Centralizer properties of the affine Hecke algebra of type C

Zajj Daugherty
(joint with Arun Ram)

Dartmouth College

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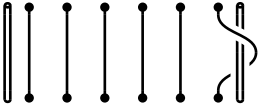
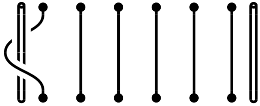
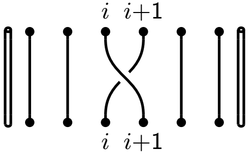
Pictorially, the generators of $\mathcal{B}_k$ are identified with the diagrams

$T_k =$  , $T_0 =$  ,

and

$T_i =$ 

for $i = 1, \ldots, k-1$.

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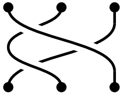
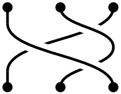
$T_iT_{i+1}T_i$ =  =  = $T_{i+1}T_iT_{i+1}$

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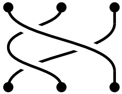
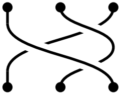
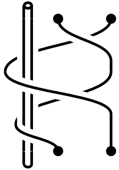
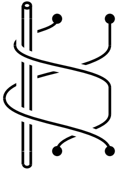
$T_iT_{i+1}T_i$ =  =  = $T_{i+1}T_iT_{i+1}$

$T_0T_1T_0T_1$ =  =  = $T_1T_0T_1T_0$

(similar picture for $T_kT_{k-1}T_kT_{k-1} = T_{k-1}T_kT_{k-1}T_k$)

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1. Fix $a_1, a_2, b_1, b_2, q \in \mathbb{C}^\times$. The affine Hecke algebra $\mathcal{H}_k$ of type C is the quotient of $\mathbb{C}\mathcal{B}_k$ by

$$(*) \quad (T_0 - a_1)(T_0 - a_2) = 0, \quad (T_k - b_1)(T_k - b_2) = 0, \quad (T_i - q)(T_i + q^{-1}) = 0.$$

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2. Let A, B, C be finite dim'l $U_q\mathfrak{g}$-modules. Then $\mathbb{C}\mathcal{B}_k$ acts on

$$B \otimes \underbrace{C \otimes \cdots \otimes C}_{k \text{ factors}} \otimes A$$

Under good (to be defined) conditions, this action factors through the quotient $(*)$.

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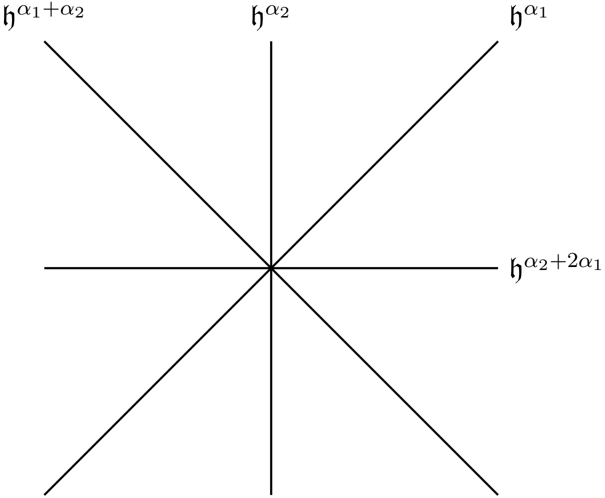

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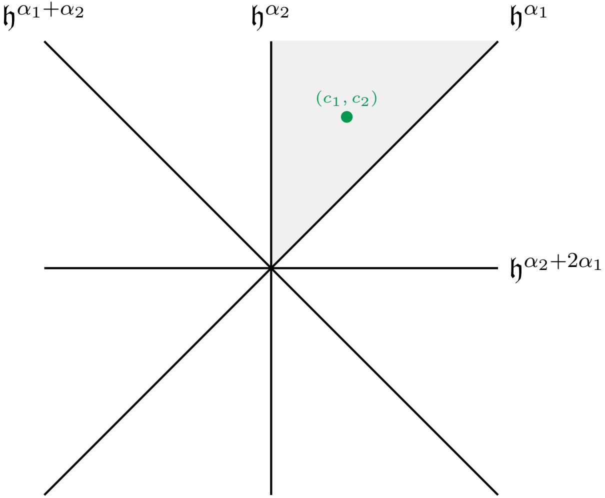

Points correspond to central characters.

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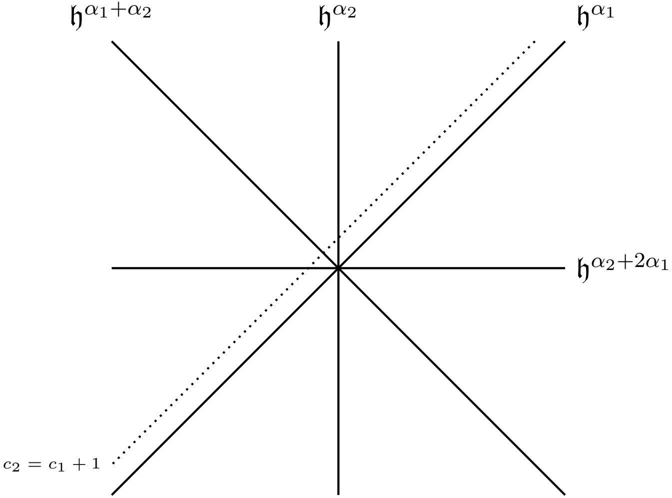

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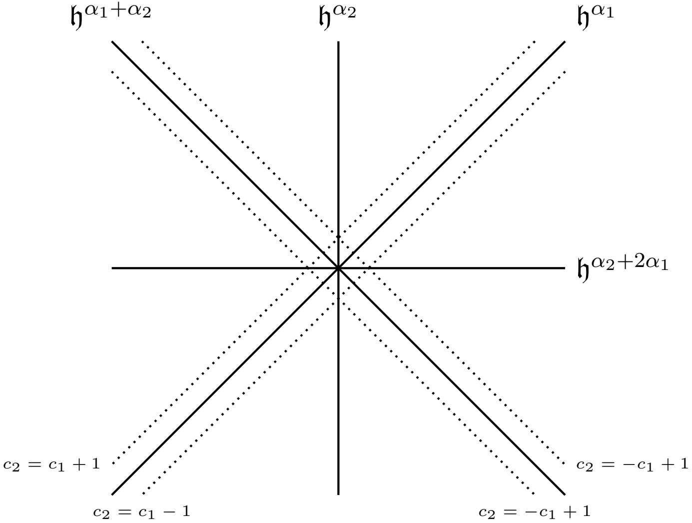

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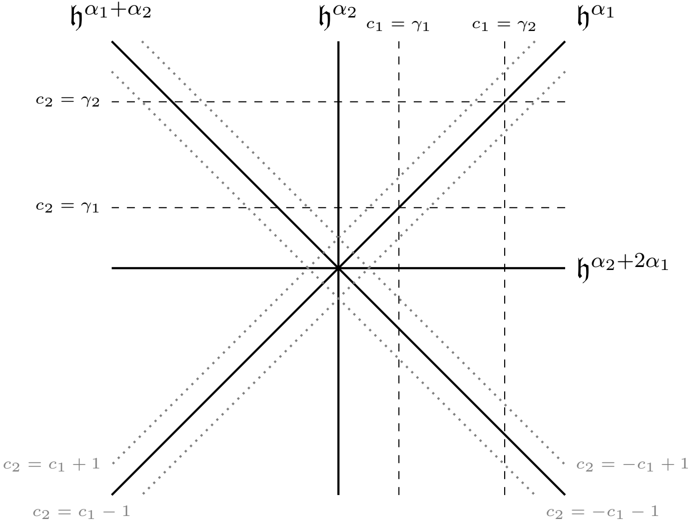

