

Combinatorics of affine Hecke algebras of type C.

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(joint with Arun Ram)

Dartmouth College and ICERM

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The affine type C Hecke algebra H_k is generated by invertible elements T_0, T_1, \dots, T_k with relations

$$\begin{array}{ccccccc}
 T_0 & T_1 & T_2 & & T_{k-2} & T_{k-1} & T_k \\
 \circ \text{---} \circ & \text{---} \circ & \text{---} \circ & \text{---} \cdots \text{---} \circ & \text{---} \circ & \text{---} \circ & \text{---} \circ \\
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Goal today:

Tell you 3 descriptions of calibrated irreducible reps of H_k , where “calibrated” means $\mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]$ is simultaneously diagonalized.

Central characters

The center of H_k is symmetric Laurent polynomials

$$Z(H_k) = \mathbb{C}[Y_1^{\pm 1}, \dots, Y_k^{\pm 1}]^{W_0}$$

w.r.t. the Weyl group W_0 of type C.

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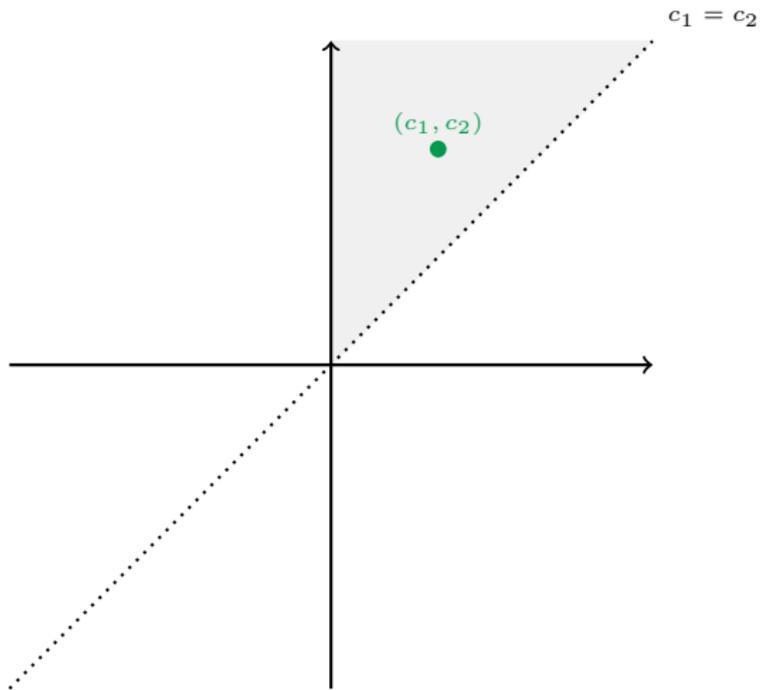
Description 1: Central characters are indexed by points \mathbf{c} in \mathbb{C}^k .

Central characters as points

Restrict to real points.

Fav equivalence class reps: $0 \leq c_1 \leq \cdots \leq c_k$. (W_0 acts by signed permutations)

When $k = 2$:

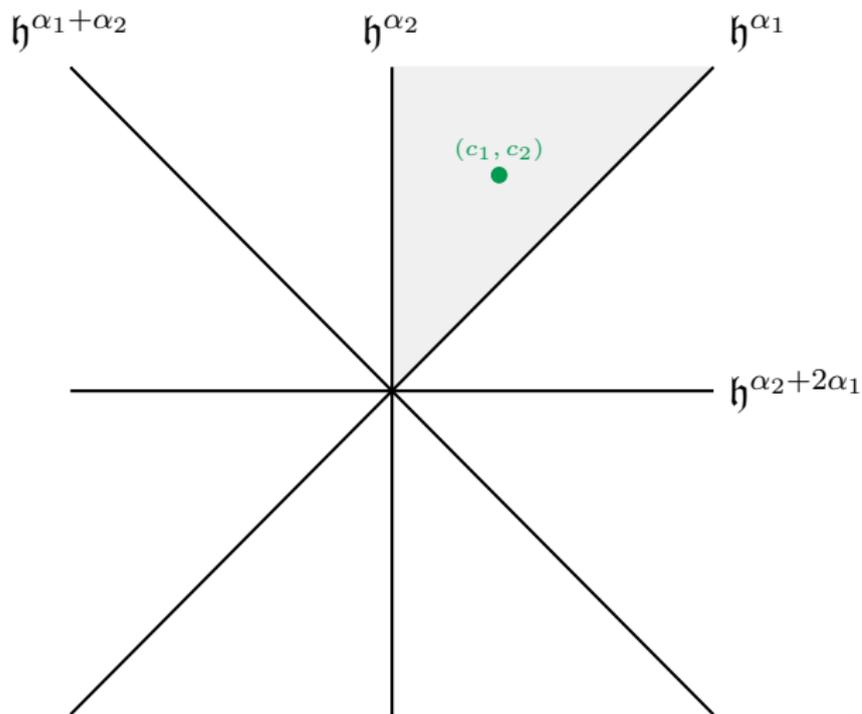


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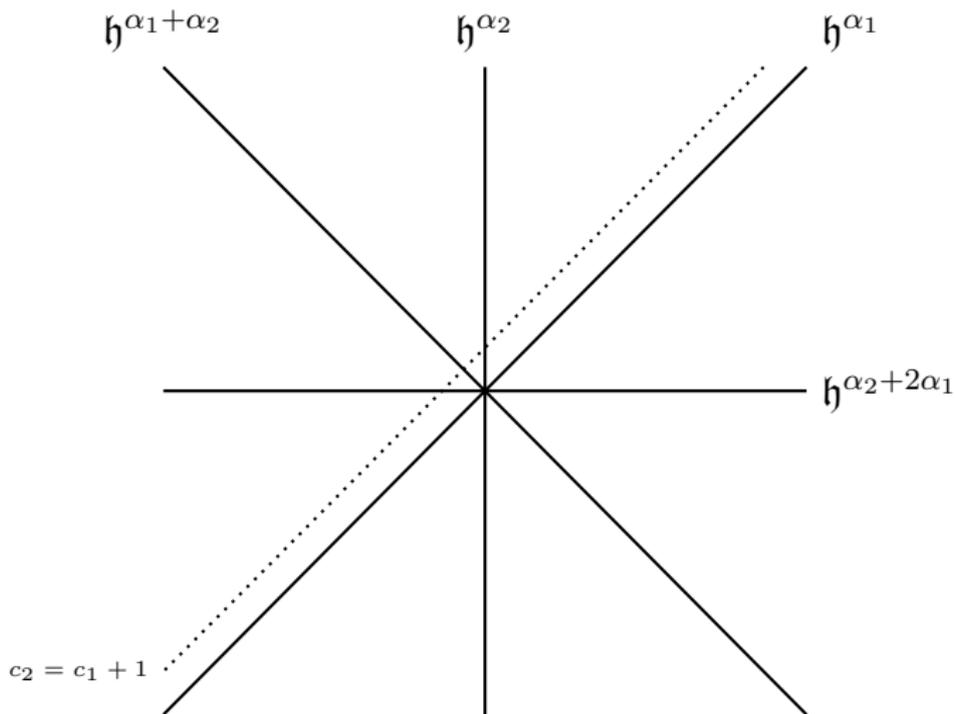


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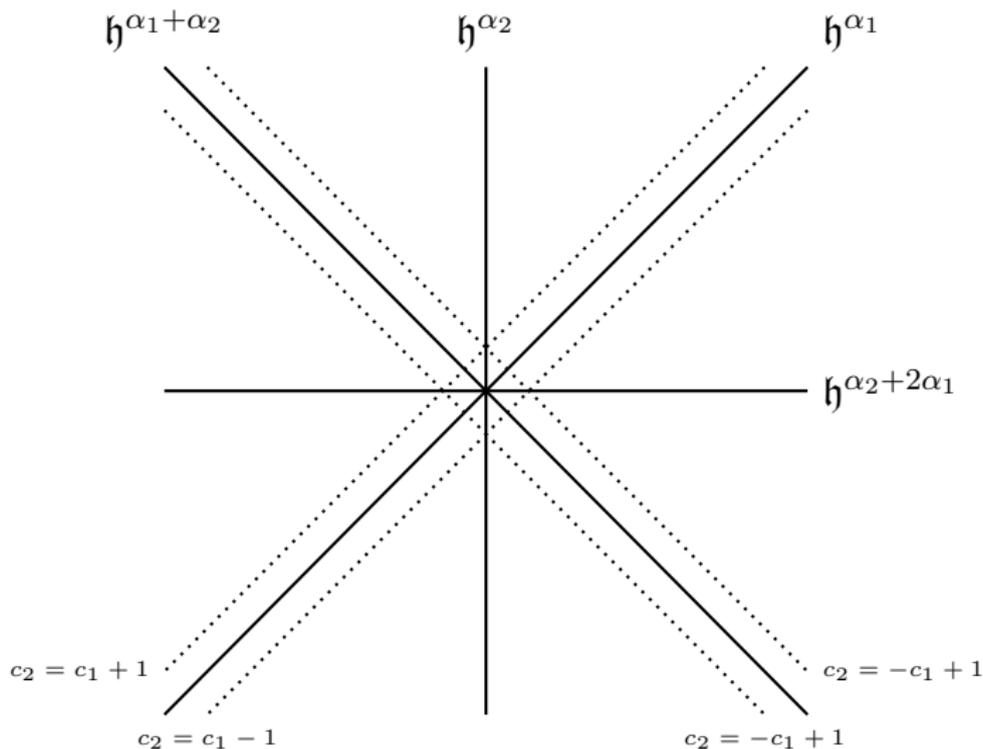


