Type C symmetry of two-boundary Hecke algebras

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Joint with Arun Ram

Dartmouth College

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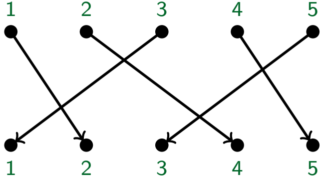
(Happy π day!)

A quick tour of some diagram algebras

Everyone's favorite diagram algebra:
Group algebra of the **symmetric group** S_k



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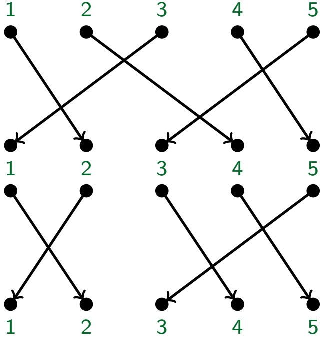

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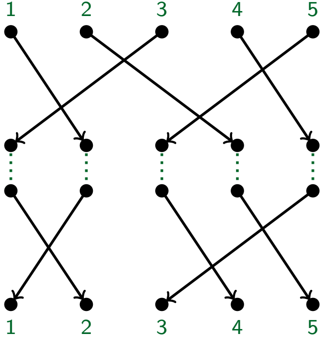

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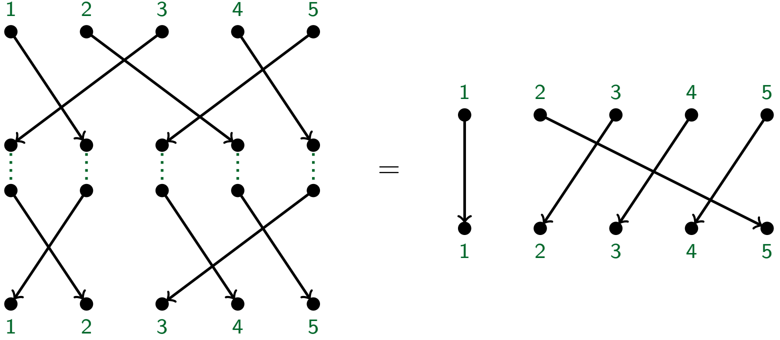

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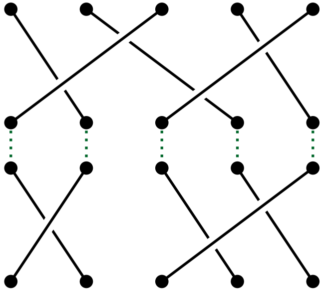

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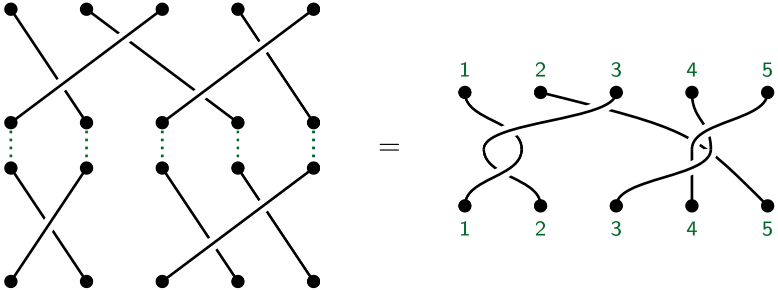

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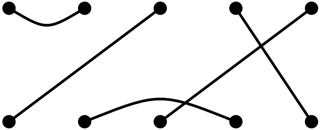
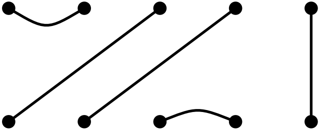
Brauer algebras

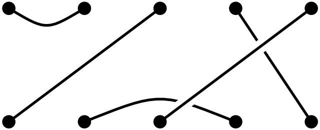


Temperley-Lieb algebras



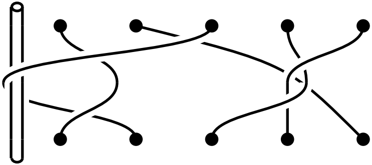
(with relations on closed loops)

BMW algebras



(With relations on closed loops and crossings, a la Skein relations)

Affine braid group



(Affine) **Hecke algebras** of type A are quotients of the (affine) braid group by relations on double twists.