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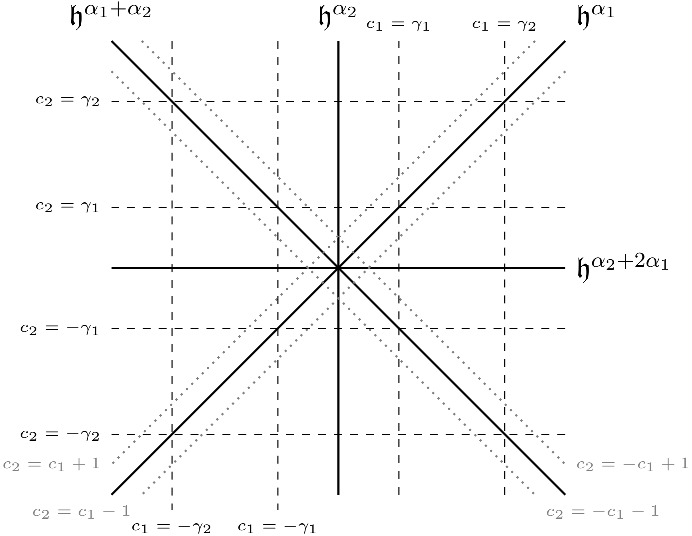

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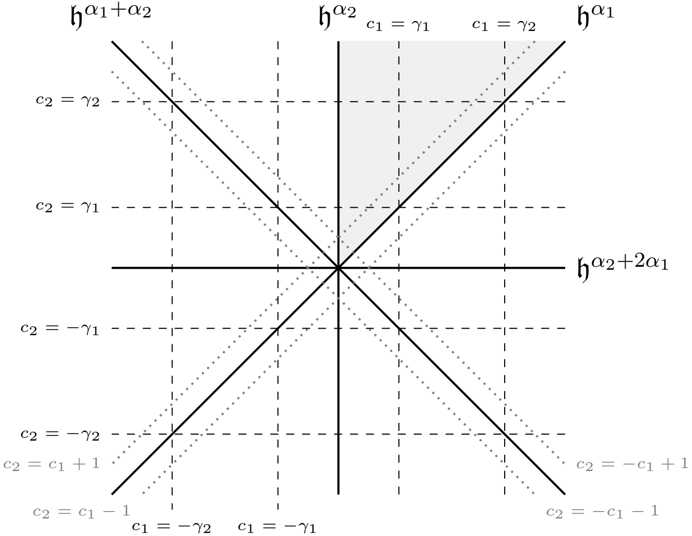

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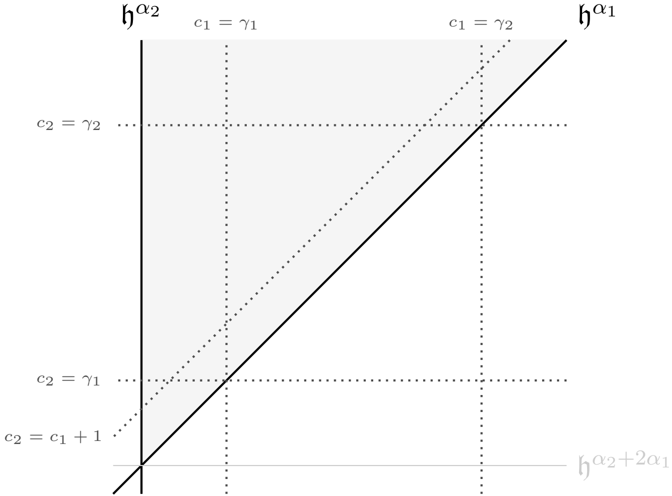

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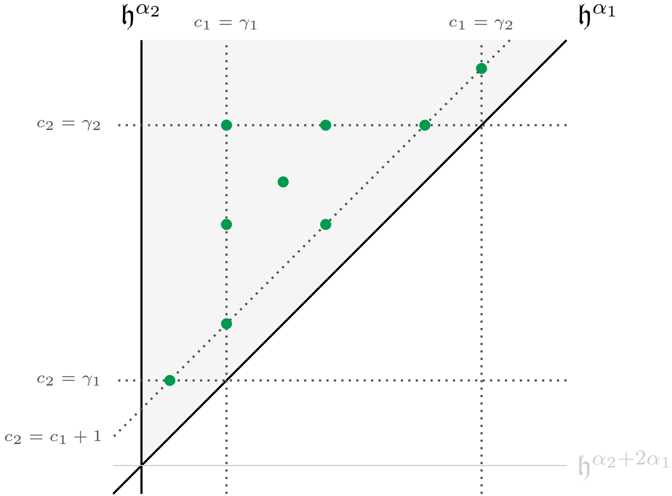

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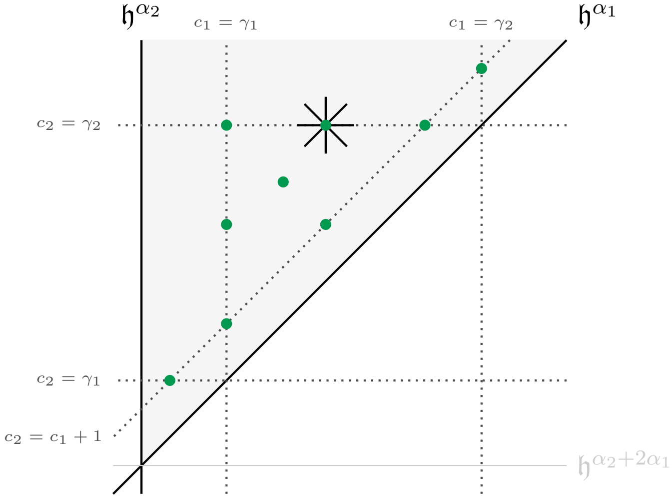

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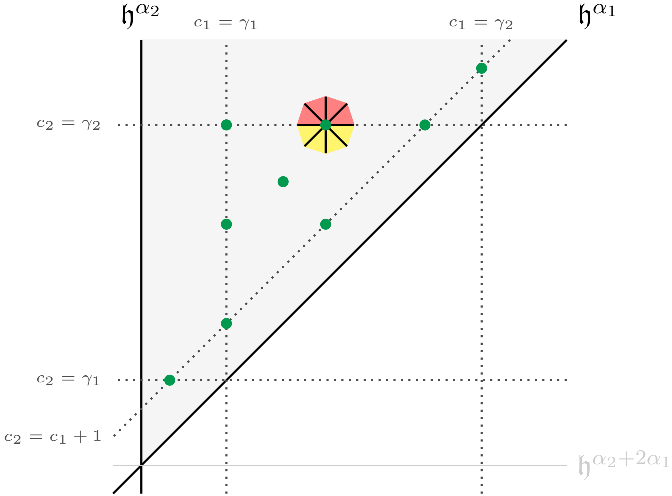