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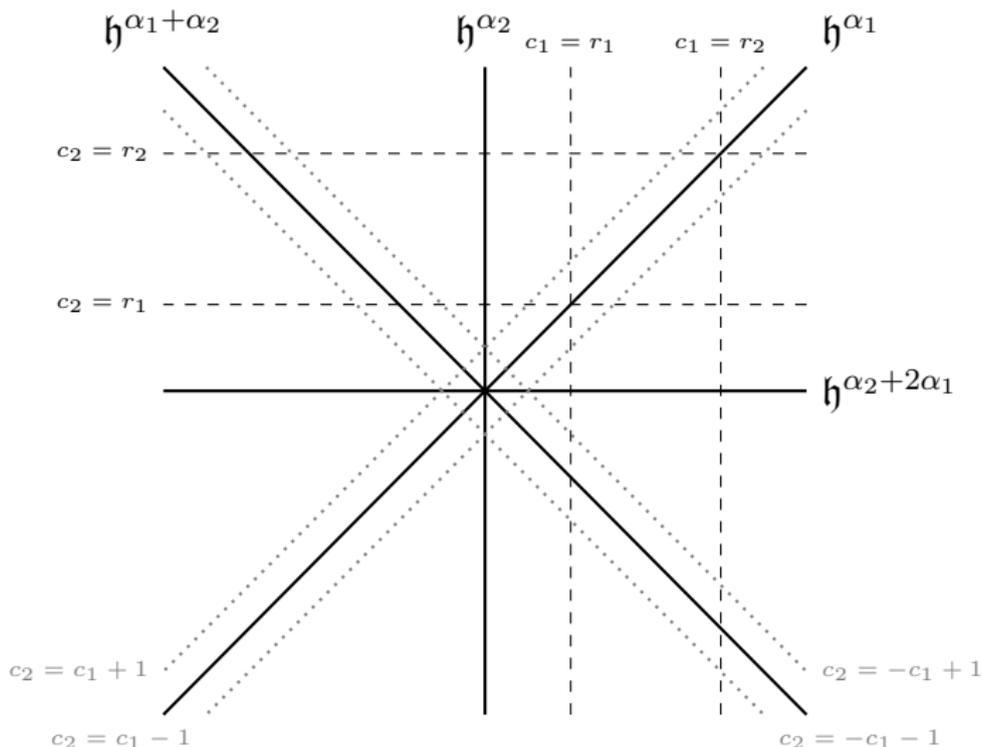


