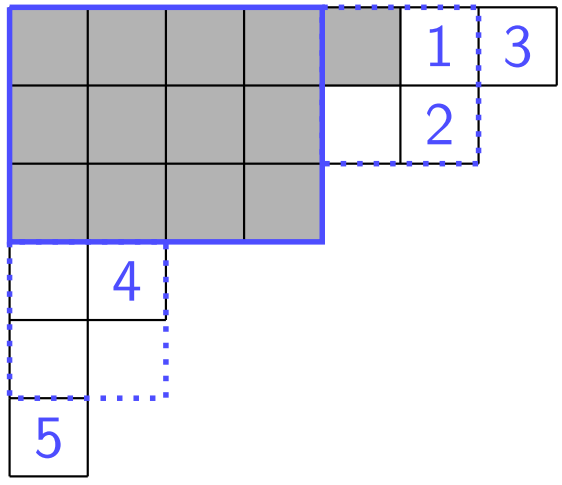
From {partitions in tensor space} to {box arrangements}



From {partitions in tensor space} to {box arrangements}

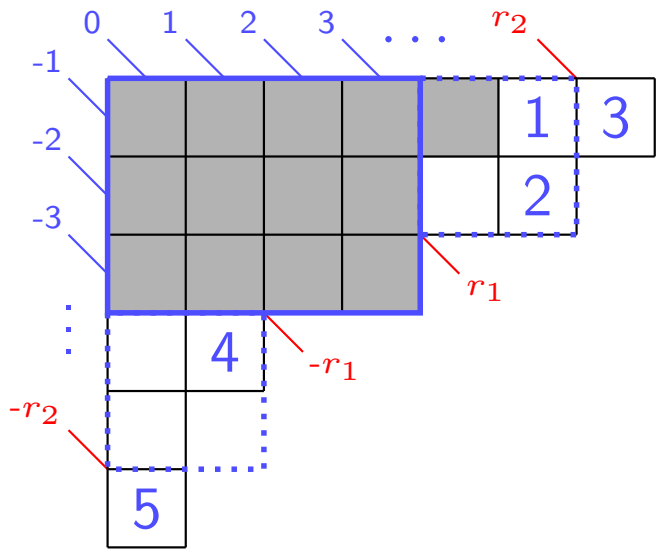


From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

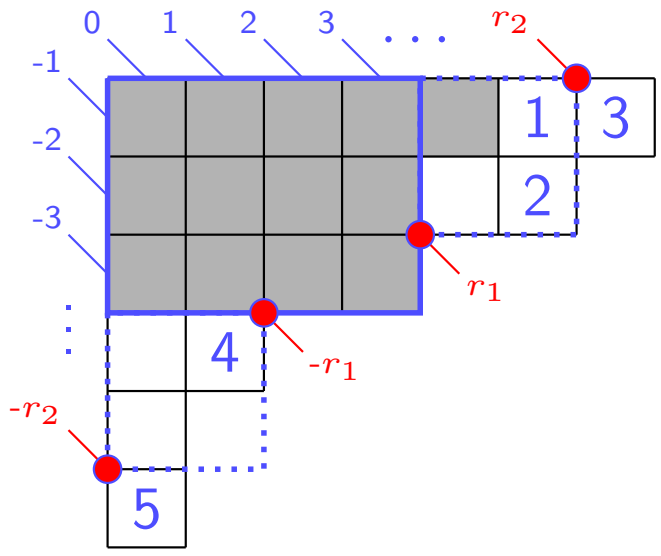
From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

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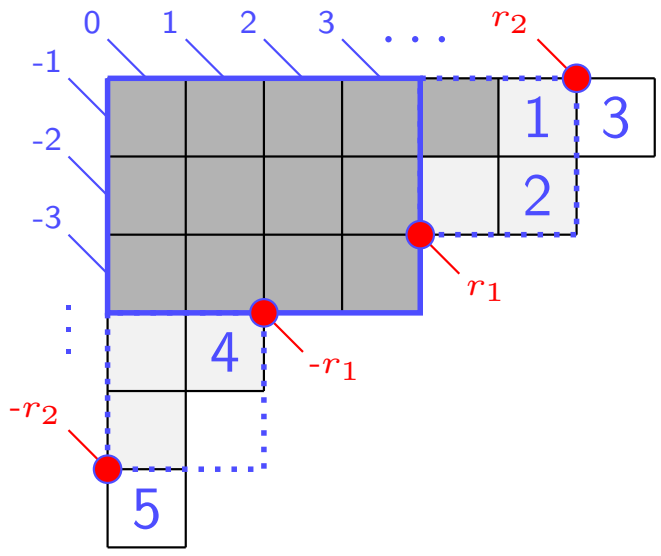
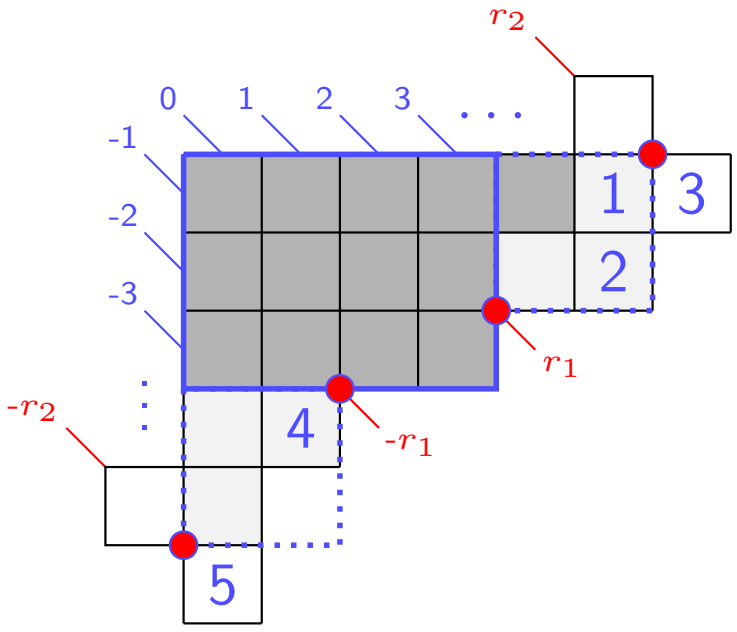
From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

