Combinatorics of affine Hecke algebras of type C.

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The affine type C Hecke algebra H_k is generated by invertible elements $T_0, T_1, \ldots, T_k$ with relations

$$(T_0 - t_0)(T_0 - t_0^{-1}) = 0 = (T_k - t_k)(T_k - t_k^{-1})$$

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Goal today:

Tell you 3 descriptions of calibrated irreducible reps of H_k, where "calibrated" means $\mathbb{C}[Y_1^{\pm 1}, \ldots, Y_k^{\pm 1}]$ is simultaneously diagonalized.

Central characters

The center of H_k is symmetric Laurent polynomials

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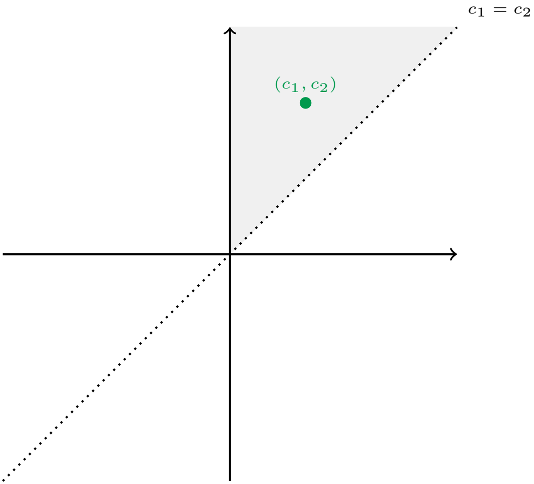
Central characters as points

Restrict to real points.

Fav equivalence class reps: $0 \leq c_1 \leq \cdots \leq c_k$. ($W_0$ acts by signed permutations)
When $k = 2$:



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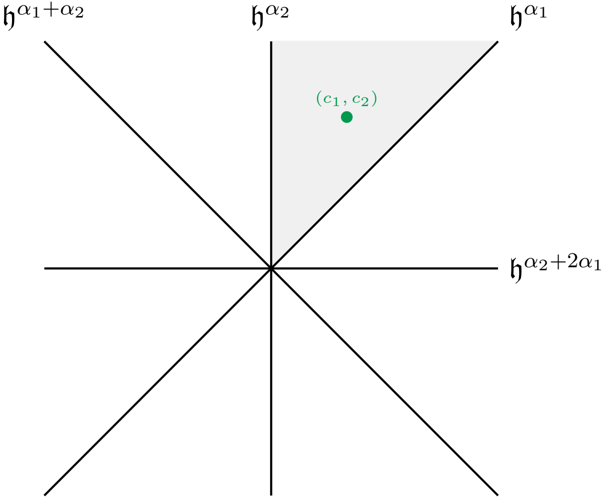

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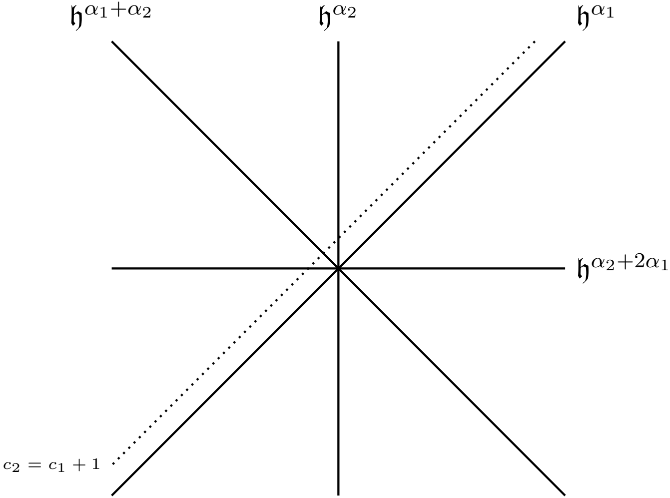

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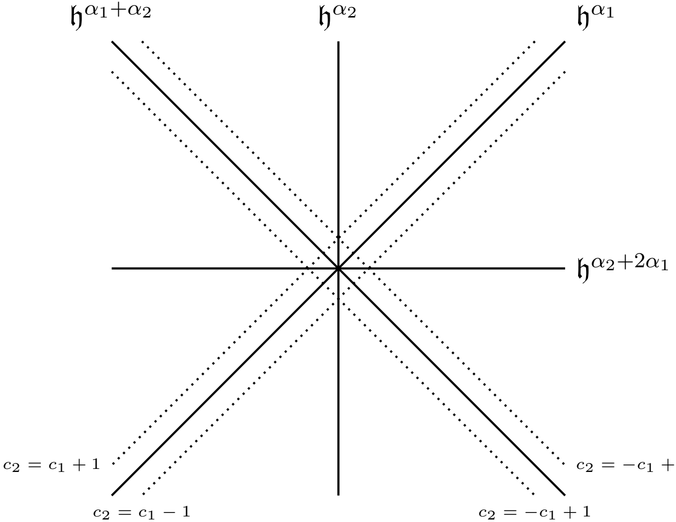