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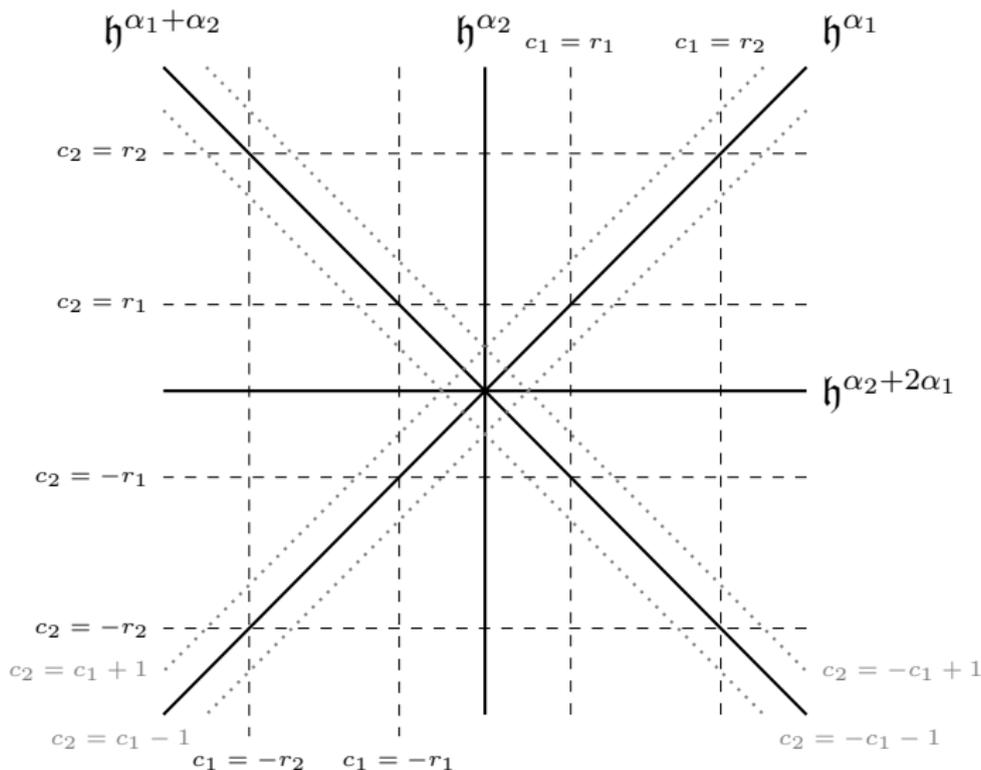
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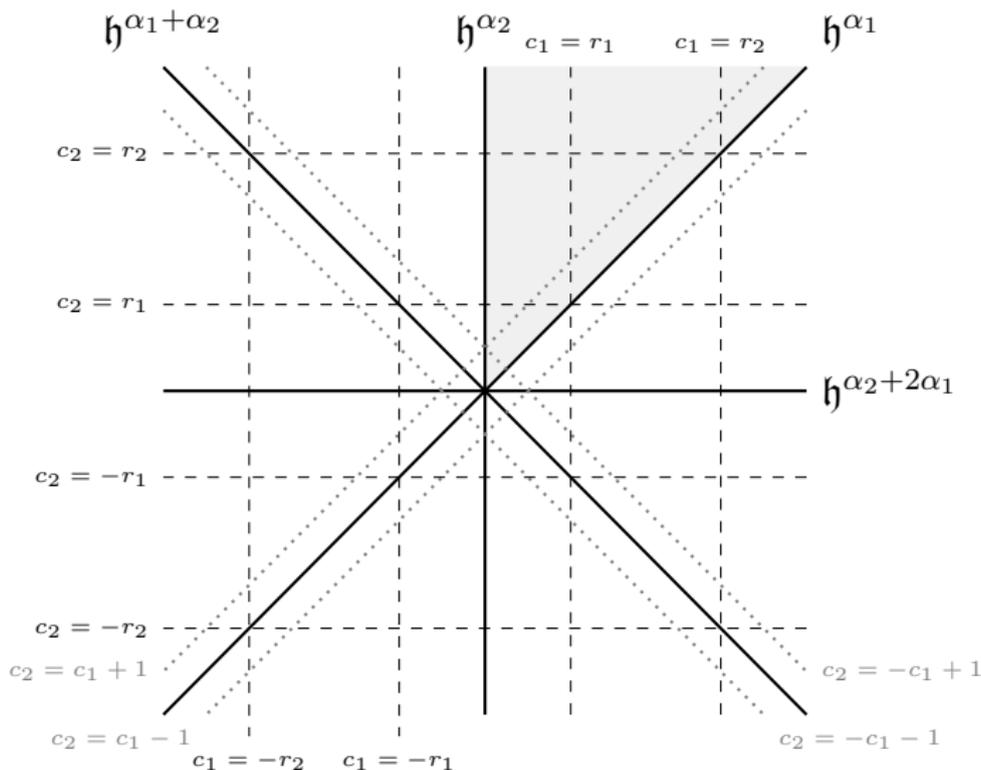
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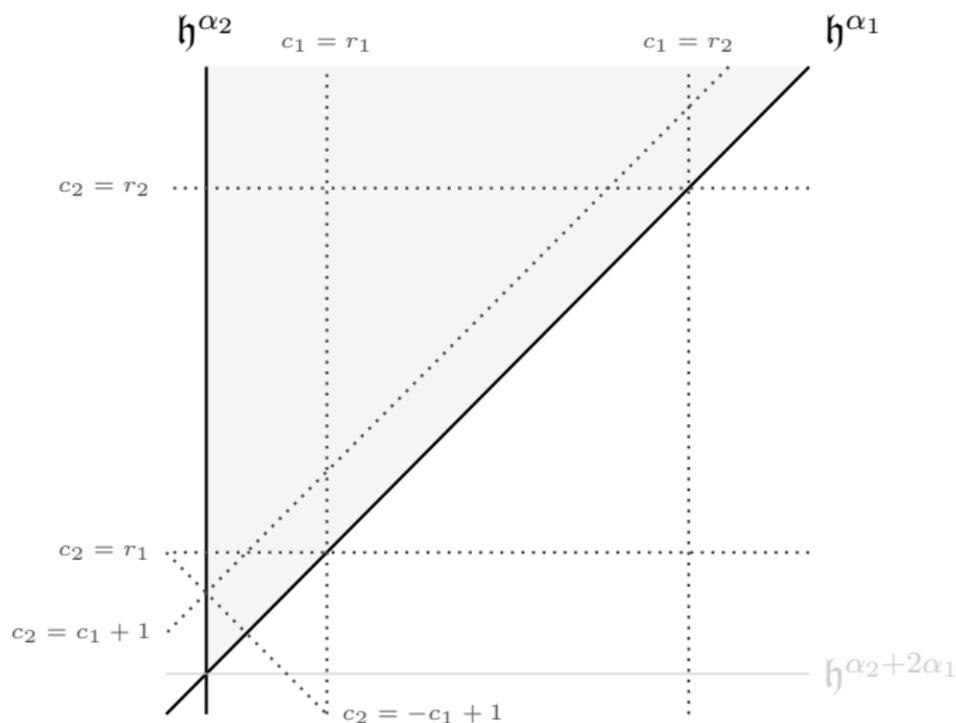
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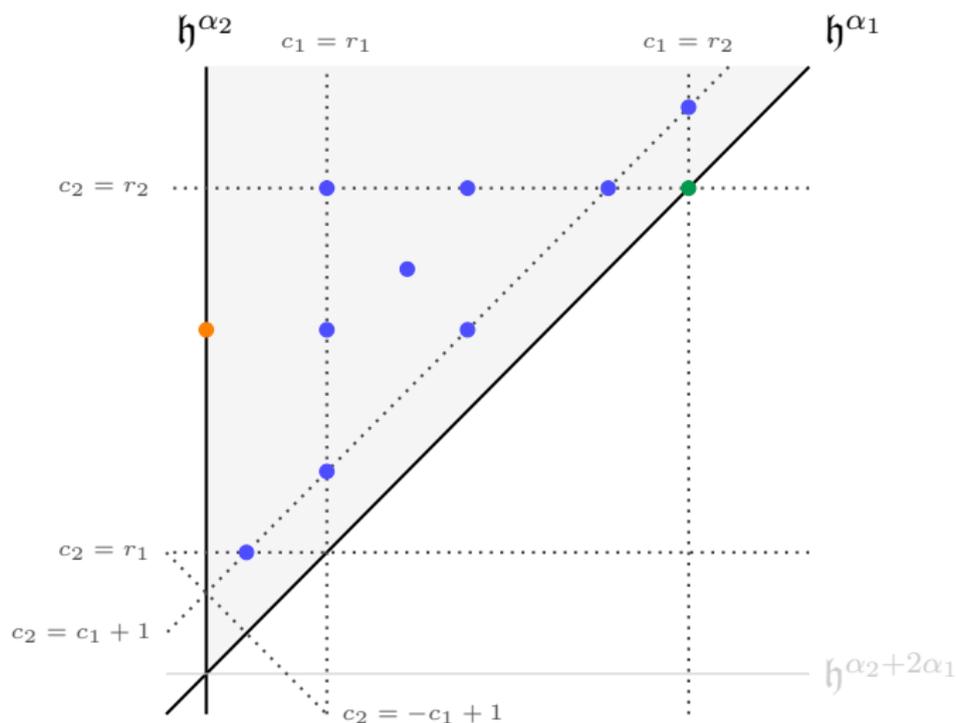
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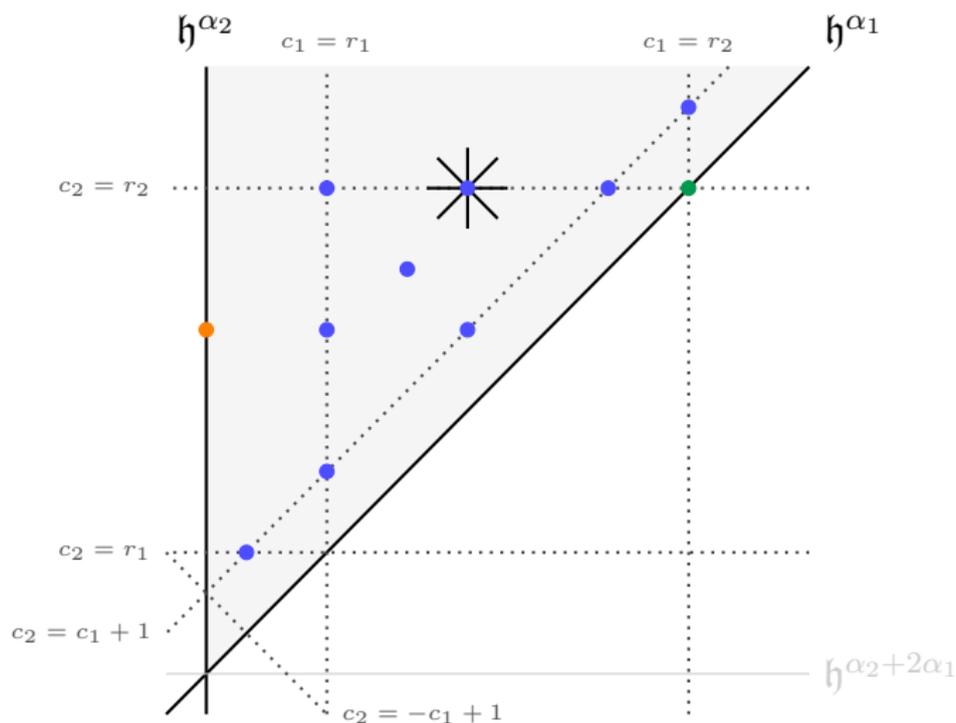
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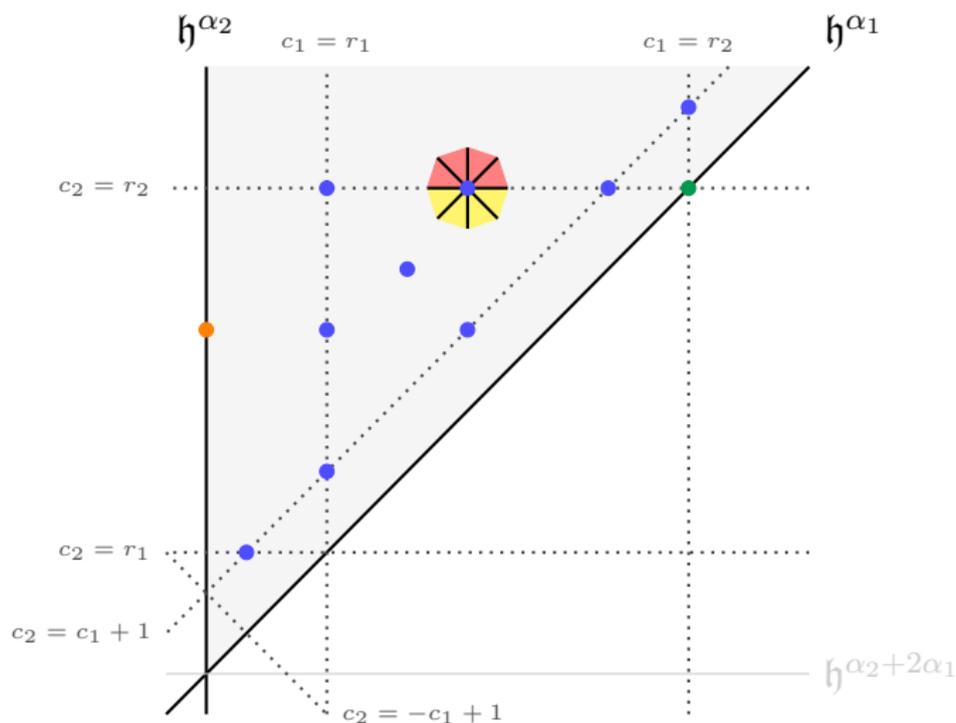
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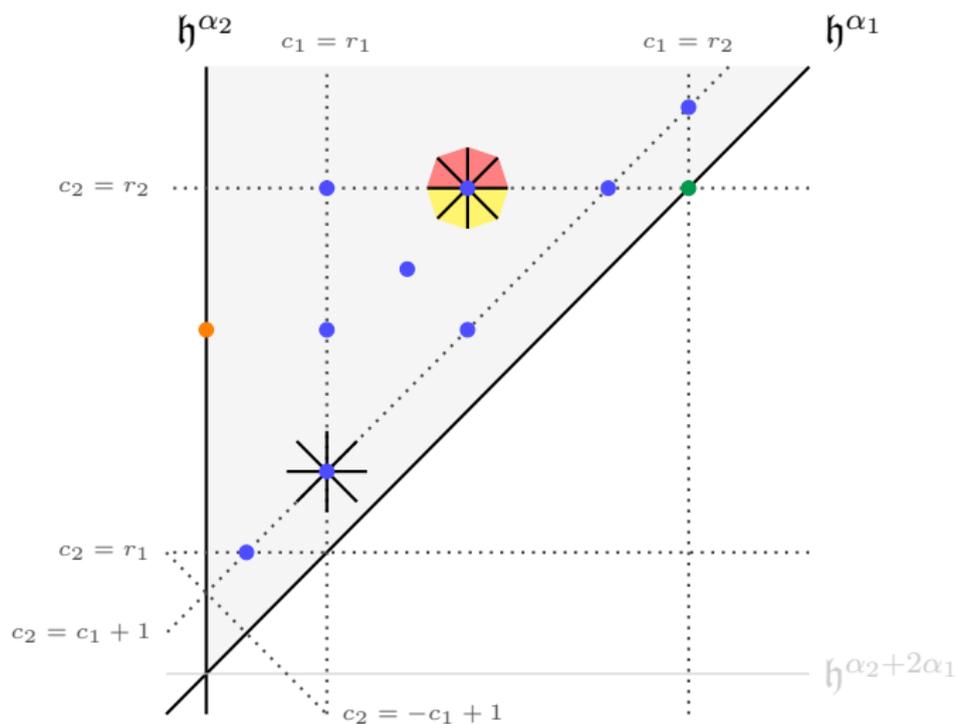
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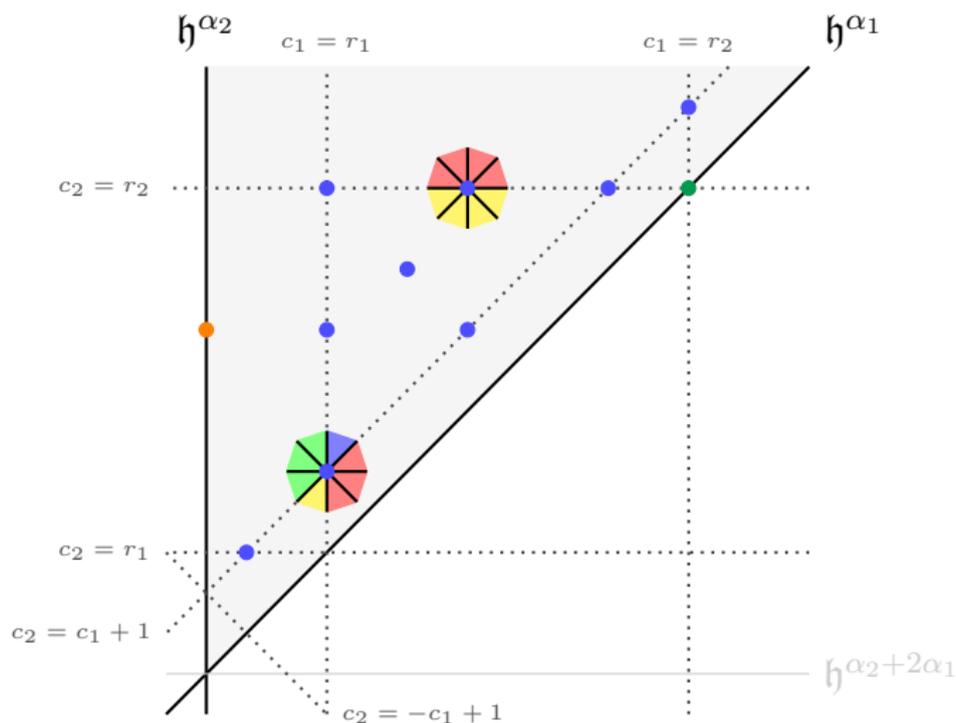
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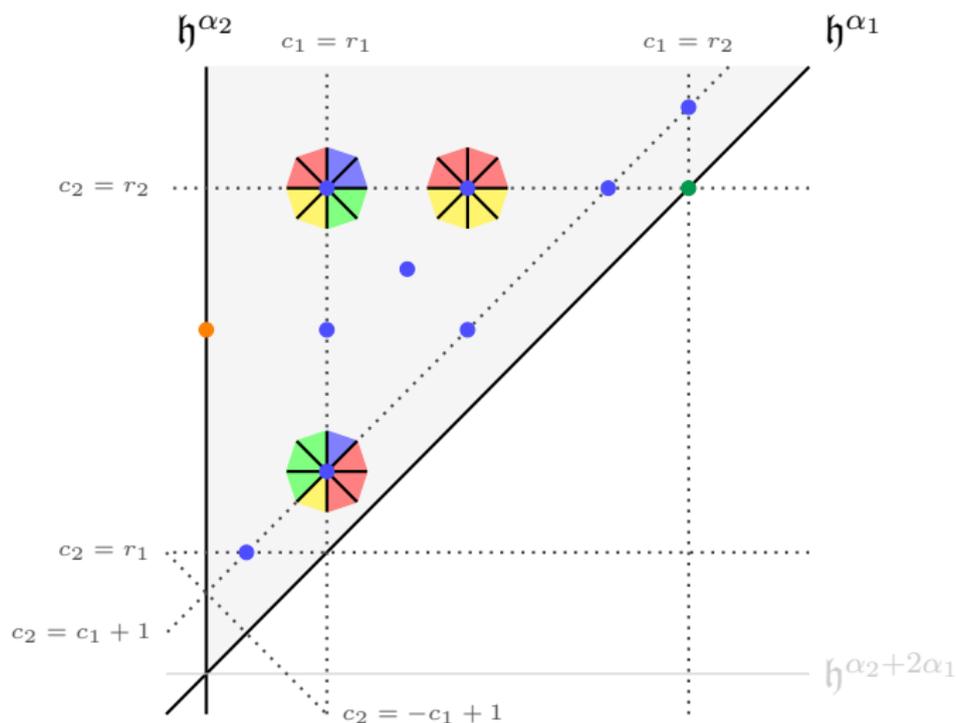
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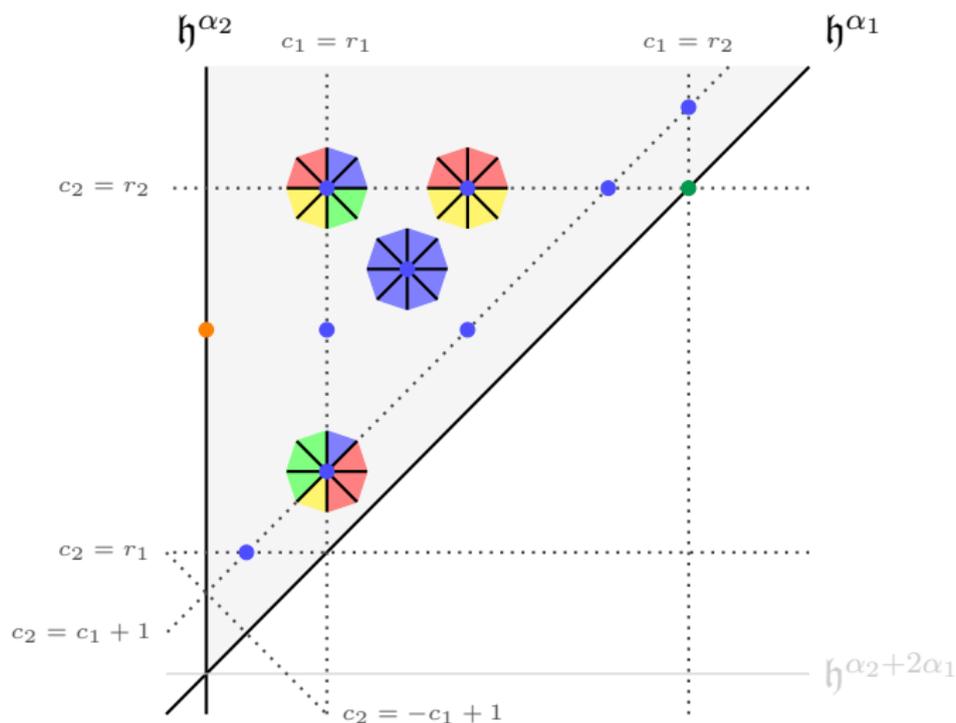
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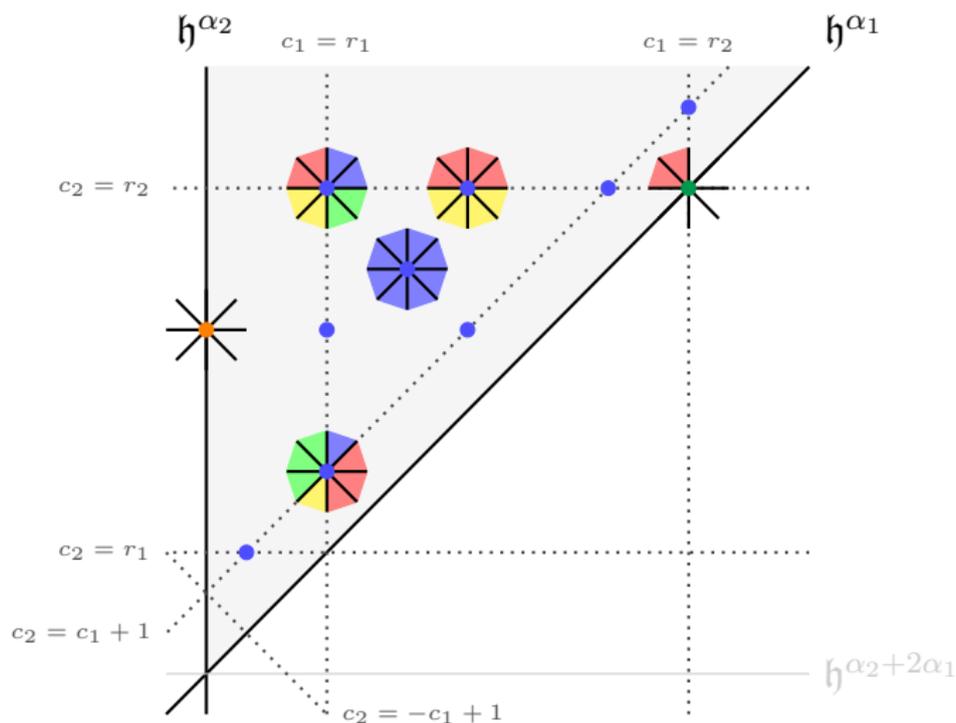
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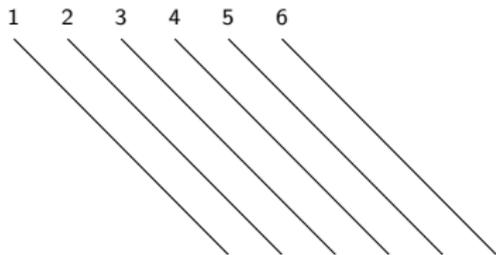
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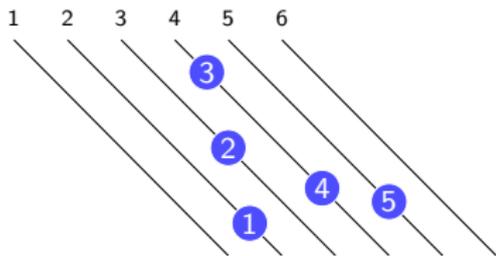
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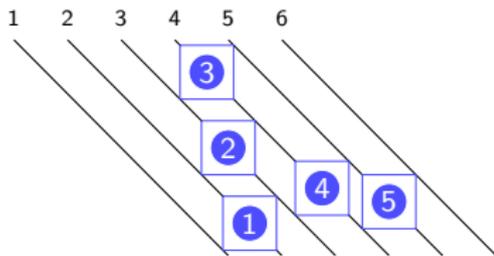
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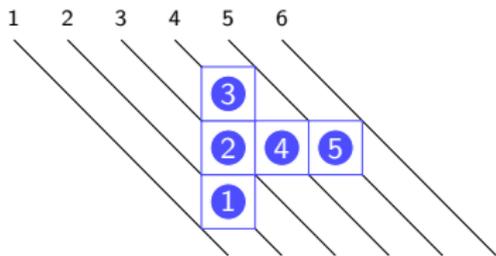
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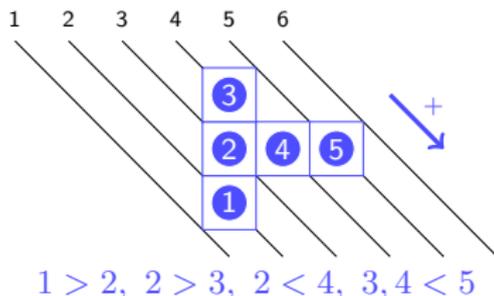
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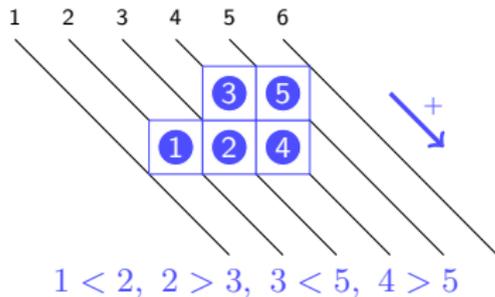
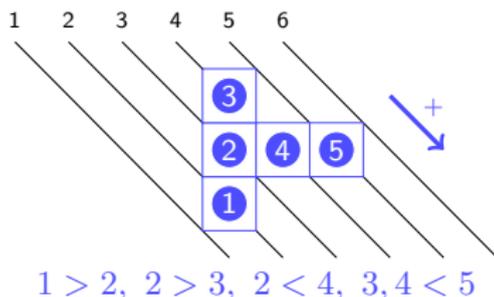
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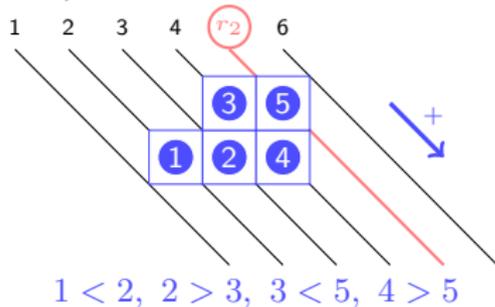
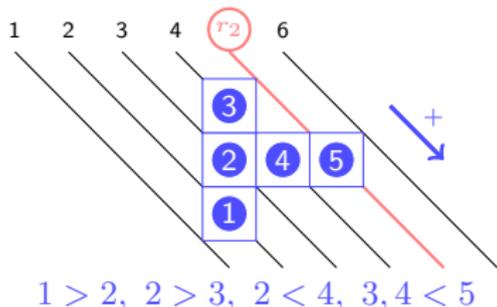
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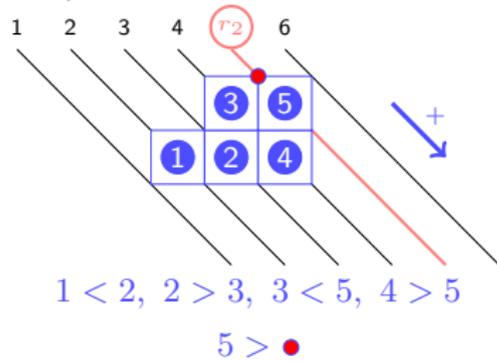
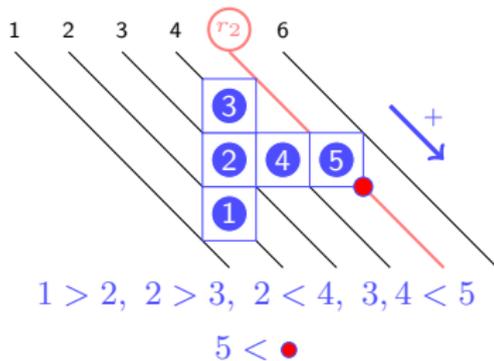
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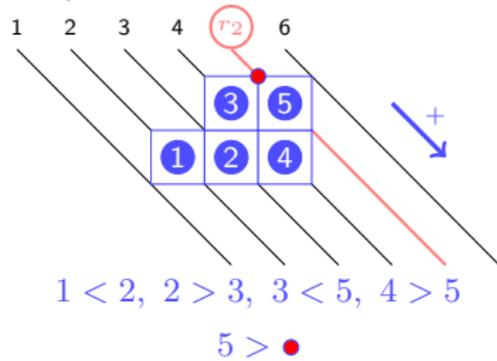
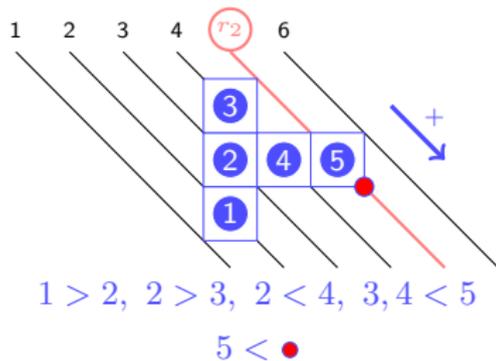
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Basis indexed by standard fillings with $\{\pm 1, \dots, \pm k\}$ with restrictions:

- (1) Exactly one of i or $-i$ appears.
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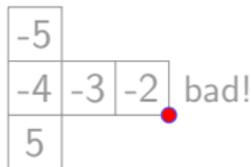
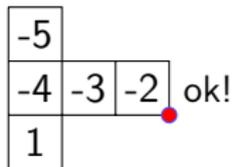
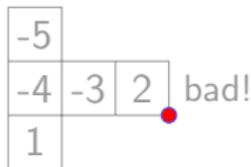
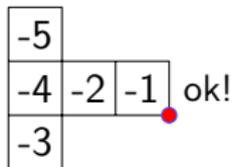
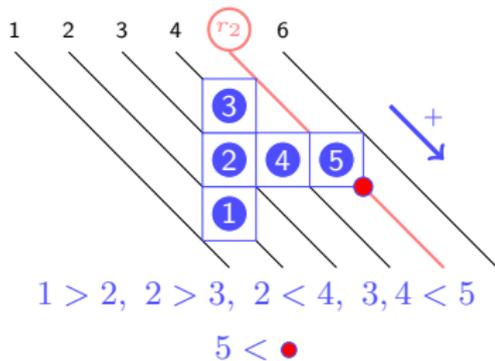
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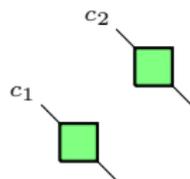
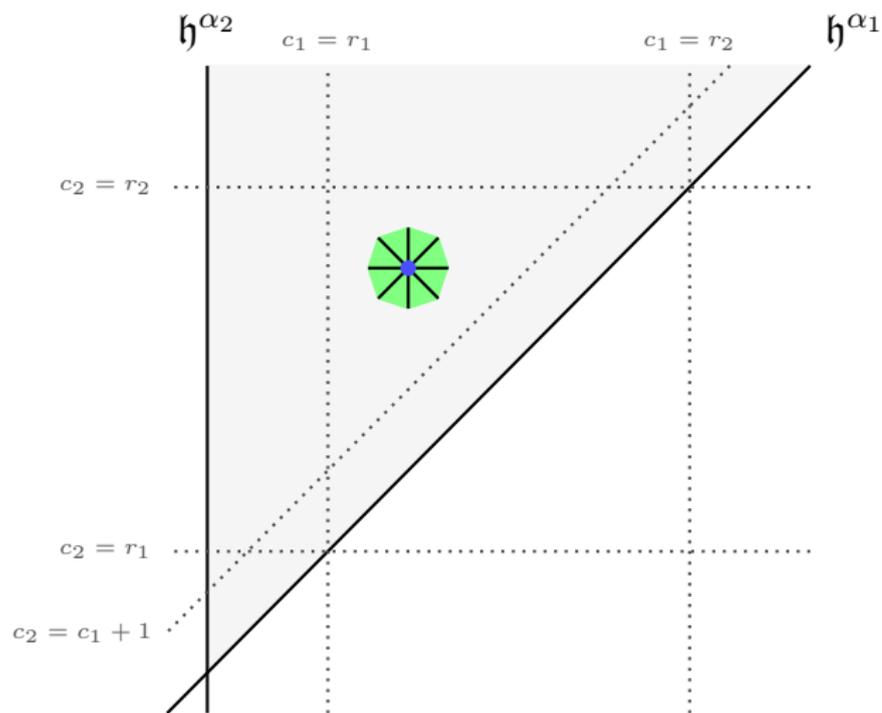
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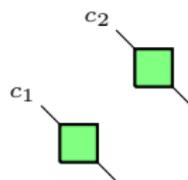
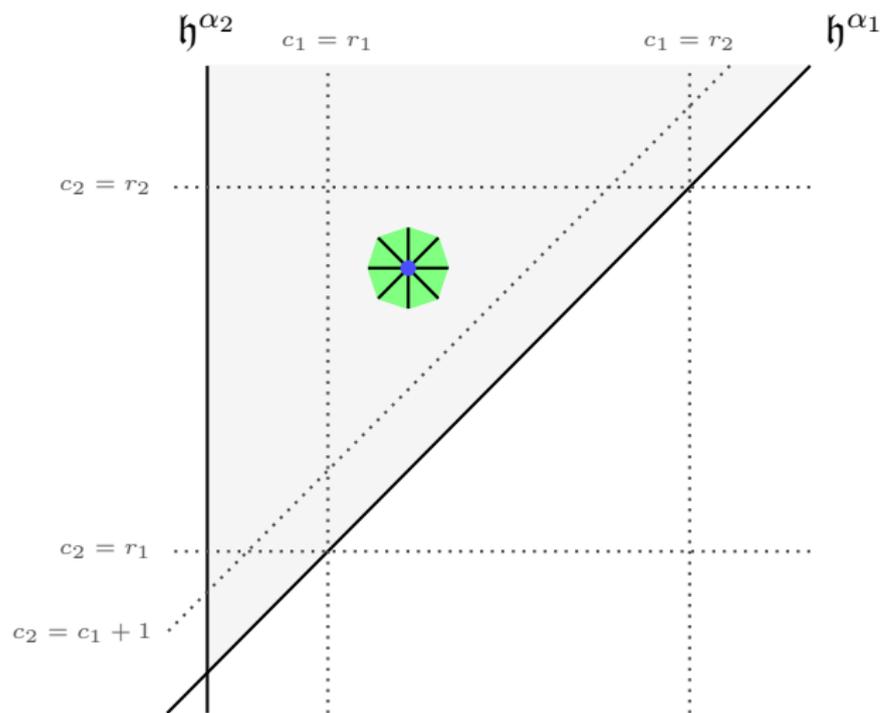
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Points versus box arrangements