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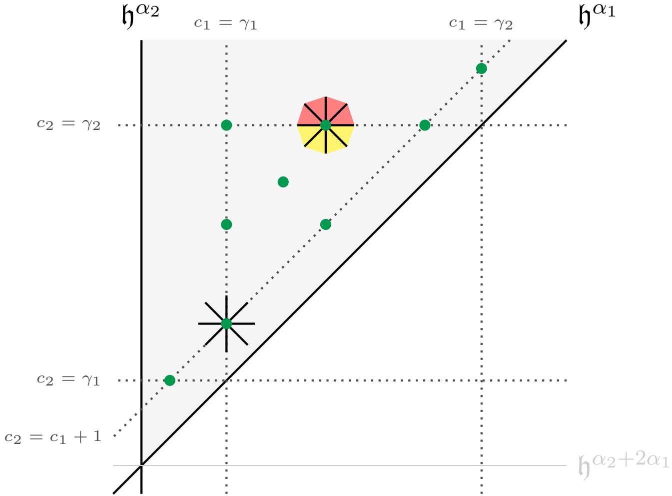

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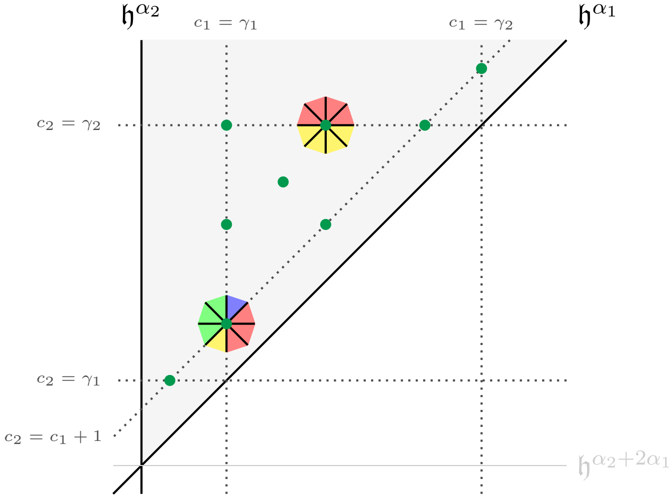

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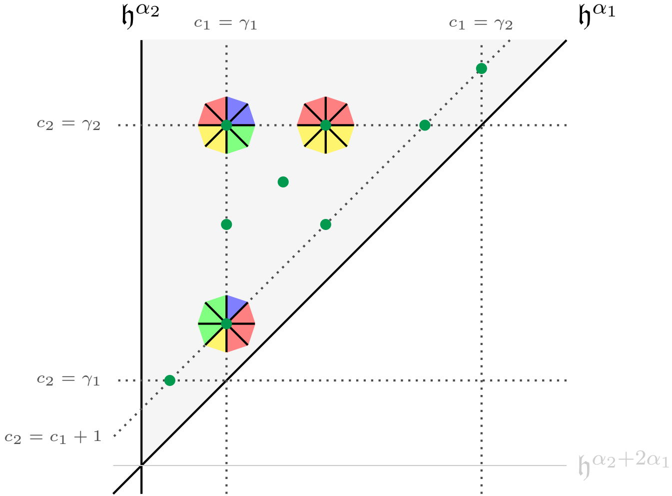

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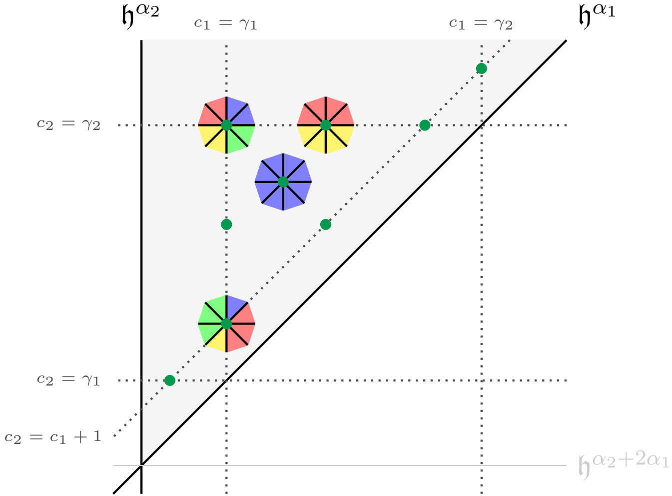

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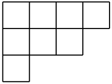
Centralizer properties

Let $U = U_q\mathfrak{g}$ be the quantum group for a finite dim'l reductive Lie algebra. We're interested in certain finite dimensional simple U-modules $L(\lambda)$ indexed by partitions:

$\lambda =$ 

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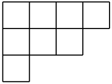
$\lambda =$ 

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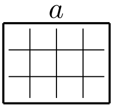
In particular, rectangular partitions:

$(a^c) = c$ 

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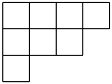
$\lambda =$ 

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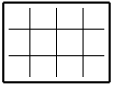
$(a^c) = c$ 

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-2			

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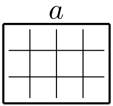

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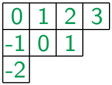
$\lambda =$ 

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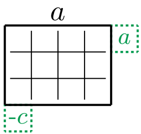
$(a^c) = c$ 

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Theorem (D.-Ram)

1. *Let $U = U_q\mathfrak{g}$, and let A, B, and C be finite dim'l U-modules. The two-boundary braid group $\mathcal{B}_k$ acts on $B \otimes (C)^{\otimes k} \otimes A$ (via R-matrices) and this action commutes with that of U.*

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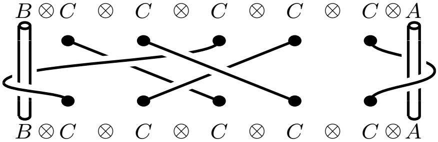

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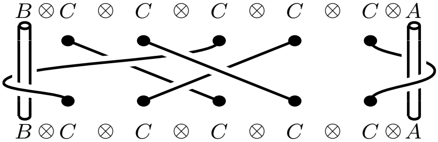
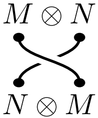


R-matrices: U has an associated invertible element $R = \sum_{\mathcal{R}} R_1 \otimes R_2$ of $U \otimes U$ that gives us a map

$$\check{R}_{MN} \colon M \otimes N \longrightarrow N \otimes M$$



This map acts a component $L(\lambda)$ of $L(\mu) \otimes L(\square)$ by $q^{c(\lambda/\mu)}$.