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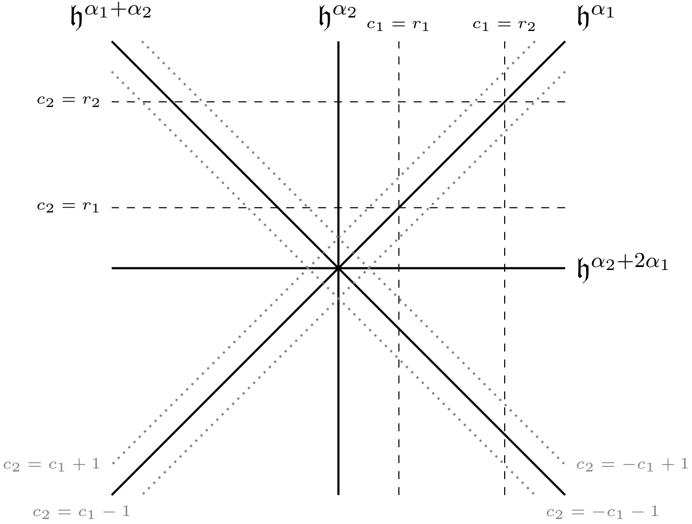

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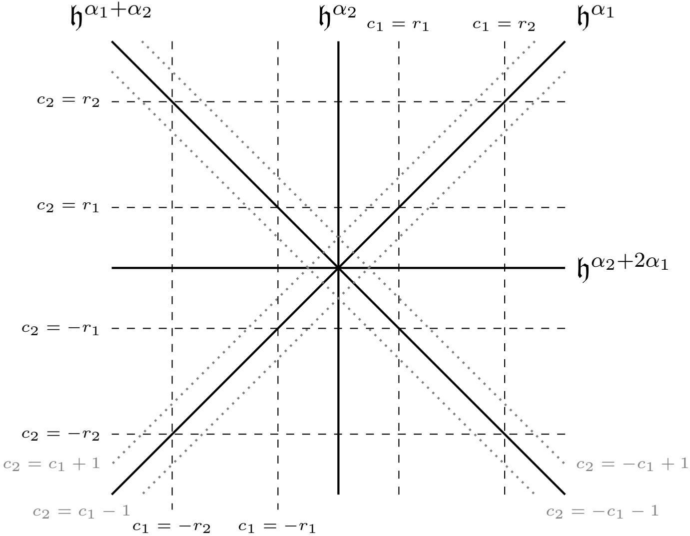

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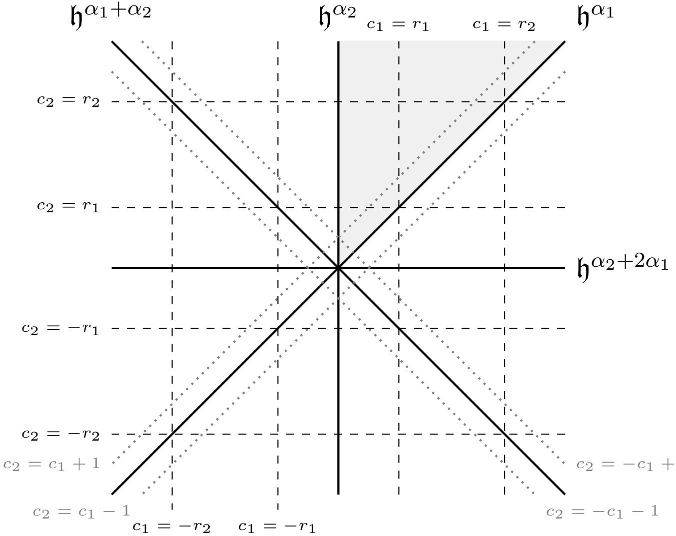

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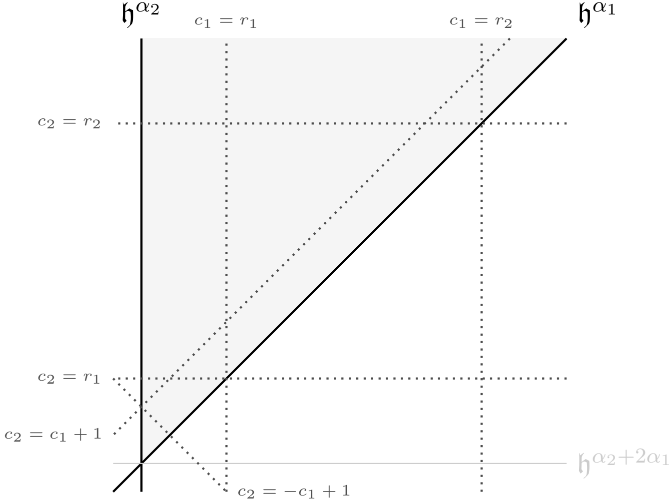

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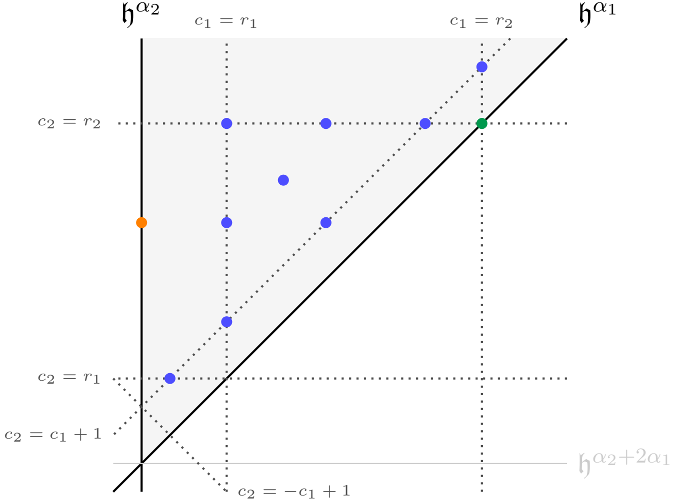