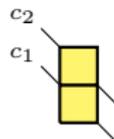
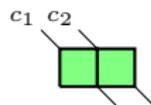
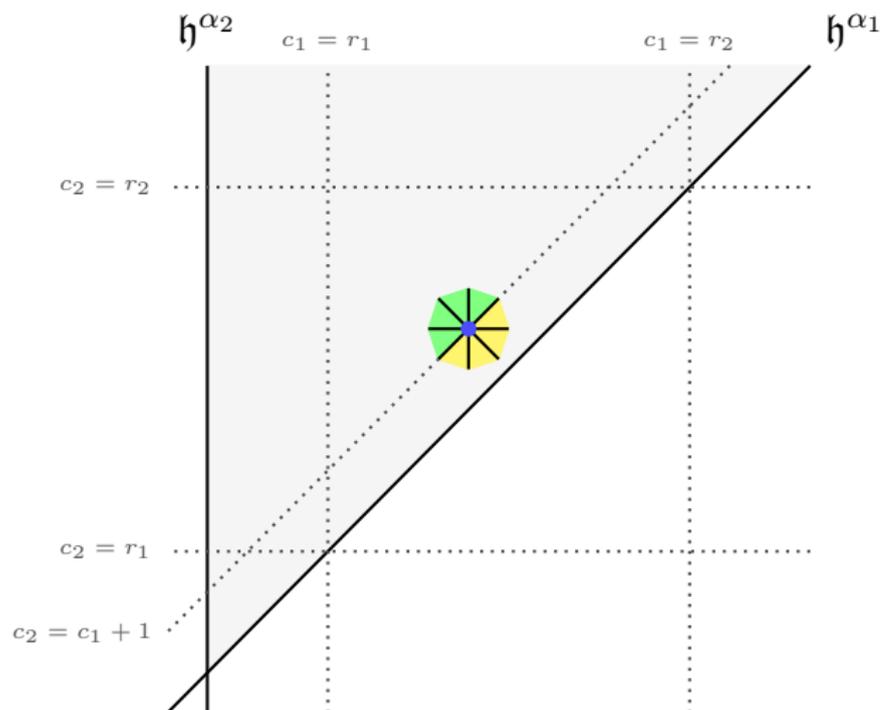