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$$(T_0 - q^{2b})(T_0 - q^{-2d}) = 0, \ (T_k - q^{2a})(T_k - q^{-2c}) = 0, \ (T_i - q)(T_i + q^{-1}) = 0.$$

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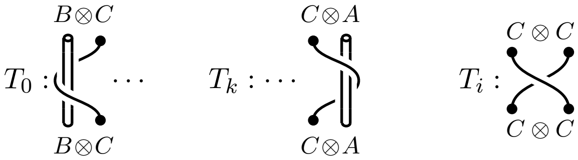
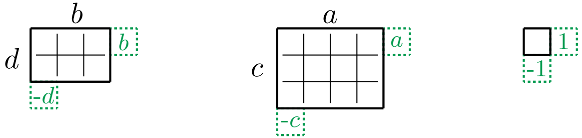
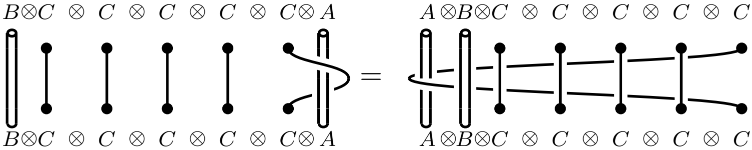




Exploring our new favorite tensor space, $A \otimes B \otimes C^{\otimes k}$

Move the right pole to the left:



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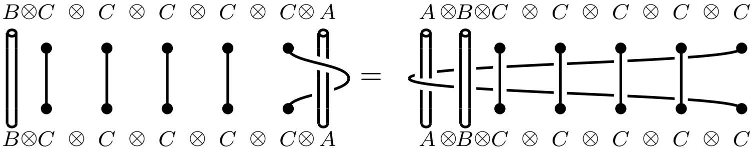
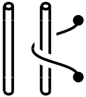
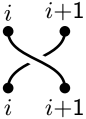
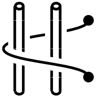
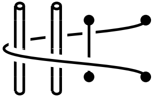


New favorite generators:

$T_0 =$  $T_i =$ 

and $Y_1 =$  .

Let $Y_2 = T_1 Y_1 T_1 =$  .

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Let $A = L((a^c))$ and $B = L((b^d))$. Then

$$A \otimes B = \bigoplus_{\lambda \in \Lambda} L(\lambda) \qquad \text{(multiplicity one!)}$$

where Λ is the following set of partitions:

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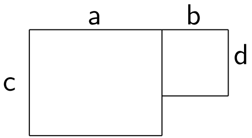

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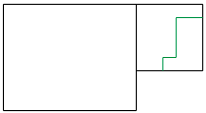

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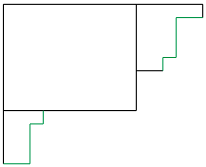
$$A \otimes B = \bigoplus_{\lambda \in \Lambda} L(\lambda) \qquad \text{(multiplicity one!)}$$

where Λ is the following set of partitions:

(Littlewood-Richardson, Okada)



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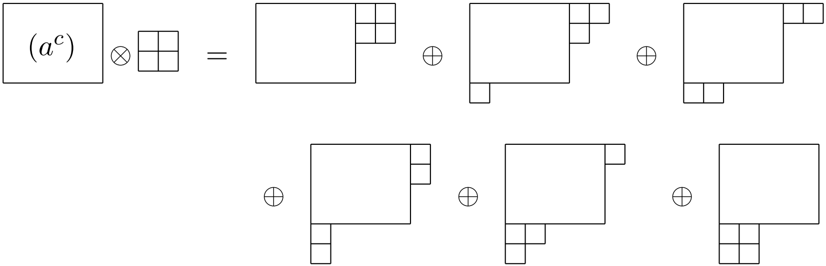

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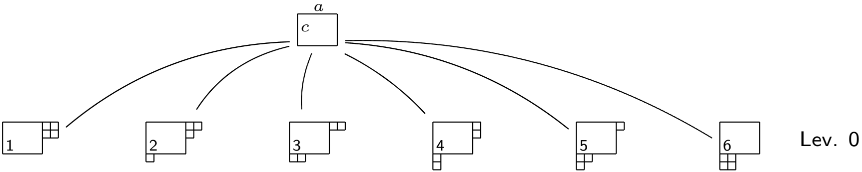

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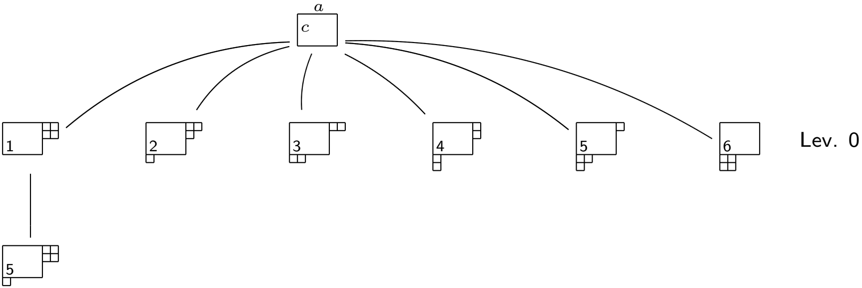

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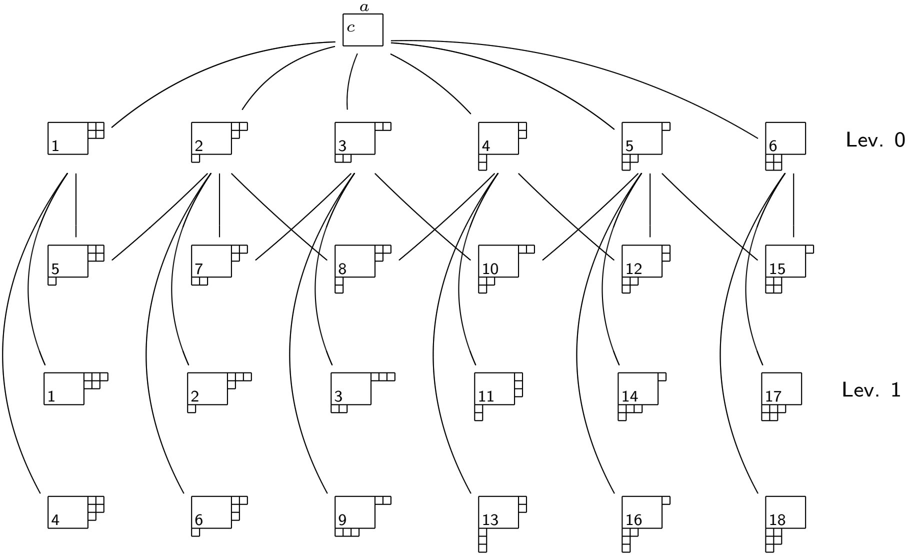

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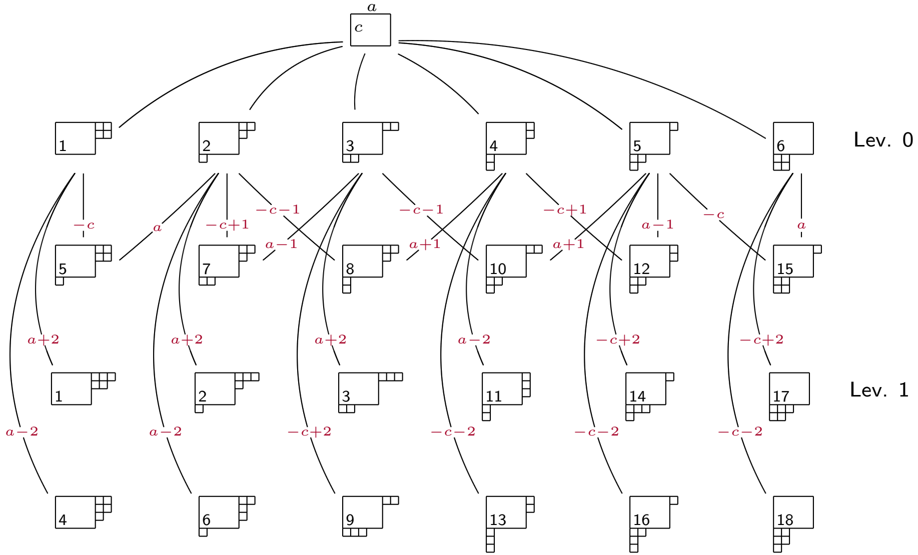