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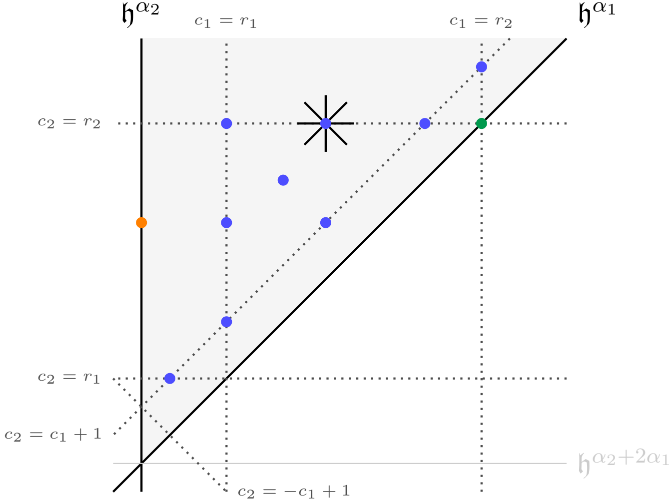

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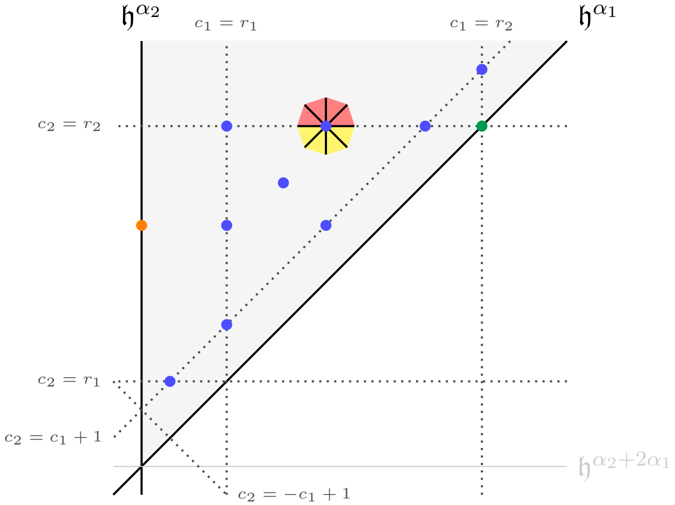

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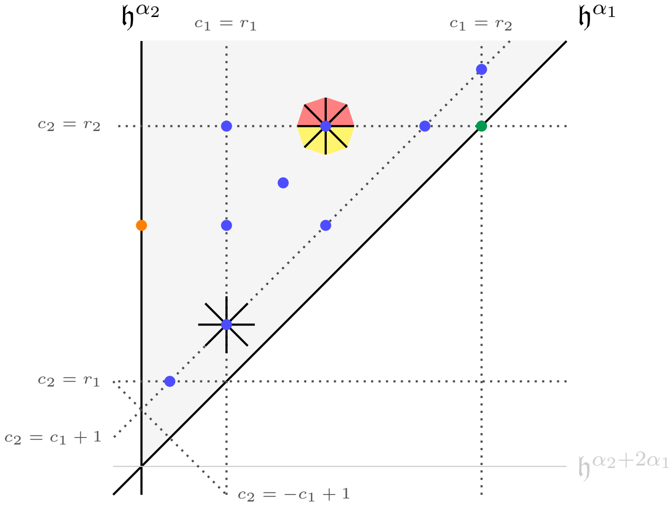

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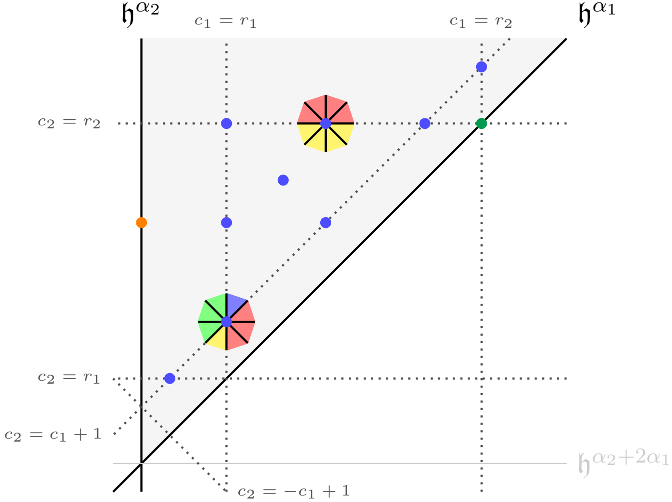

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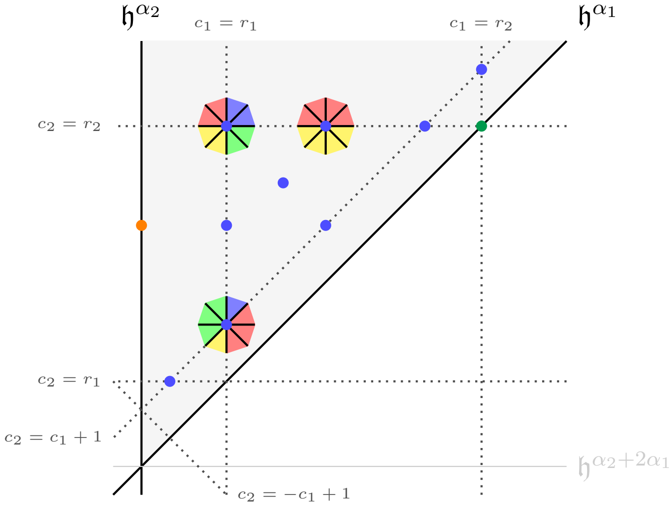