Points versus box arrangements

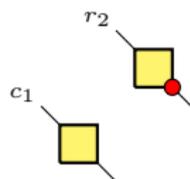
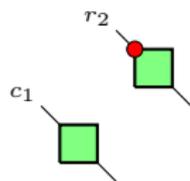
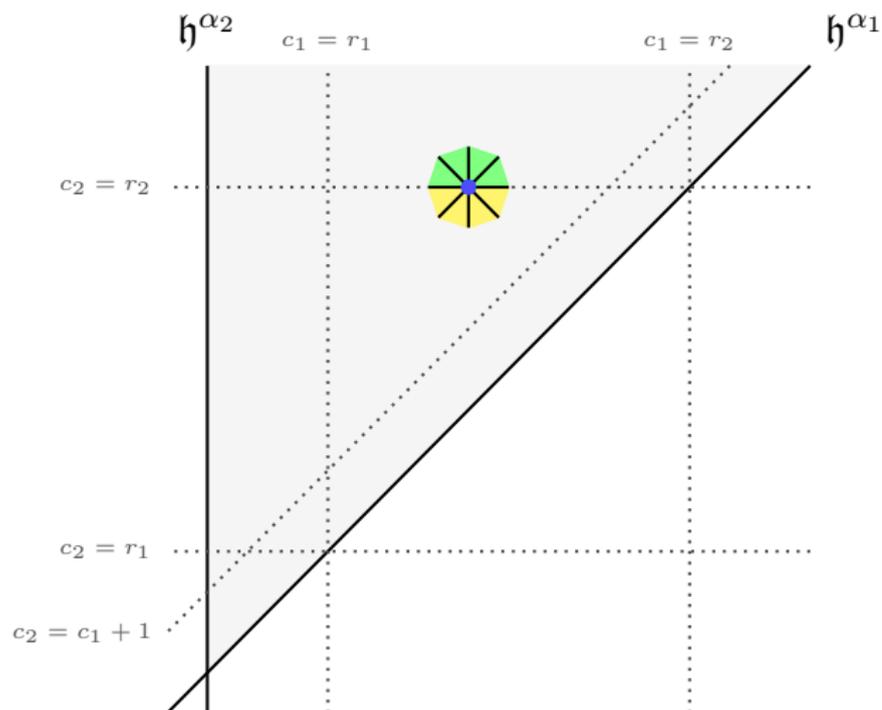


1	2	-1	2
2	1	2	-1
-1	-2	1	-2
-2	-1	-2	1

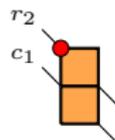
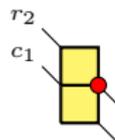
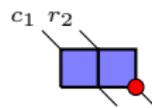
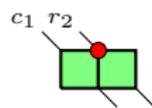
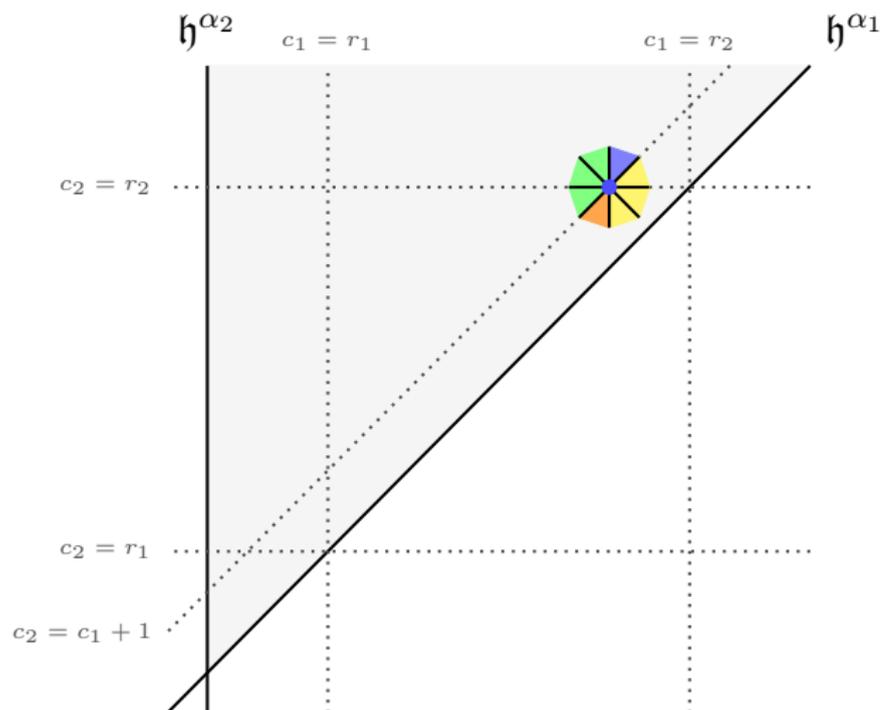
Points versus box arrangements



Points versus box arrangements



Points versus box arrangements



Description 1: Central characters are indexed by points \mathbf{c} in \mathbb{C}^k .
Representations of H_k are indexed by skew local regions.
Basis indexed by chambers.

Description 2: Marked box arrangements.
Basis indexed by good fillings.

Description 1: Central characters are indexed by points \mathbf{c} in \mathbb{C}^k .
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Description 3: Partitions.
Representation arise in Schur-Weyl duality with certain $U_q \mathfrak{gl}_n$ reps.

Centralizer properties

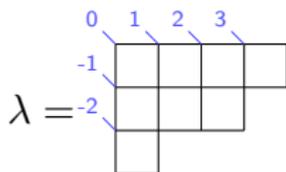
Let $U = U_q \mathfrak{gl}_n$ be the quantum group for $\mathfrak{gl}_n(\mathbb{C})$. We're interested in certain finite dimensional simple U -modules $L(\lambda)$ indexed by partitions:

$$\lambda = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \\ \hline \square & & & \\ \hline \end{array}$$

(drawn as a collection of boxes piled up and to the left)

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In particular, rectangular partitions:

$$(a^c) = c \begin{array}{|c|c|c|} \hline & & a \\ \hline & & \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

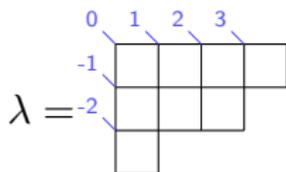
H_k has a commuting action with U on the space

$$L((a^c)) \otimes L((b^d)) \otimes (L(\square))^{\otimes k}.$$

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The eigenvalues of T_0 and T_k are controlled by the contents of addable boxes to (a^c) and (b^d) .