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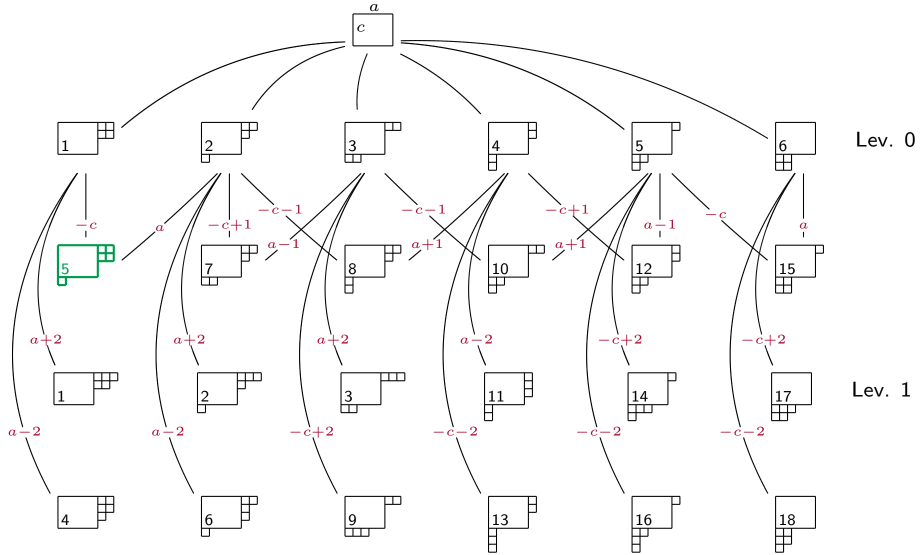

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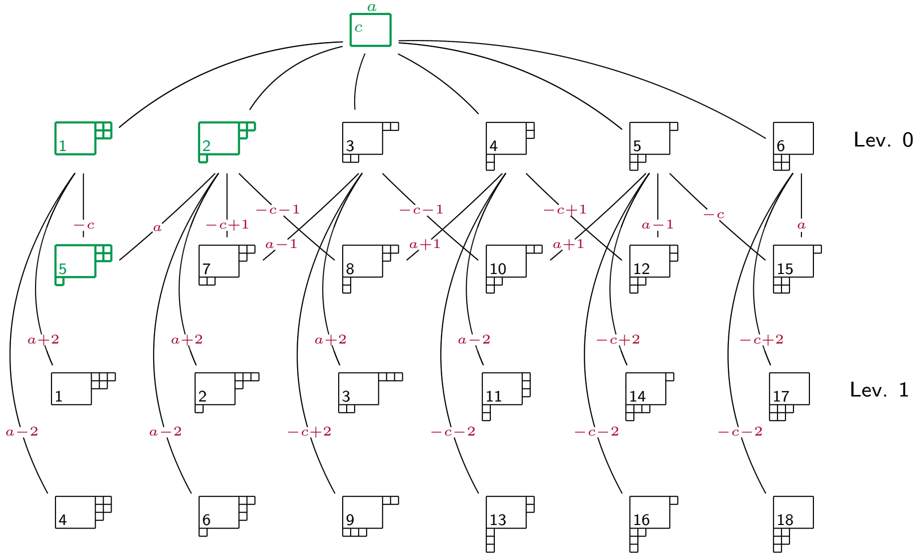
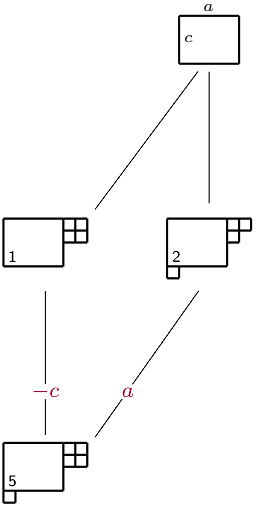
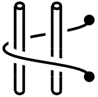
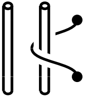


A two-dimensional Hecke module ($k=1$): Generators: Y_1 and T_0

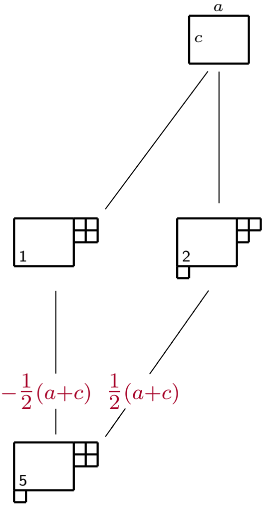


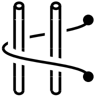
$Y_1 =$  $= \begin{pmatrix} q^{2(-c)} & 0 \\ 0 & q^{2(a)} \end{pmatrix}$

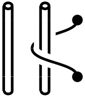
$T_0 =$  $\sim \begin{pmatrix} q^{2*(-2)} & 0 \\ 0 & q^{2*(2)} \end{pmatrix}$

(formulas for T_0 given in terms of contents of added boxes)

A two-dimensional Hecke module ($k=1$): Generators: Y_1 and T_0



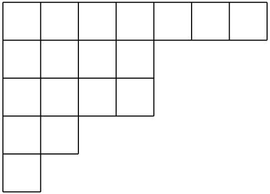
$Y_1 =$  $= \begin{pmatrix} q^{2(-\frac{1}{2}(a+c))} & 0 \\ 0 & q^{2(\frac{1}{2}(a+c))} \end{pmatrix}$

$T_0 =$  $\sim \begin{pmatrix} q^{2*(-2-\frac{1}{2}(a-c))} & 0 \\ 0 & q^{2*(2-\frac{1}{2}(a-c))} \end{pmatrix}$

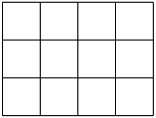
(formulas for T_0 given in terms of contents of added boxes)

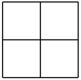
Shift! Shift contents by $-\frac{1}{2}(a-c+b-d) = -\frac{1}{2}(a-c)$

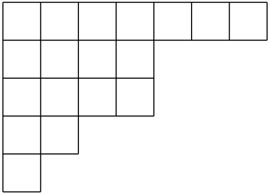
A concrete example of a component of $A \otimes B \otimes C \otimes C$

$\lambda =$ 

A concrete example of a component of $A \otimes B \otimes C \otimes C$

$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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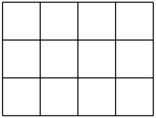
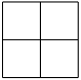
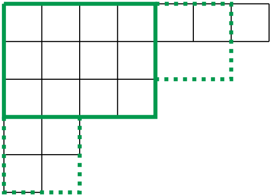
$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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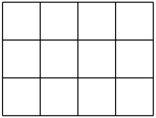
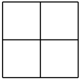
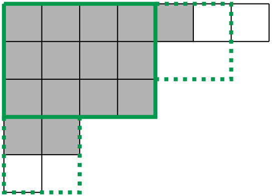
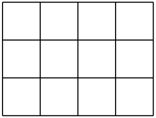
$(a^c) = (4^3) =$ 

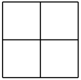
$(b^d) = (2^2) =$ 

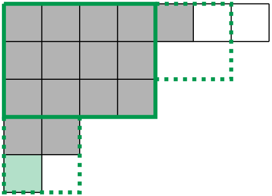
$\lambda =$ 

 = boxes that must appear in the partition at level 0.