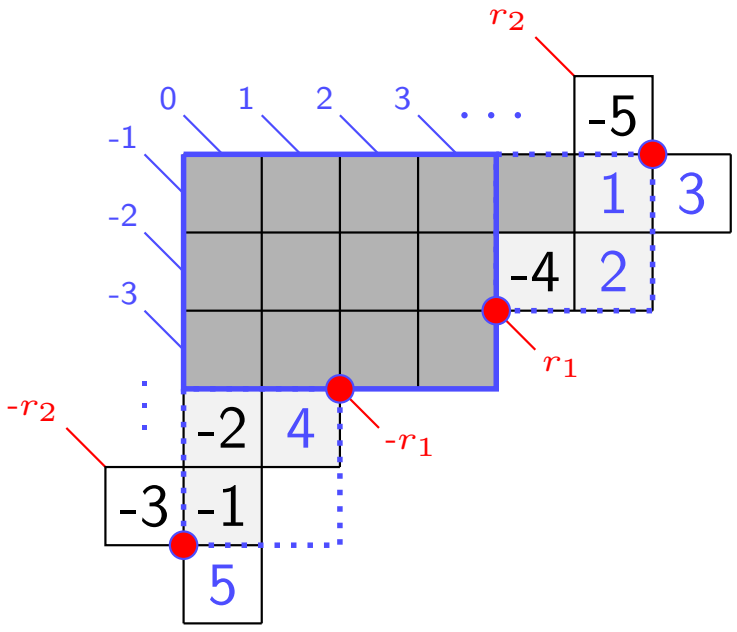
From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

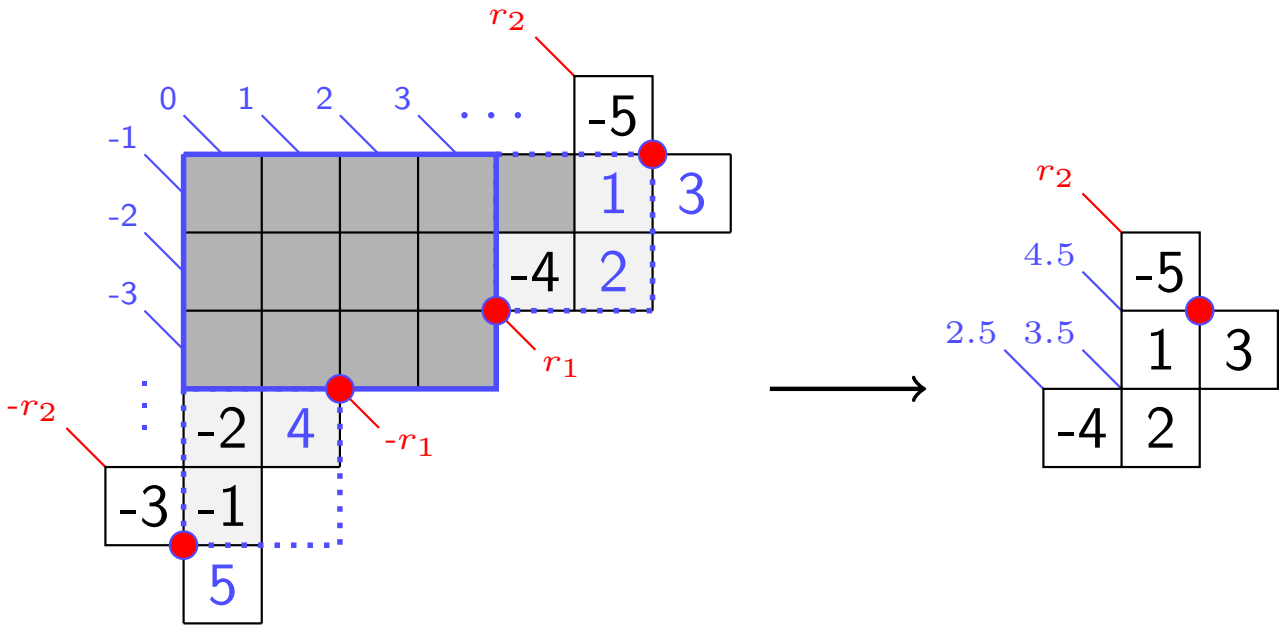
From {partitions in tensor space} to {box arrangements}



$\square$ = boxes that must appear in the partition at level 0.