Exploring $L((a^c)) \otimes L((b^d)) \otimes (L(\square))^{\otimes k}$

Products of rectangles:

$$L((a^c)) \otimes L((b^d)) = \bigoplus_{\lambda \in \Lambda} L(\lambda) \quad (\text{multiplicity one!})$$

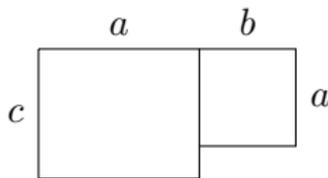
where Λ is the following set of partitions:
(Littlewood-Richardson, Okada)

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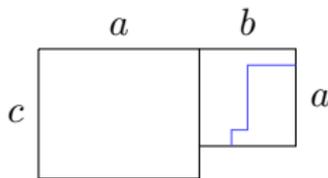


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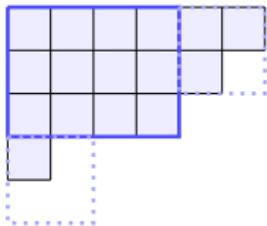
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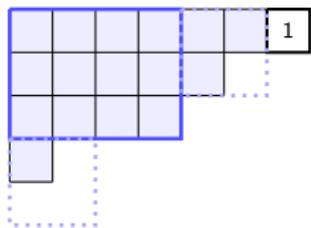
The diagram shows the decomposition of the tensor product of a rectangle (a^c) and a 2×2 square into six Young diagrams. The first row contains three diagrams, and the second row contains three diagrams, all separated by \oplus symbols. The diagrams represent the following partitions:

- Row 1, Diagram 1: A rectangle with a 2×2 square attached to its top-right corner.
- Row 1, Diagram 2: A rectangle with a 1×2 horizontal bar attached to its top-right corner and a 1×1 square attached to its bottom-left corner.
- Row 1, Diagram 3: A rectangle with a 1×1 square attached to its top-right corner and a 2×1 vertical bar attached to its bottom-left corner.
- Row 2, Diagram 1: A rectangle with a 1×1 square attached to its top-right corner and a 2×1 vertical bar attached to its bottom-left corner.
- Row 2, Diagram 2: A rectangle with a 1×1 square attached to its top-right corner and a 1×2 horizontal bar attached to its bottom-left corner.
- Row 2, Diagram 3: A rectangle with a 2×2 square attached to its bottom-right corner.

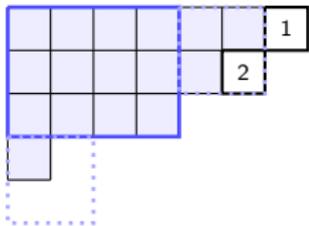
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right)$$



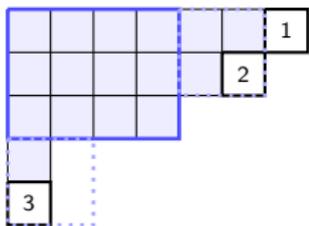
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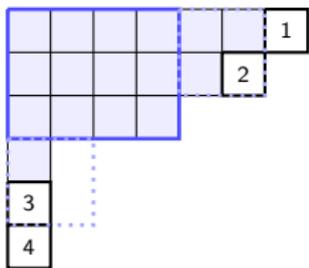
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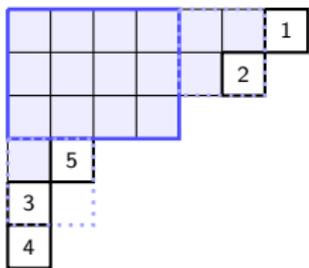
$$L\left(\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



$$L\left(\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$

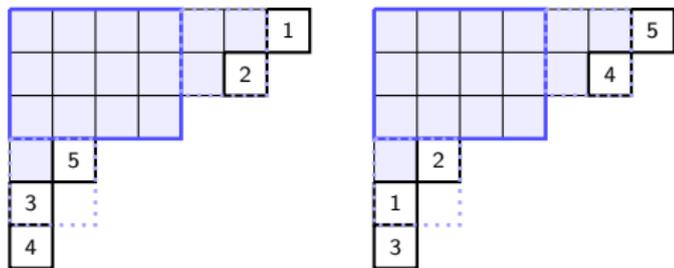


$$L\left(\begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \square & \square & \square & \square \\ \hline \end{array}\right) \otimes L\left(\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}\right) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square) \otimes L(\square)$$



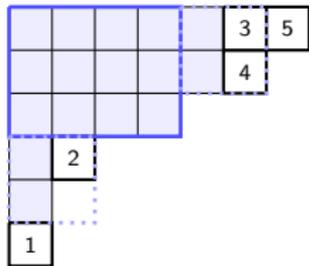
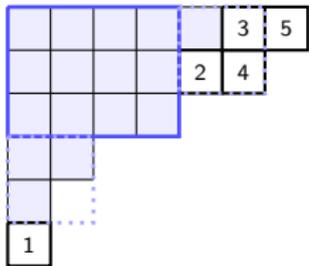
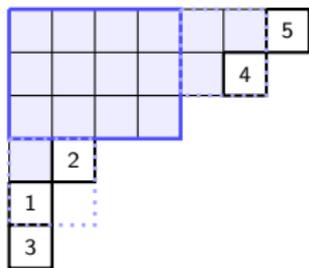
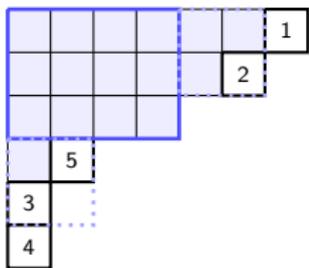
(*) H_k representations in tensor space are labeled by certain partitions λ .

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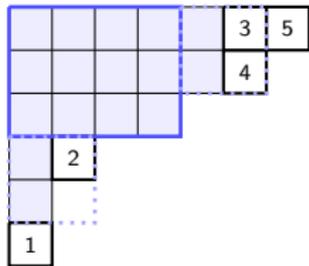
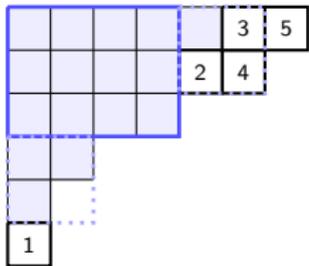
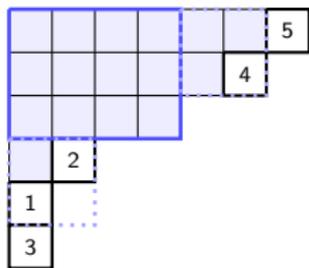
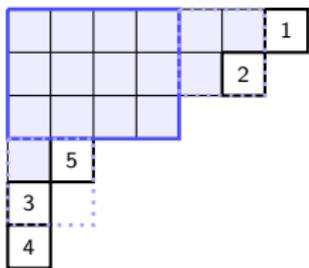
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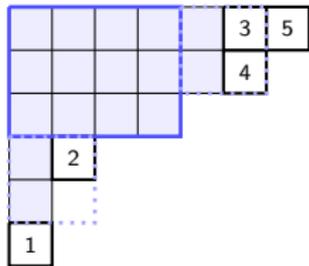
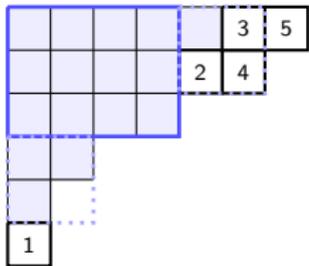
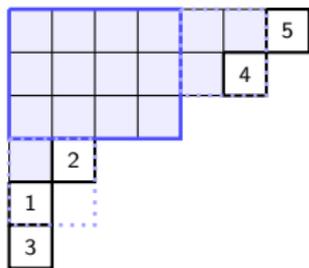
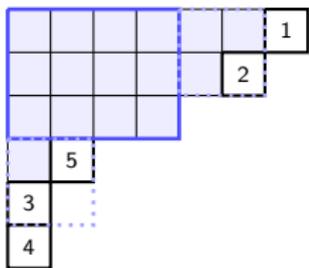
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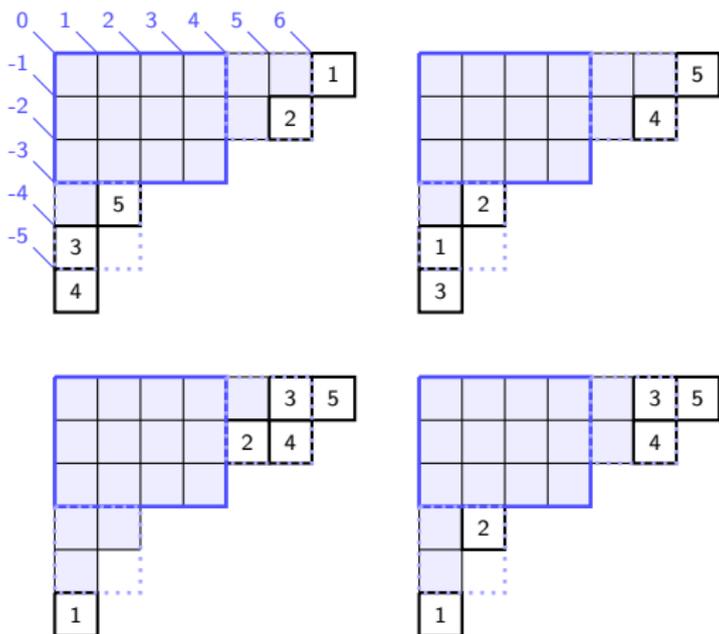
- (*) H_k representations in tensor space are labeled by certain partitions λ .
- (*) Basis labeled by tableaux from *some* partition μ in $(a^c) \otimes (b^d)$ to λ .