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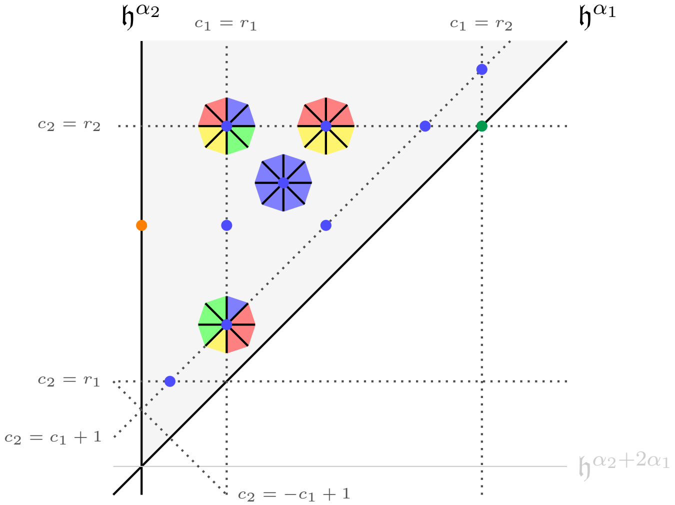

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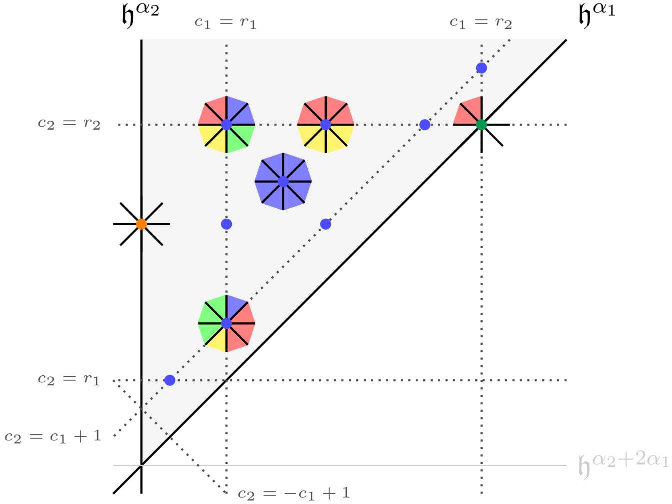

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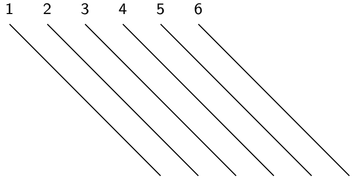

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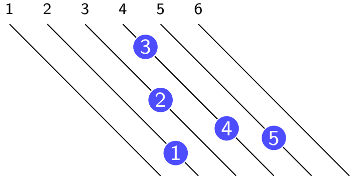

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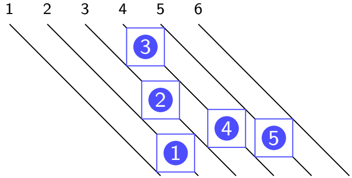

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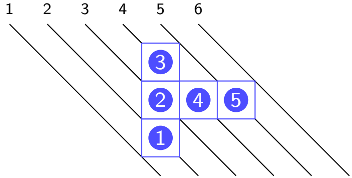

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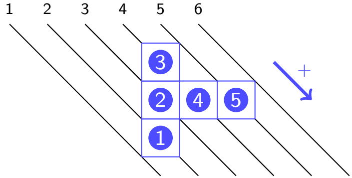

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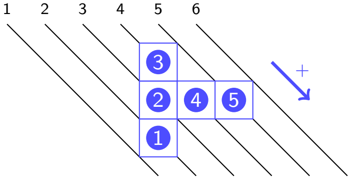

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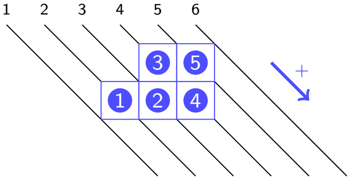

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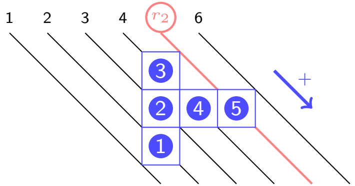

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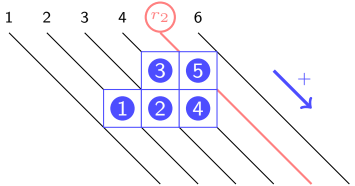

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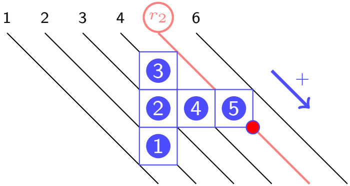

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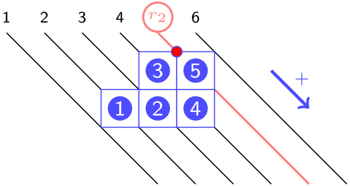

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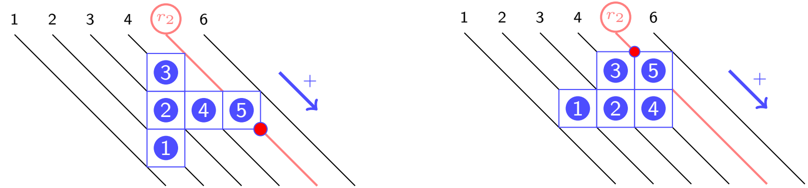

