HELLO

my name is

Zajj Daugherty

John Wesley Young Research Instructor
Dartmouth College

Postdoctoral Researcher, ICERM

A little about me

- **Harvey Mudd College**, BS 2005
  Thesis: "An algebraic approach to Voting Theory"
- **University of WI – Madison**, Ph.D 2010
  Advisor: Arun Ram
  Thesis: "Two-boundary centralizer algebras"
- **St. Olaf College**, Visiting Assistant Professor 2010-2011
- **Dartmouth College**, JWY Research Instructor 2011-2014

More at `http://www.math.dartmouth.edu/~zdaugherty/`

A few of my favorite things: Schur-Weyl duality

Classical example: (Schur 1901)

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$$g \cdot (v_1 \otimes v_2 \otimes \cdots \otimes v_k) = gv_1 \otimes gv_2 \otimes \cdots \otimes gv_k.$$

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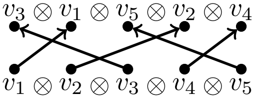
S_k also acts on $(\mathbb{C}^n)^{\otimes k}$ by place permutation.



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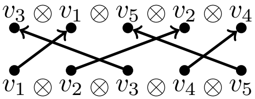

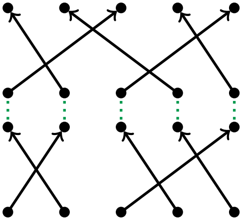

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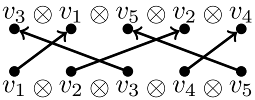

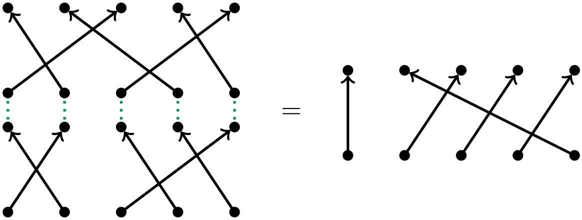

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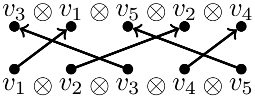

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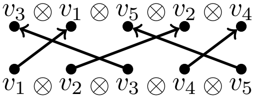

These actions commute! Moreover, they centralize each other.

Why this is exciting:

In general, if

- V is a vector space,
- $\mathcal{A} \subseteq \mathrm{End}(V)$, and
- $\mathcal{B} = \mathrm{End}_{\mathcal{A}}(V) = \{b \in \mathrm{End}(V) \mid ab = ba\}$.

Then under 'good' conditions, commuting actions provides a duality between the $\mathcal{A}$-modules and the $\mathcal{B}$-modules inside of V.

A few of my favorite things: Diagram algebras

Start with an algebra $\mathcal{A}$ (e.g.

$\mathcal{A}$ is a group algebra, like $\mathbb{C}S_n$, or

$\mathcal{A} = U\mathfrak{g}$ is the enveloping algebra of a favorite Lie algebra, or

$\mathcal{A} = U_q\mathfrak{g}$ is the quantum group associated to a fav Lie alg, etc..)

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Pick some $\mathcal{A}$-modules, and build a tensor space.
What operators commute with the action of $\mathcal{A}$?

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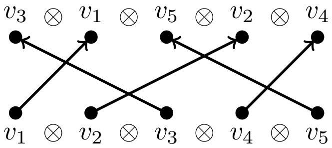
Pick some $\mathcal{A}$-modules, and build a tensor space.
What operators commute with the action of $\mathcal{A}$?

Example: Let $\mathcal{A} = U\mathfrak{gl}_n$, $M = \mathbb{C}^n$, and $V = M^{\otimes k}$ $(n > k)$.
Then commuting operators can be encoded as permutation diagrams:



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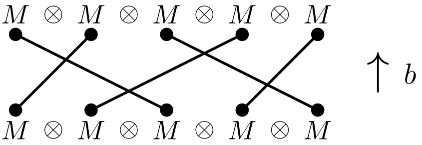


(Take the algebra generated linearly, with multiplication given by concatenation)

A few of my favorite things: Diagram algebras

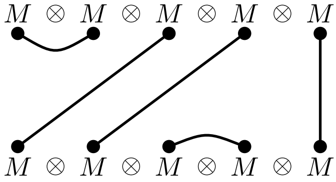
Recall: we're starting with an algebra $\mathcal{A}$ with module V,
and looking for an algebra $\mathcal{B}$ so that $ab = ba$ in $\mathrm{End}(V)$.

More examples:

$\mathcal{A} = U\mathfrak{sl}_2$, and

$M = \mathbb{C}^2$ is the defining representation and $V = M^{\otimes k}$.

Then $\mathcal{B}$ is the *Temperley-Lieb algebra*:



non-crossing pairings

(Take the algebra generated linearly,
with multiplication given by concatenation,
and relations on closed loops)

A few of my favorite things: Diagram algebras

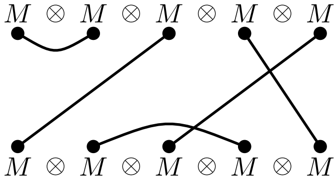
Recall: we're starting with an algebra $\mathcal{A}$ with module V, and looking for an algebra $\mathcal{B}$ so that $ab = ba$ in $\mathrm{End}(V)$.