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- (*) Calibrated

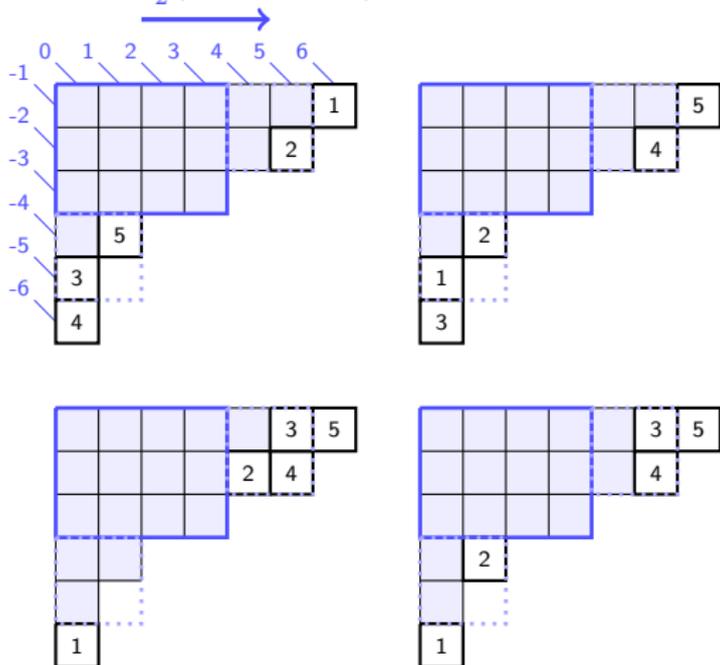
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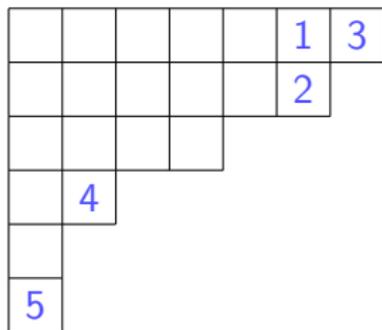
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Shift by $\frac{1}{2}(a-c+b-d)$

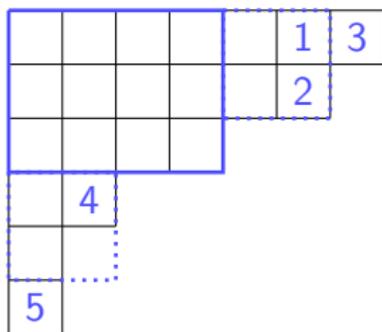


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- (*) Calibrated: Y_i acts by t to the shifted content of box_i .

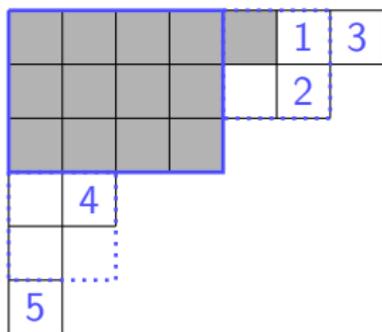
From {partitions in tensor space} to {box arrangements}



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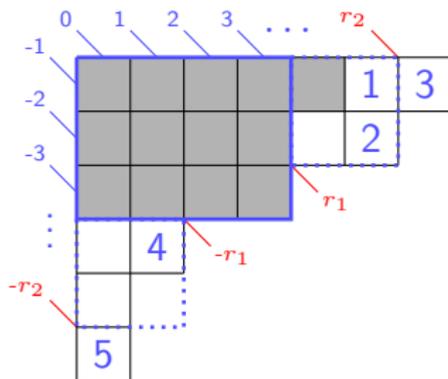


From {partitions in tensor space} to {box arrangements}



■ = boxes that must appear in the partition at level 0.

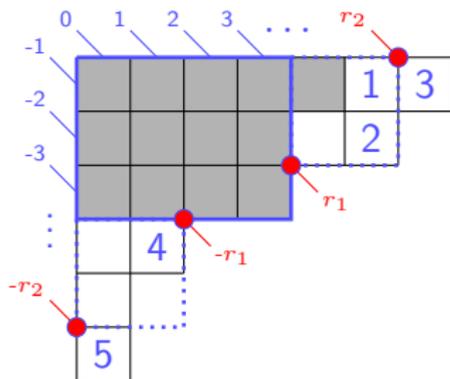
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$$\gamma(Y_1) = t^{4.5}, \quad \gamma(Y_2) = t^{3.5}, \quad \gamma(Y_3) = t^{r_2}, \quad \gamma(Y_4) = t^{-2.5}, \quad \gamma(Y_5) = t^{-r_2}.$$

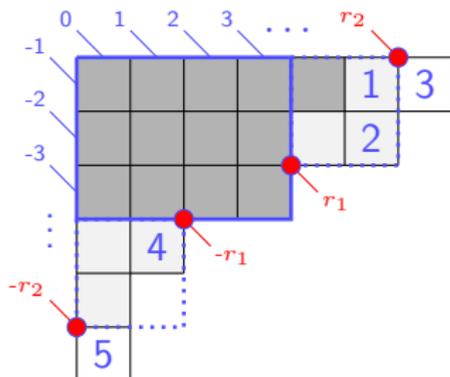
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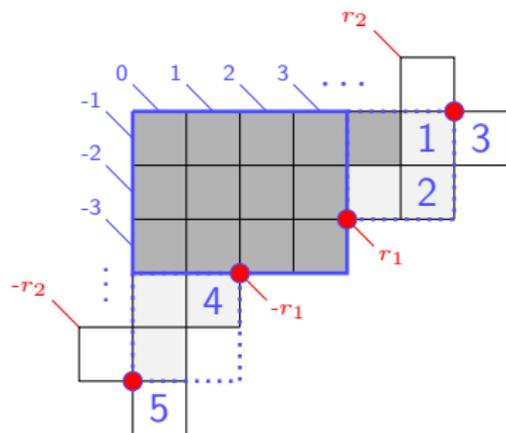
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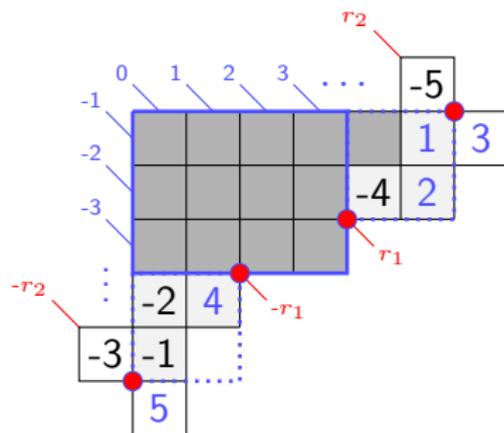
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From {partitions in tensor space} to {box arrangements}



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Thanks!



$$P(c) = \{ \xi \in \mathbb{Z}^3 \mid \xi \in \mathbb{Z}^3 \setminus \{ \xi \mid (a+c+b+d)/4 \mid a-c-b-d \} \cup \{ \xi \in \mathbb{Z}^3 \mid \xi - c_j = \pm 1 \} \}$$

$$= \{ \xi \in \mathbb{Z}^3 \mid \xi \in \mathbb{Z}^3 \setminus \{ \xi \mid (a+c+b+d)/4 \mid a-c-b-d \} \cup \{ \xi \in \mathbb{Z}^3 \mid \xi - c_j = \pm 1 \} \}$$

$$\frac{1}{2}(a-c+b-d) = \frac{1}{2}(8-6+6-5) = \frac{3}{2} = 1.5$$

$$\frac{1}{2}(a+c+b+d) = \frac{1}{2}(8+6+6+5) = 12.5$$

central character = $(-1)^{s_1} \dots (-1)^{s_n}$

$$Z(c) = \{ \xi \in \mathbb{Z}^3 \mid \xi \in \mathbb{Z}^3 \setminus \{ \xi \mid \xi - c_j = \pm 1 \} \}$$

$$R = \{ \xi \in \mathbb{Z}^3 \mid \xi \in \mathbb{Z}^3 \setminus \{ \xi \mid \xi - c_j = \pm 1 \} \cup \{ \xi \in \mathbb{Z}^3 \mid \xi - c_j = \pm 1 \} \}$$

"standard tableaux" =

$$\begin{pmatrix} 87 & 86 & 85 & 84 & 83 & 82 \\ 81 & 80 & 79 & 78 & 77 & 76 \\ 70 & 69 & 68 & 67 & 66 & 65 \\ 59 & 58 & 57 & 56 & 55 & 54 \\ 43 & 42 & 41 & 40 & 39 & 38 \\ 22 & 21 & 20 & 19 & 18 & 17 \\ 10 & 9 & 8 & 7 & 6 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix} = w$$

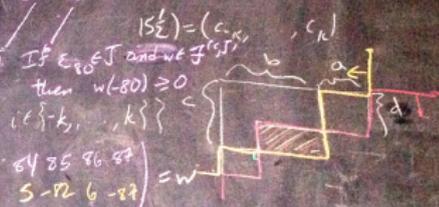


standard tableaux as shape (6,3)

local region must be on pos side of solid hyps
 $J = \{ \text{dashed hyps local region is on neg side of} \}$

$$\xi \in w(c, J) \text{ is } R(w) \cap Z(c) = \emptyset, R(w) \cap P(c) = J$$

(pos side) Solid



Conjecture