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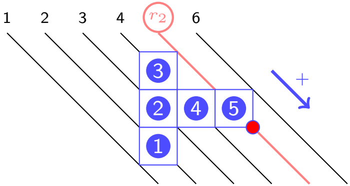

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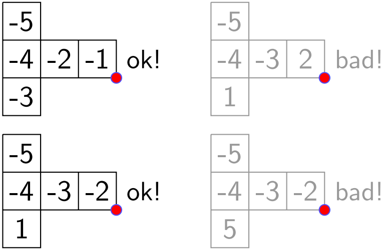

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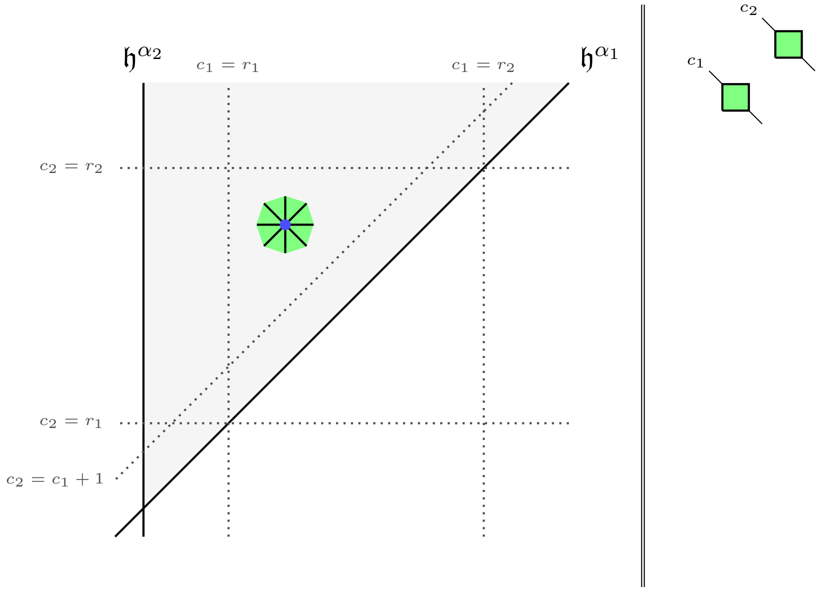

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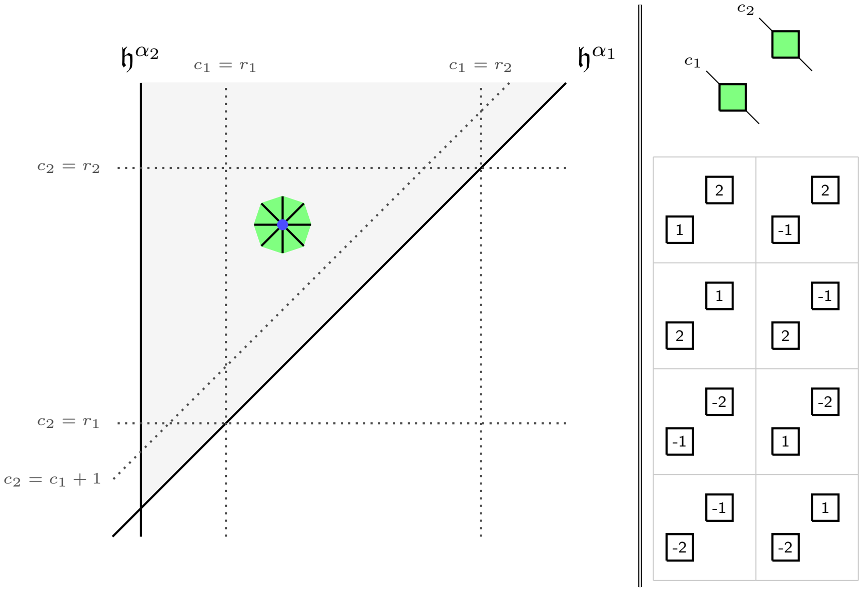

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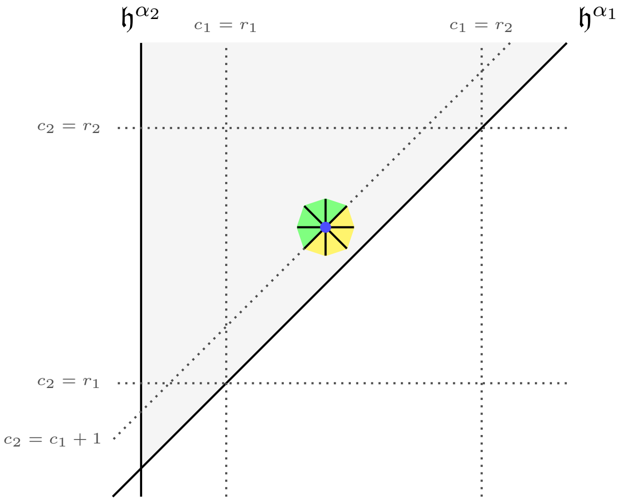