Actions on tensor space

Classical example: (Schur 1901)

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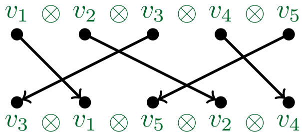

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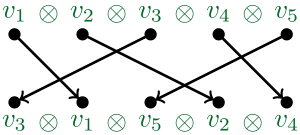
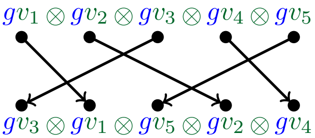


3. These actions commute!



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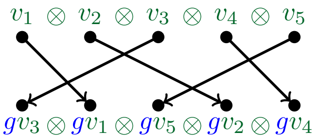

Schur-Weyl duality

Classical example: S_k and GL_n have commuting actions on $(\mathbb{C}^n)^{\otimes k}$. Even better, if $k \leq n$,

$$\mathrm{End}_{\mathbb{C}\mathrm{GL}_n}\left((\mathbb{C}^n)^{\otimes k}\right) = \mathbb{C}S_k \quad \text{and} \quad \mathrm{End}_{\mathbb{C}S_k}\left((\mathbb{C}^n)^{\otimes k}\right) = \mathbb{C}\mathrm{GL}_n.$$

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Why this is exciting:

Centralizer relationship produces

$$(\mathbb{C}^n)^{\otimes k} \cong \bigoplus_{\lambda \vdash k} G^\lambda \otimes S^\lambda \quad \text{as a } \mathrm{GL}_n\text{-}S_k \text{ bimodule,}$$

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Punchline: Knowing a lot about symmetric group modules now produces information about GL_n-modules.

Diagram algebras as centralizer algebras

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Hecke algebras arise if $\mathfrak{g} = \mathfrak{gl}_n$ or $\mathfrak{sl}_n$
BMW algebras arise if $\mathfrak{g} = \mathfrak{so}_n$ or $\mathfrak{sp}_{2n}$

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Idea: the picture encodes a map from $V \otimes \cdots \otimes V$ to itself.

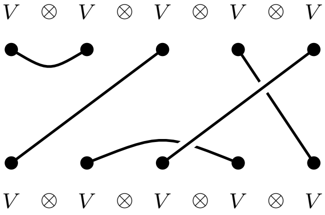

Diagram algebras as centralizer algebras

1. The Brauer algebras centralize the actions of O_n and SP_n (n even) on $(\mathbb{C}^n)^{\otimes k}$. (Brauer 1937)
2. The group algebra of the (affine) braid group commutes with the quantum group $U_q\mathfrak{g}$ on $M \otimes V^{\otimes k}$, and has centralizers as quotients. If $V = L(\square)$ (See Orellana-Ram 2007)

(affine) Temperley-Lieb algebras arise if $\mathfrak{g} = \mathfrak{gl}_2$ or $\mathfrak{sl}_2$
(affine) Hecke algebras arise if $\mathfrak{g} = \mathfrak{gl}_n$ or $\mathfrak{sl}_n$
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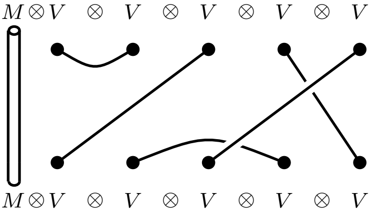
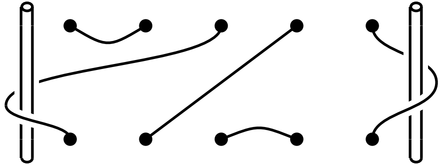


Two-boundary algebras

J. de Gier, A. Nicols, et. al. (2009):
Two-boundary Temperley-Lieb algebra $\mathcal{T}_k$



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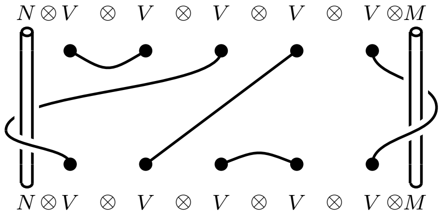