More examples:

$\mathcal{A} = U\mathfrak{g}$ where $\mathfrak{g} = \mathfrak{so}_n$ or $\mathfrak{sp}_{2n}$, and

M is the defining representation and $V = M^{\otimes k}$.

Then $\mathcal{B}$ is the *Brauer algebra*:



possibly crossing pairings

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A few of my favorite things: Diagram algebras

Recall: we're starting with an algebra $\mathcal{A}$ with module V,
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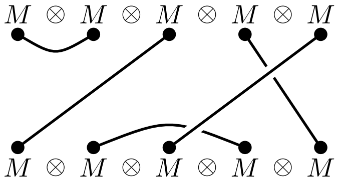
More examples:

$\mathcal{A} = U_q\mathfrak{g}$ (quantum group) where $\mathfrak{g} = \mathfrak{so}_n$ or $\mathfrak{sp}_{2n}$, and M is the defining representation and $V = M^{\otimes k}$.

Then $\mathcal{B}$ is the *Birman-Murakami-Wenzl (BMW) algebra*:



tangles

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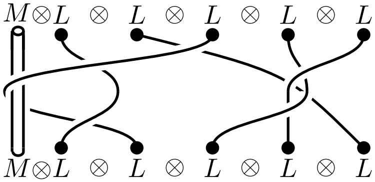
A few of my favorite things: Diagram algebras

Recall: we're starting with an algebra $\mathcal{A}$ with module V,
and looking for an algebra $\mathcal{B}$ so that $ab = ba$ in $\mathrm{End}(V)$.

More examples:

$\mathcal{A} = U_q\mathfrak{g}$ (quantum group) where $\mathfrak{g}$ can be many things and L and M are $\mathcal{A}$-modules, and $V = M \otimes L^{\otimes k}$.

Then $\mathcal{B}$ is a quotient of the grp alg of the *affine braid group*:



braids in a space with one puncture

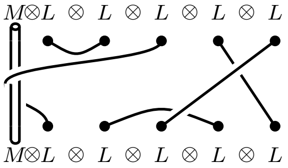
(Take the algebra generated linearly,
with multiplication given by concatenation)

My pet examples

$\mathcal{A} = U\mathfrak{g}$ or $U_q\mathfrak{g}$ where $\mathfrak{g} = \mathfrak{so}_n$ or $\mathfrak{sp}_{2n}$,
L is the defining representation and $V = M \otimes L^{\otimes k}$.

Quantum group case:

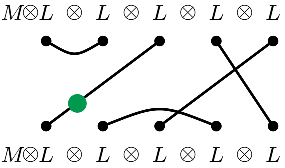
Affine BMW algebra



tangles in a space with one puncture

Enveloping algebra case:

Degenerate affine BMW algebra



decorated pairings

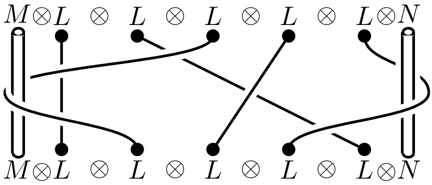
Papers:

1. Affine and degenerate affine BMW algebras: The center (with Arun Ram and Rahbar Virk), to appear in Osaka Journal of Mathematics. arXiv:1105.4207
2. Affine and degenerate affine BMW algebras: Actions on tensor space (with Arun Ram and Rahbar Virk), to appear in Selecta Mathematica. arXiv:1205.1852

My pet examples

$\mathcal{A} = U\mathfrak{g}$ or $U_q\mathfrak{g}$ where $\mathfrak{g}$ can be many things,
M, N, and L are $\mathcal{A}$-modules and $V = M \otimes L^{\otimes k} \otimes N$.

The two-boundary braid group
and its degenerate version



braids in a space with two punctures

Papers:

1. Degenerate two-boundary centralizer algebras, Pac. J. of Math., 258-1 (2012) 91–142. arXiv:1007.3950; see also doctoral dissertation, UW–Madison, June 2010.
2. In progress: Two boundary Hecke Algebras and the combinatorics of type $(C_n^\vee, C)$ Hecke algebras (with Arun Ram)

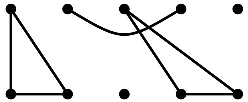
My pet examples

$\mathcal{A} = \mathbb{C}S_n$,

M is the permutation module or its large irreducible submodule.

Permutation module:
Partition algebra

$M \otimes M \otimes M \otimes M \otimes M$

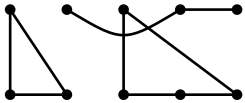


$M \otimes M \otimes M \otimes M \otimes M$

connected components

Irreducible submodule:
Quasi-partition algebra

$M \otimes M \otimes M \otimes M \otimes M$



$M \otimes M \otimes M \otimes M \otimes M$

c.c w/ no isolated vertices

Paper:
Quasi-partition algebra (with Rosa Orellana), arXiv:1212.2596