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

From {partitions in tensor space} to {box arrangements}



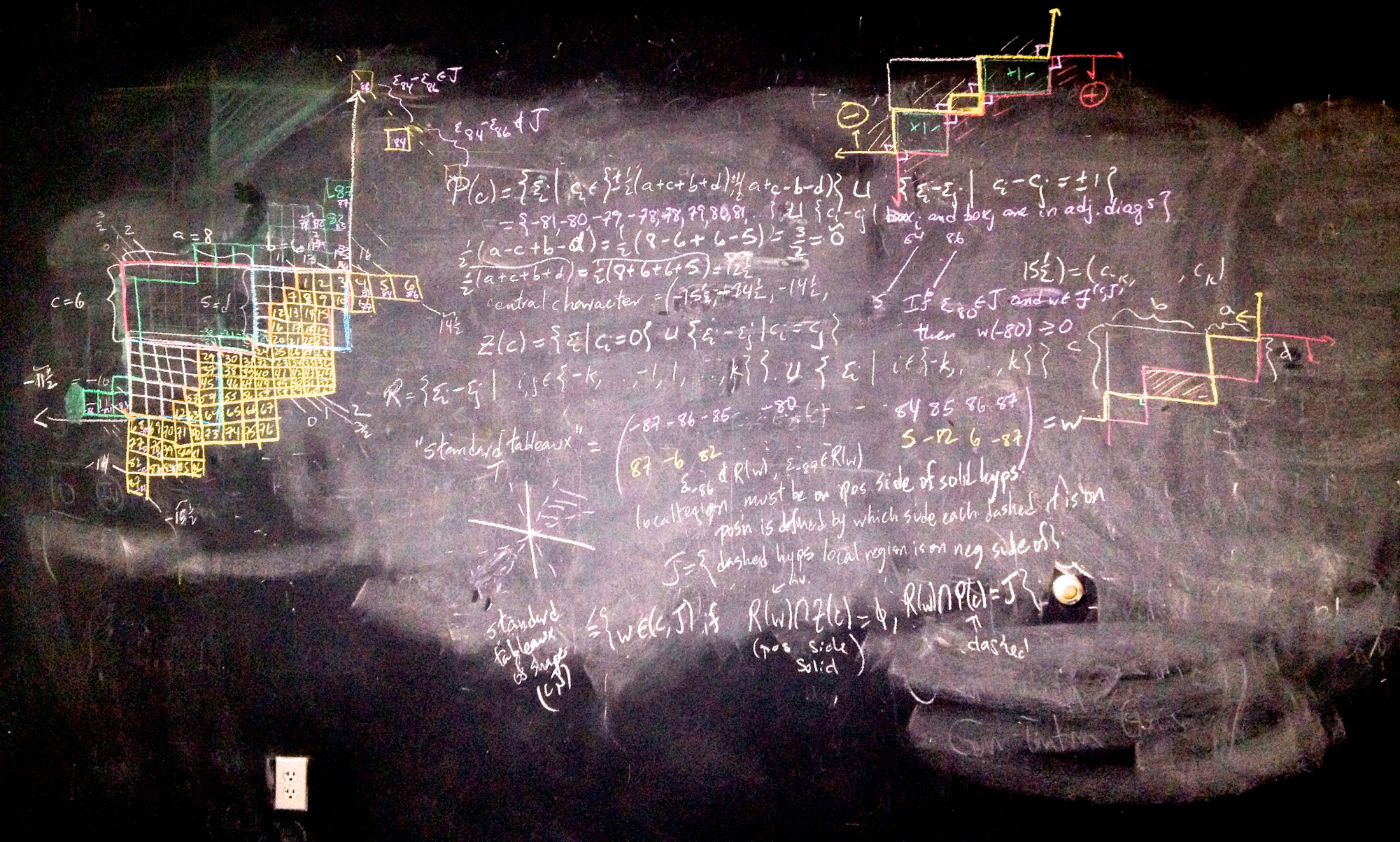
$\square$ = boxes that must appear in the partition at level 0.

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

versus

$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4^{-1}) = t^{2.5},\ \gamma(Y_5^{-1}) = t^{r_2}.$$

Thanks!



central character
"standard tableaux"
localregion must be on pos side of solid hyps
posn is defined by which side each dashed it is on
J = {dashed hyps local region is on neg side of}
standard tableaux of shape (c,J)
(pos side solid)
dashed