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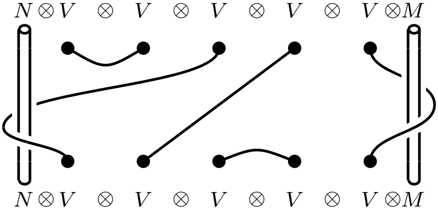

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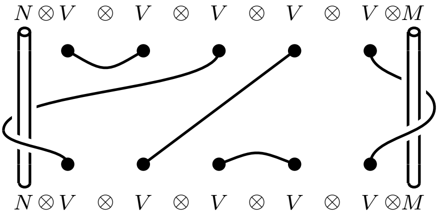

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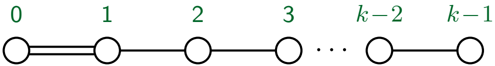
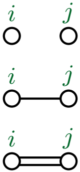
Question: Can we lift the pictures and commutator results up to the Hecke algebra by studying tensor products of $U_q\mathfrak{sl}_n$-modules?

Type C affine Hecke algebra



$m_{i,j} =$ 2 if; 3 if; 4 if



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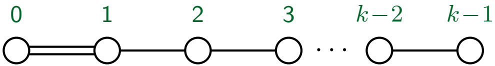
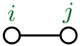
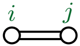


The **Weyl group of type C** is generated by $s_0, \ldots, s_{k-1}$ with relations $s_i^2 = 1$ and

$$\underbrace{s_i s_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{s_j s_i \ldots}_{m_{i,j} \text{ factors}} \qquad \text{where} \qquad m_{i,j} = \begin{cases} 2 & \text{if} \\ 3 & \text{if} \\ 4 & \text{if} \end{cases}$$





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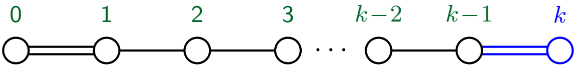

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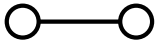
3 if 

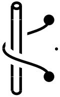
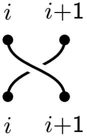
4 if 

Fix constants a_0, a_k, and $a_1 = \cdots = a_{k-1}$. The **affine Hecke algebra of type C** is generated by $T_0, T_1, \ldots, T_k$ with relations

$$T_i^2 = (a_i - a_i^{-1})T_i + 1, \qquad \underbrace{T_i T_j \ldots}_{m_{i,j} \text{ factors}} = \underbrace{T_j T_i \ldots}_{m_{i,j} \text{ factors}}.$$

Why the two-boundary braid group is type C

The **two-boundary (two-pole) braid group** is generated by

$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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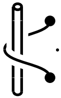
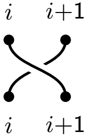
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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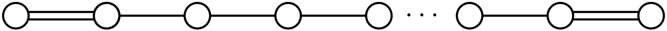

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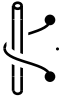
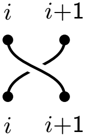
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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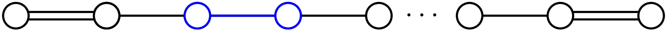
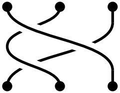
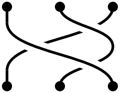


$T_i T_{i+1} T_i \ =$  $=$  $= \ T_{i+1} T_i T_{i+1}$

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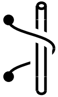
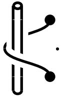
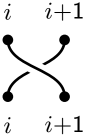
$T_k = \cdots$  $T_0 =$  $\cdots$ and $T_i =$ 

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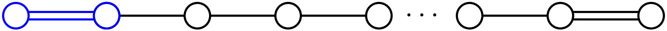
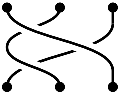
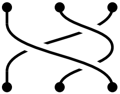
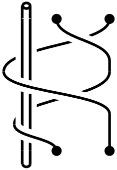
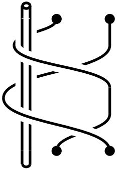