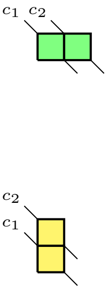

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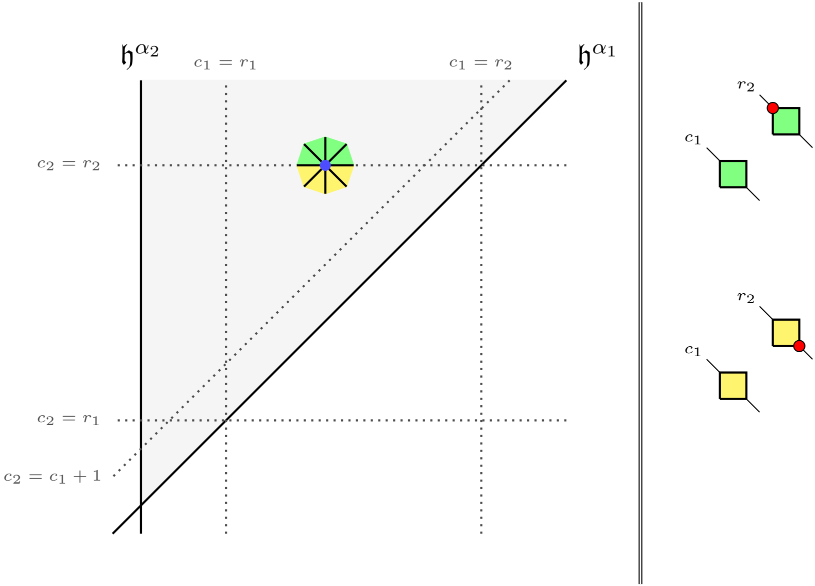

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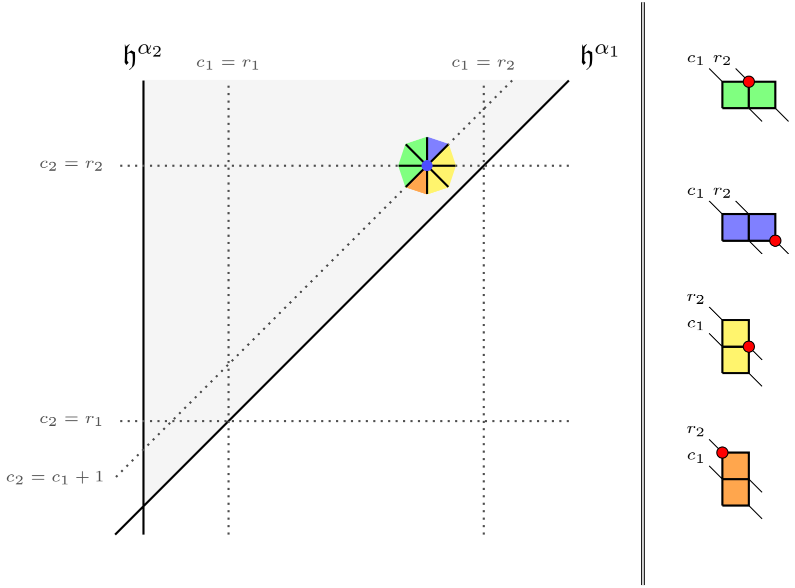

Description 1: Central characters are indexed by points $\mathbf{c}$ in $\mathbb{C}^k$.
Representations of H_k are indexed by skew local regions.
Basis indexed by chambers.

Description 2: Marked box arrangements.
Basis indexed by good fillings.

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Description 2: Marked box arrangements.
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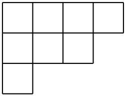
Description 3: Partitions.
Representation arise in Schur-Weyl duality with certain $U_q\mathfrak{gl}_n$ reps.

Centralizer properties

Let $U = U_q\mathfrak{gl}_n$ be the quantum group for $\mathfrak{gl}_n(\mathbb{C})$. We're interested in certain finite dimensional simple U-modules $L(\lambda)$ indexed by partitions:

$\lambda =$ 

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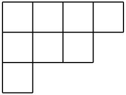
$\lambda =$ 

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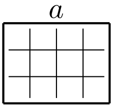
In particular, rectangular partitions:

$(a^c) = c$ 

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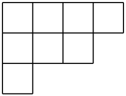
$\lambda =$ 

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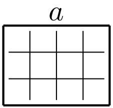
$(a^c) = c$ 

H_k has a commuting action with U on the space

$$L((a^c)) \otimes L((b^d)) \otimes \left(L(\square)\right)^{\otimes k}.$$

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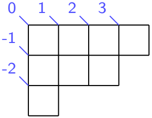
$\lambda =$ 

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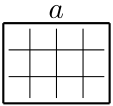
$(a^c) = c$ 

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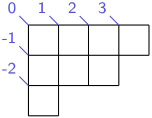
$\lambda =$ 

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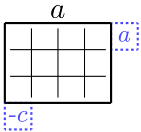
$(a^c) = c$ 

H_k has a commuting action with U on the space

$$L((a^c)) \otimes L((b^d)) \otimes \left(L(\square)\right)^{\otimes k}.$$

The content of a box is its diagonal number.

The eigenvalues of T_0 and T_k are controlled by the contents of addable boxes to (a^c) and (b^d).