A concrete example of a component of $A \otimes B \otimes C \otimes C$

$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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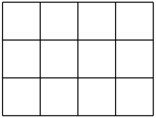
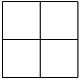
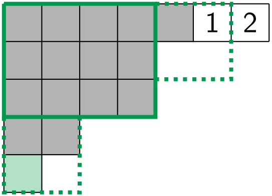
$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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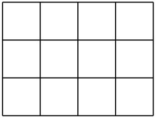
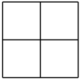
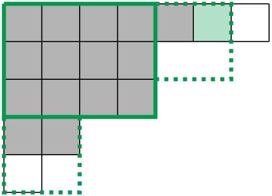
$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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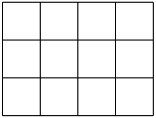
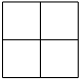
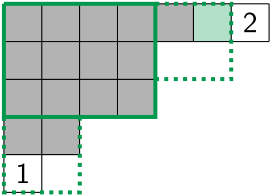
$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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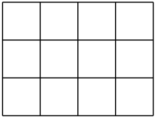
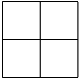
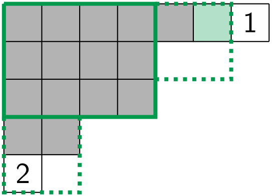
$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

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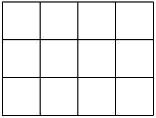
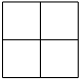
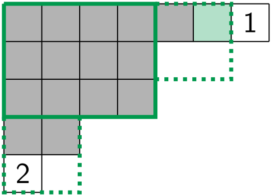
$(a^c) = (4^3) =$ 

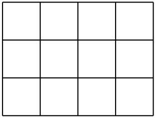
$(b^d) = (2^2) =$ 

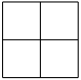
$\lambda =$ 

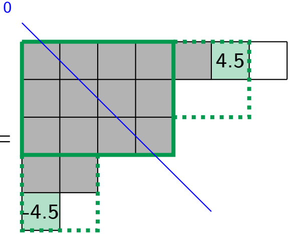
= boxes that must appear in the partition at level 0.

So the $\mathcal{H}_k$-module in $A \otimes B \otimes C \otimes C$ indexed by λ is 3-dimensional.

A concrete example of a component of $A \otimes B \otimes C \otimes C$

$(a^c) = (4^3) =$ 

$(b^d) = (2^2) =$ 

$\lambda =$ 

Shifting by $\frac{1}{2}(a - c + b - d) = -\frac{1}{2}$

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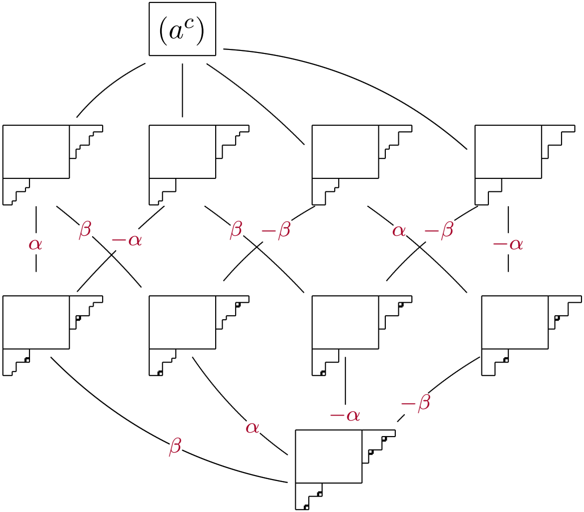
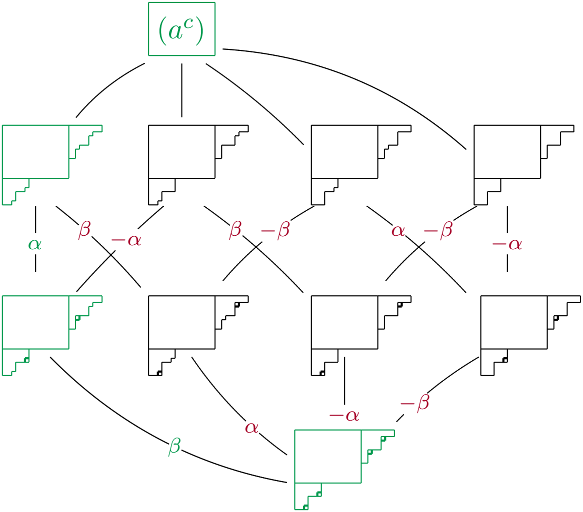
Largest Hecke module when $k = 2$:



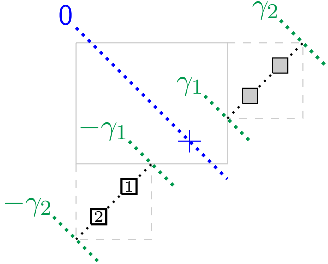
Shift: Label edges by action of $q^{-(a-c+b-d)}Y_1$ and $q^{-(a-c+b-d)}Y_2$

Largest Hecke module when $k = 2$:



Shift: Label edges by action of $q^{-(a-c+b-d)}Y_1$ and $q^{-(a-c+b-d)}Y_2$

Basis of the 8-dim'l module seen as tableaux



where $\gamma_1 = \frac{1}{2}(a+c) - \frac{1}{2}(b+d)$ and $\gamma_2 = \frac{1}{2}(a+c) + \frac{1}{2}(b+d)$,

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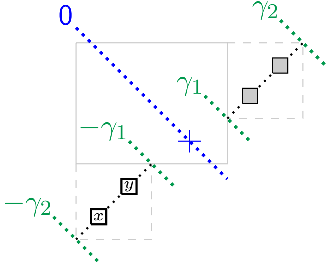

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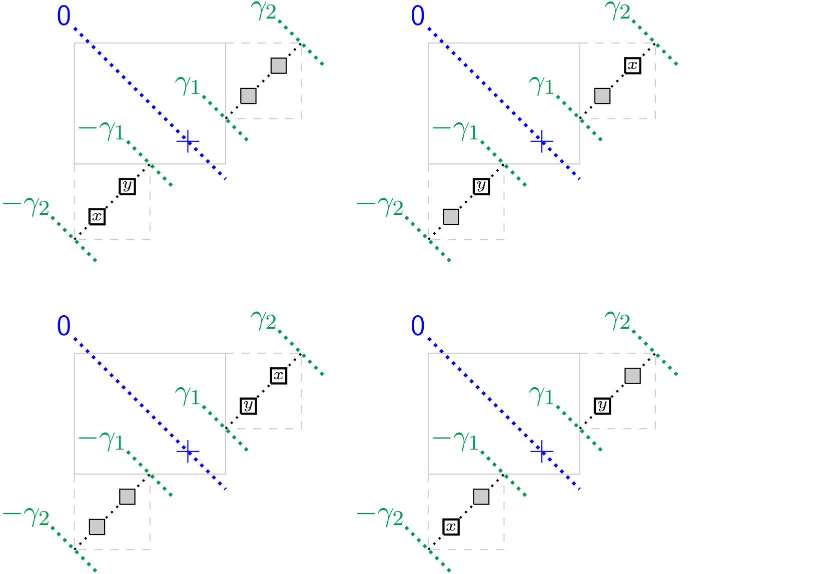
and x and y are entries of 1 or 2.