$T_i T_{i+1} T_i =$  $=$  $= T_{i+1} T_i T_{i+1}$

$T_0 T_1 T_0 T_1 =$  $=$  $= T_1 T_0 T_1 T_0$

(similar picture for $T_k T_{k-1} T_k T_{k-1} = T_{k-1} T_k T_{k-1} T_k$)

Why the two-boundary braid group is type C

Theorem (D.-Ram, degenerate version in [Da])

Let $U = U_q\mathfrak{g}$ for any complex reductive Lie algebras $\mathfrak{g}$. Let N, M, and V be finite-dimensional modules.
The two-boundary braid group acts on

$$N \otimes (V)^{\otimes k} \otimes M$$

and this action commutes with the action of U.

How?
Quantum group setting: The action is produced via R-matrices.
Lie algebra setting: The action is produced via the Casimir.

Why the representation theory is type A

Let $U = U_q\mathfrak{sl}_n$.

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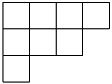
Let $U = U_q\mathfrak{sl}_n$.
The finite-dimensional U-modules $L(\lambda)$ are indexed by *partitions*:

$\lambda =$ 

4
+3
+1

(a collection of boxes piled up and to the left)

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$$\lambda = \begin{array}{|c|c|c|c|}\hline 0 & 1 & 2 & 3 \\ \hline -1 & 0 & 1 \\ \cline{1-3} -2 \\ \cline{1-1} \end{array}$$

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If a box B is in row i and column j, then the *content* of B is

$$c(B) = j - i.$$

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If $M = L(\mu)$, $N = L(\nu)$, and $V = L(\square)$,

$$\text{eigenvalues}(T_0) \sim \left\{q^{c(\text{addable boxes of } \nu)}\right\}$$
$$\text{eigenvalues}(T_i) \sim \left\{q, q^{-1}\right\} \text{ for } 1 \leq i \leq k-1$$
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Back to the Hecke algebra: The relations $T_i^2 = (a_i - a_i^{-1})T_i + 1$ say that T_i has **two** eigenvalues.

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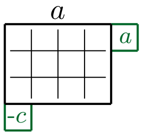
So μ and ν must be rectangles!
(Exactly two addable boxes)

$(a^c) = c$ 

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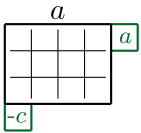
$(a^c) = c$ 

Theorem (D.-Ram in progress, degenerate version in [Da])

Let $U = U_q\mathfrak{sl}_n$ *and* $a_1 = \cdots = a_{k-1} = q$.
The Hecke algebra of type C acts on

$$L((b^d)) \otimes (L(\square))^{\otimes k} \otimes L((a^c))$$

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Why the representation theory is type A

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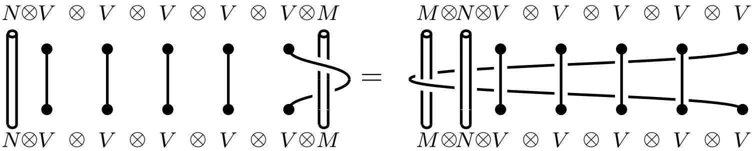

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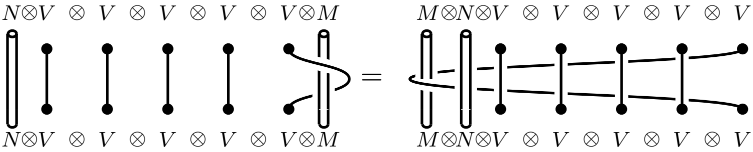
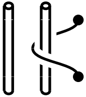
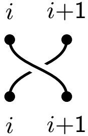
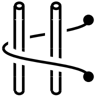
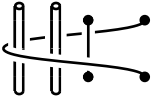


New favorite generators:

$T_0 =$  $T_i =$ 

and $Y_1 =$ .

Let $Y_2 = T_1 Y_1 T_1 =$ .