Exploring $L((a^c)) \otimes L((b^d)) \otimes \left(L(\square)\right)^{\otimes k}$

Products of rectangles:

$$L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda) \qquad \text{(multiplicity one!)}$$

where Λ is the following set of partitions:
(Littlewood-Richardson, Okada)

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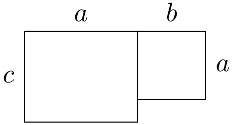

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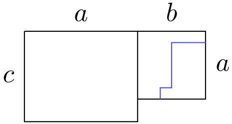

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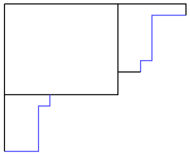

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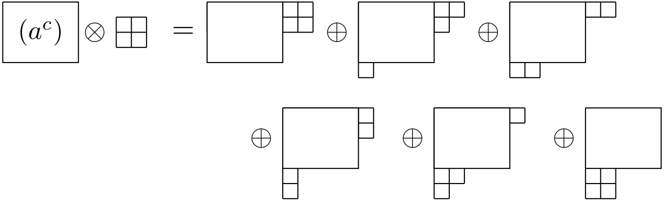
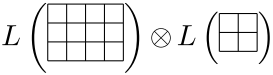
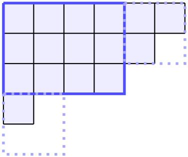
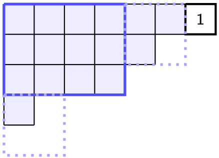






$$L\left(\ydiagram{4,4,4}\right) \otimes L\left(\ydiagram{2,2}\right) \otimes L\left(\ydiagram{1}\right)$$



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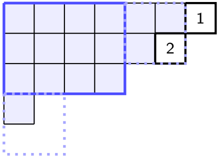

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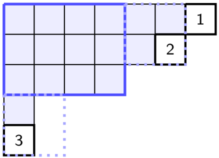

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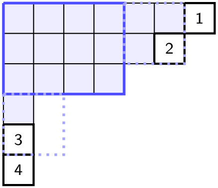

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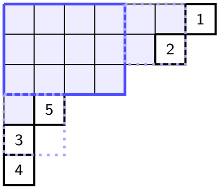
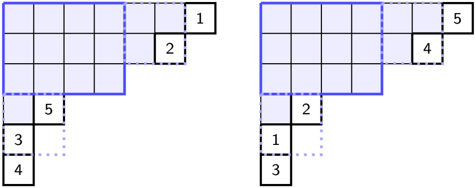


$(*)$ H_k representations in tensor space are labeled by certain partitions λ.

$$L\left(\yng(4,4,4)\right) \otimes L\left(\yng(2,2)\right) \otimes L\left(\yng(1)\right) \otimes L\left(\yng(1)\right) \otimes L\left(\yng(1)\right) \otimes L\left(\yng(1)\right) \otimes L\left(\yng(1)\right)$$



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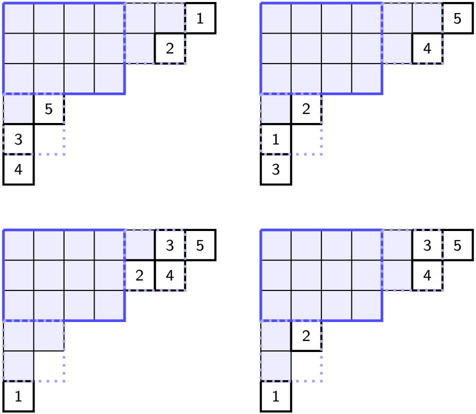

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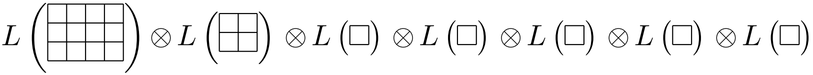
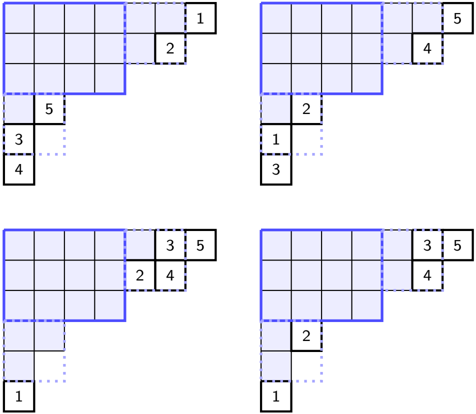
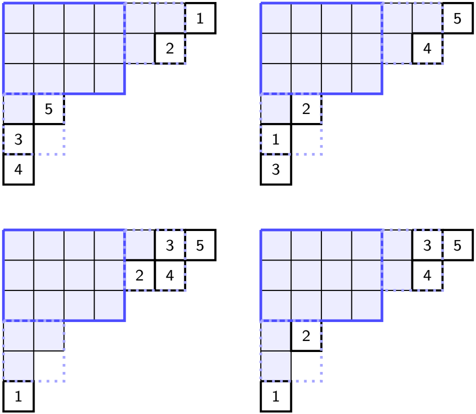




$(*)$ H_k representations in tensor space are labeled by certain partitions λ.

$(*)$ Basis labeled by tableaux from *some* partition μ in $(a^c) \otimes (b^d)$ to λ.

$$L\left(\ydiagram{4,4,4}\right) \otimes L\left(\ydiagram{2,2}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



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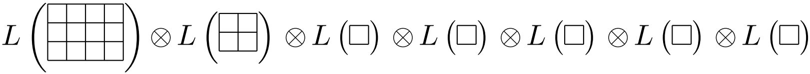
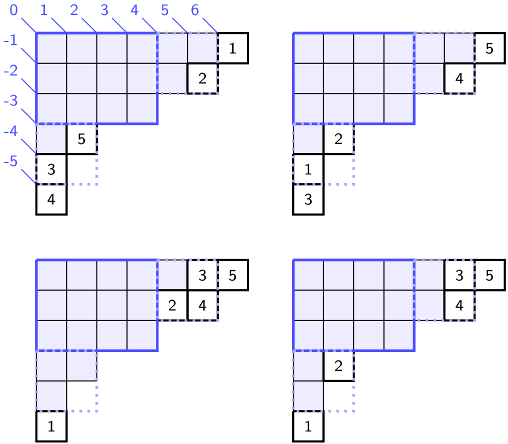
(∗) Calibrated