Basis of the 8-dim'l module seen as tableaux



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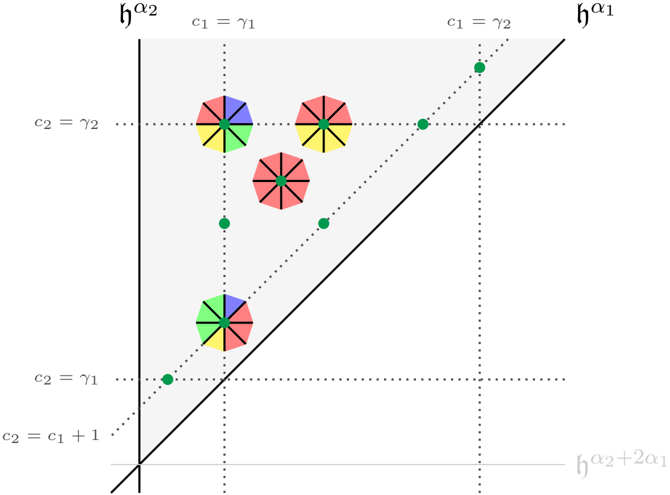
and x and y are entries of 1 or 2.

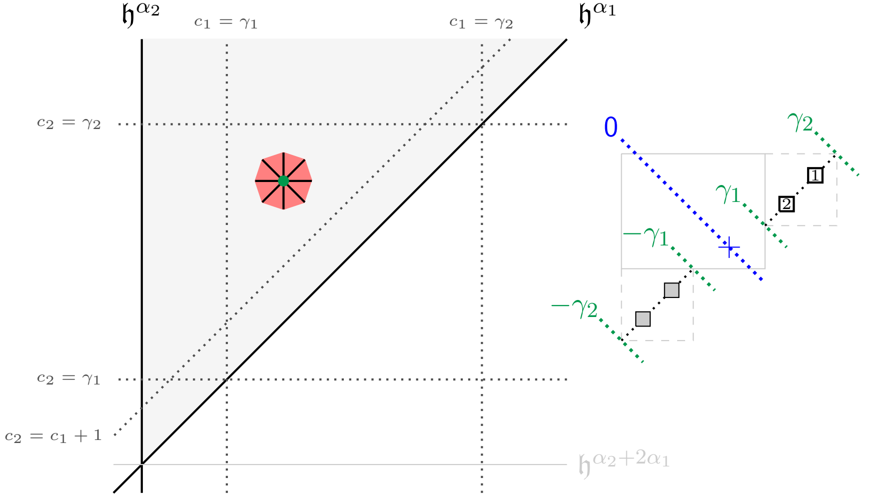
Back to the skew local regions picture:



(points correspond to central characters)

Back to the skew local regions picture:

Which appear as submodules of tensor space?



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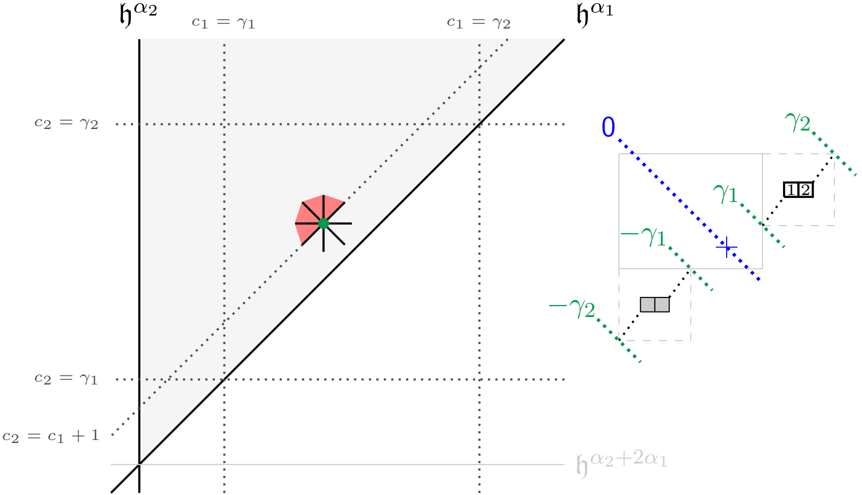

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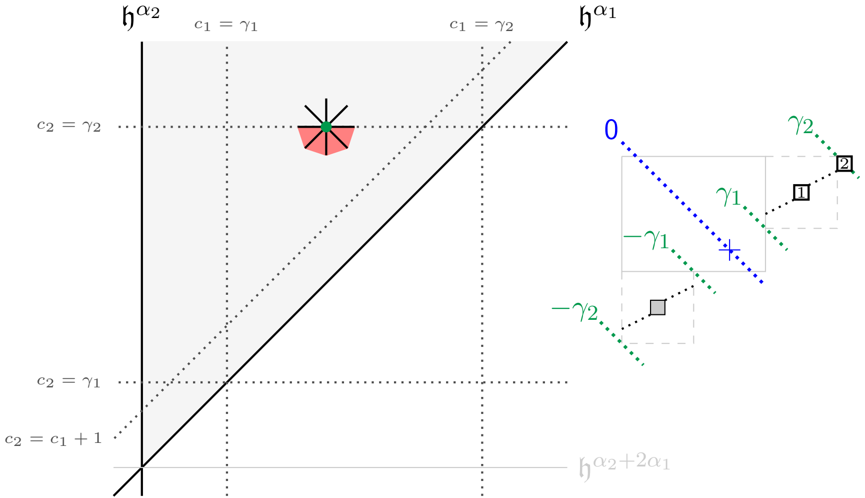

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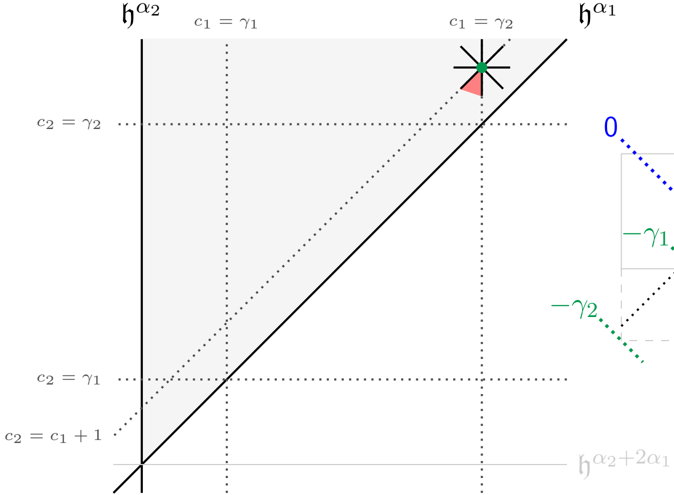

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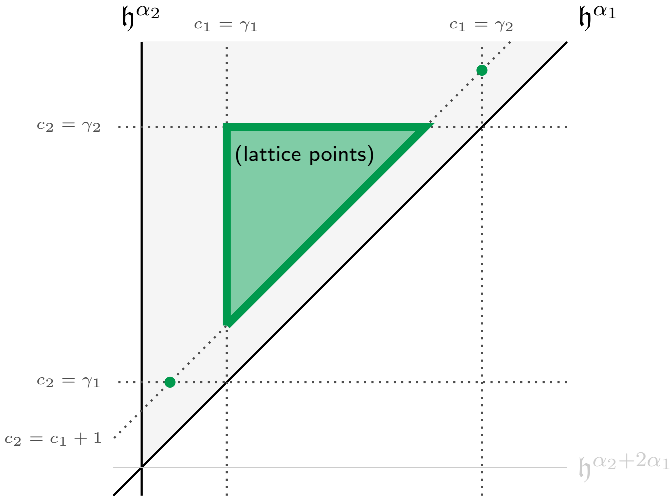

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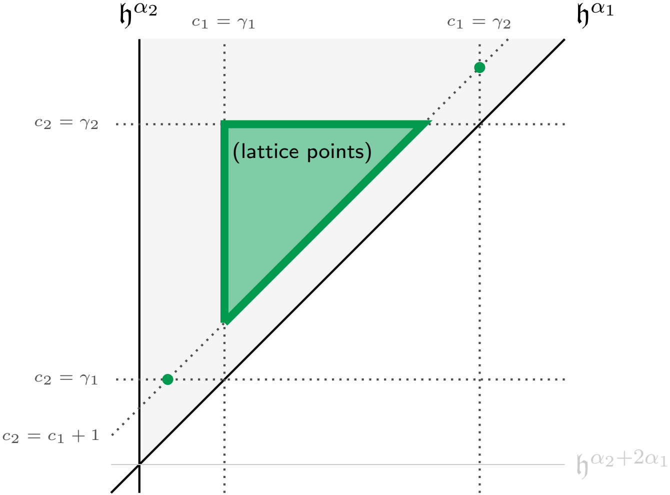
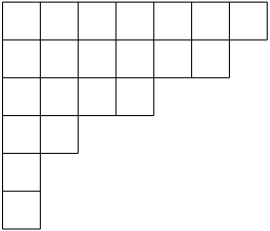


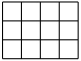
(points correspond to central characters)

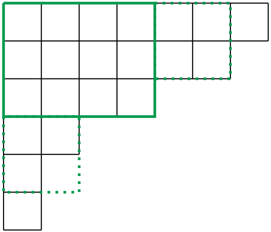
Generalizes to a bijection between points and skew shapes, regions and tableaux.

Partitions and tableaux in general

$\lambda =$ 

Partitions and tableaux in general

$(a^c) = (4^3) =$  $\qquad (b^d) = (2^2) =$ 

$\lambda =$ 

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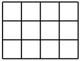
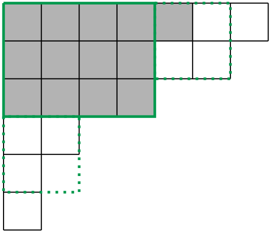
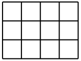
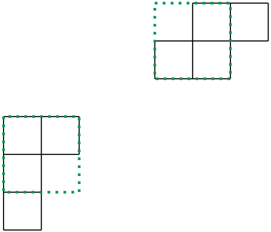
$(a^c) = (4^3) =$  $\qquad (b^d) = (2^2) =$ 

$\lambda =$ 

= boxes that must appear in the partition at level 0.

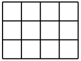
Partitions and tableaux in general

$(a^c) = (4^3) =$  $\qquad (b^d) = (2^2) =$ 

$\lambda =$ 

$(k = 5)$

Partitions and tableaux in general

$(a^c) = (4^3) =$  $\qquad (b^d) = (2^2) =$ 

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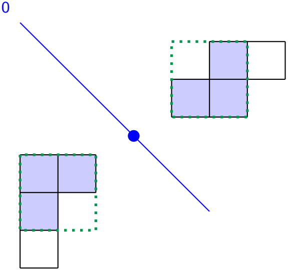

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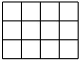
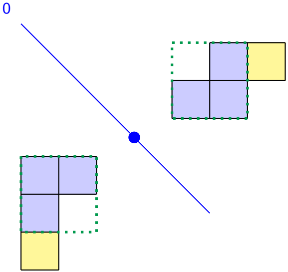
$(a^c) = (4^3) =$  $(b^d) = (2^2) =$ 

$\lambda =$ 

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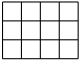
$(a^c) = (4^3) = $  $\qquad (b^d) = (2^2) = $ 

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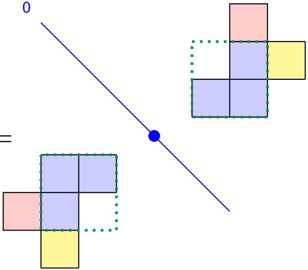

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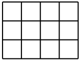
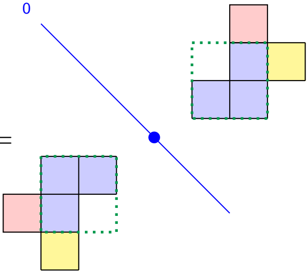
$(a^c) = (4^3) =$  $\qquad (b^d) = (2^2) =$ 

$\lambda =$ 

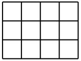
$(k = 5)$

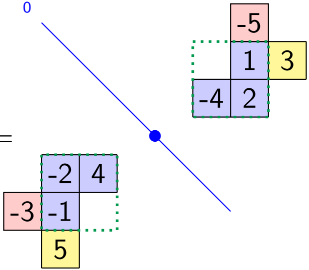
Fill with $\{-k, \ldots, -1, 1, \ldots, k\}$ so that

1. adjacent boxes increase down and to the right,
2. rotationally symmetric boxes have opposite values,
3. red boxes are negative and yellow boxes are positive.

The basis for the $\mathcal{H}_k$-module corresponding to λ is indexed by these tableaux.

Partitions and tableaux in general

$(a^c) = (4^3) =$  $(b^d) = (2^2) =$ 

$\lambda =$ 

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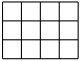
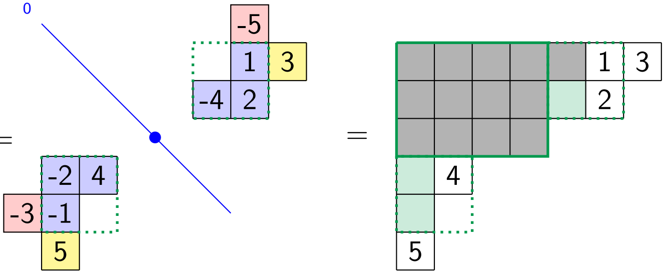
$(a^c) = (4^3) =$  $(b^d) = (2^2) =$ 

$\lambda =$ 

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[Dau] Z. Daugherty, *Degenerate two-boundary centralizer algebras*, Pac. J. Math., 258-1 (2012) 91–142.

[GN] J. de Gier and A. Nichols, *The two-boundary Temperley-Lieb algebra*, J. Algebra **321** (2009) 1132–1167.

[Ram] A. Ram, *Affine Hecke algebras and generalized standard Young tableaux*, J. Algebra **260** (2003) 367–415.

In preparation:

[DR] Z. Daugherty, A. Ram, *Two boundary Hecke Algebras and the combinatorics of type* $(C_n^\vee, C_n)$ *Hecke algebras*