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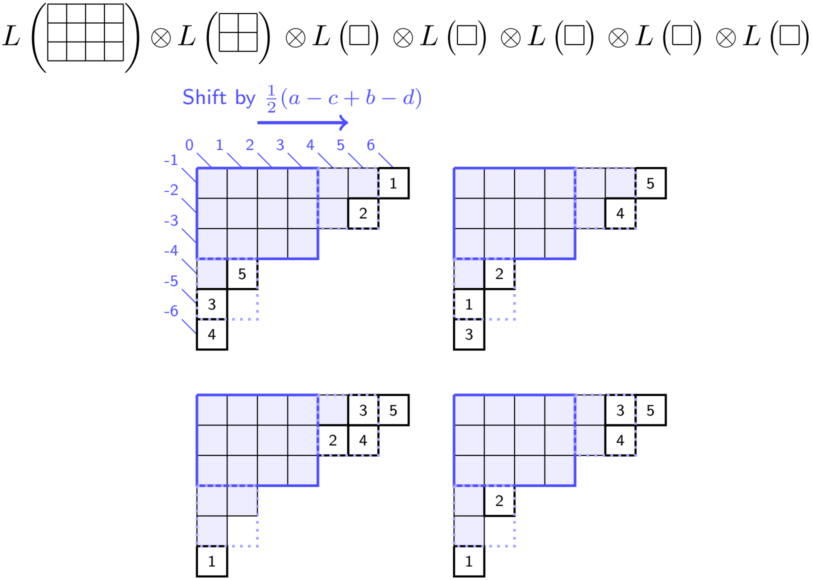

(∗) H_k representations in tensor space are labeled by certain partitions λ.

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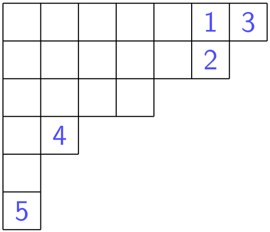
(∗) Calibrated: Y_i acts by t to the shifted content of box_i.

From {partitions in tensor space} to {box arrangements}



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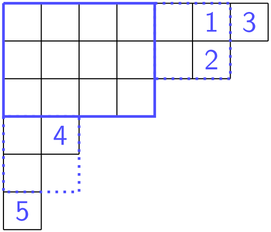

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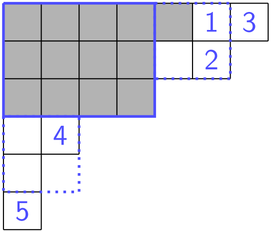

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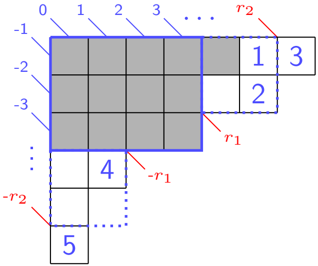

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$$\gamma(Y_1) = t^{4.5},\ \gamma(Y_2) = t^{3.5},\ \gamma(Y_3) = t^{r_2},\ \gamma(Y_4) = t^{-2.5},\ \gamma(Y_5) = t^{-r_2}.$$

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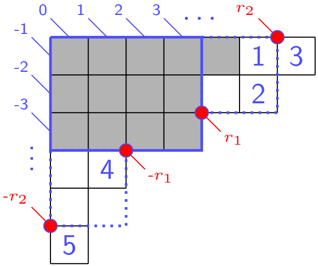

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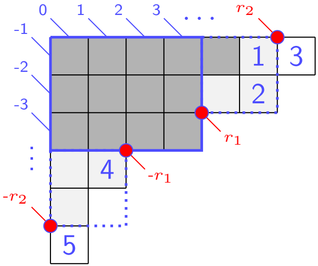

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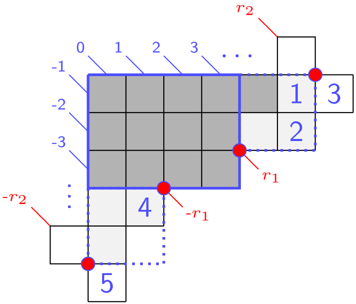

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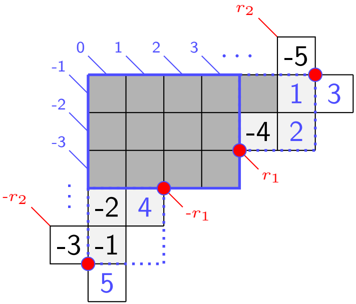

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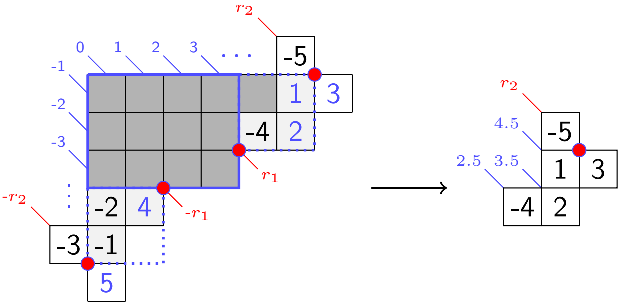

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Thanks!