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Let $M = L((a^c))$ and $N = L((b^d))$. Then

$$M \otimes N = \bigoplus_{\lambda \in \Lambda} L(\lambda) \qquad \text{(multiplicity one!)}$$

where Λ is the following set of partitions:

(Littlewood-Richardson, Okada)

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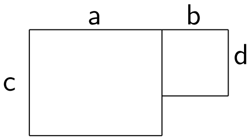

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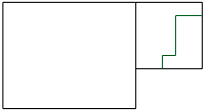

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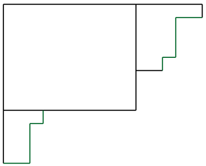

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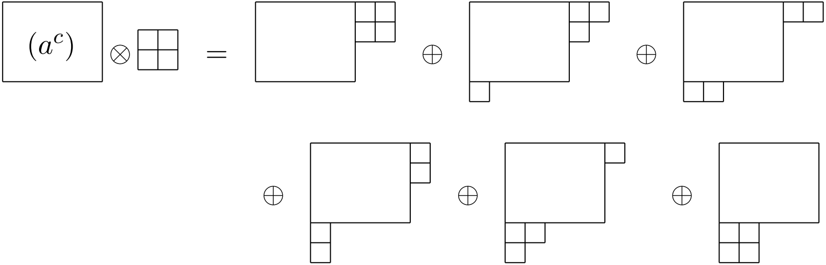

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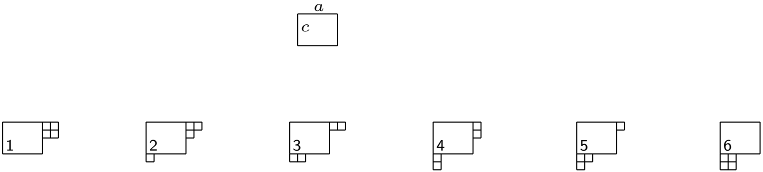

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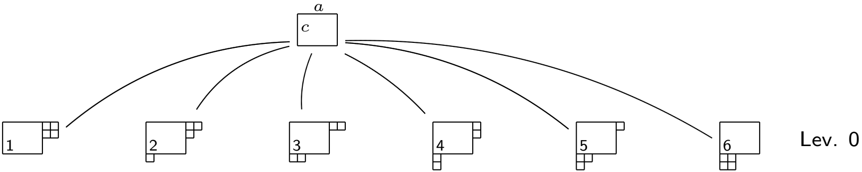

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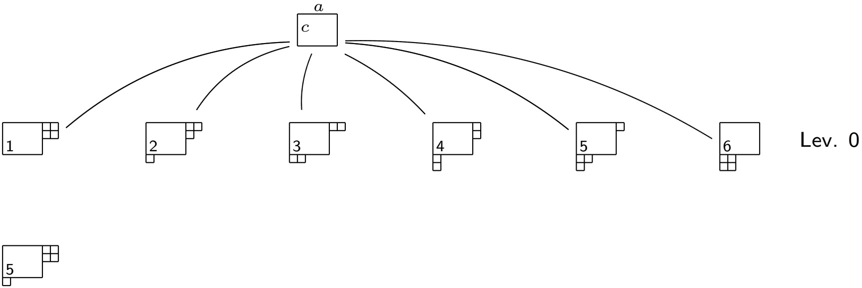

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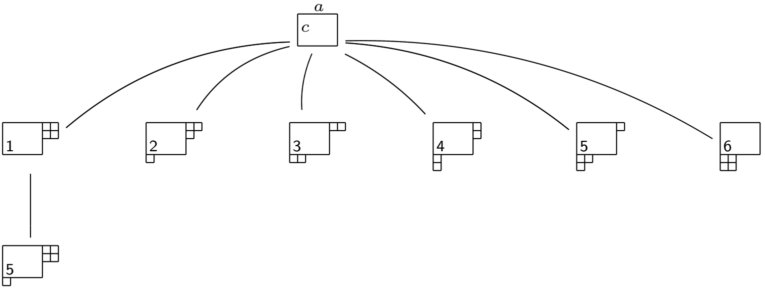

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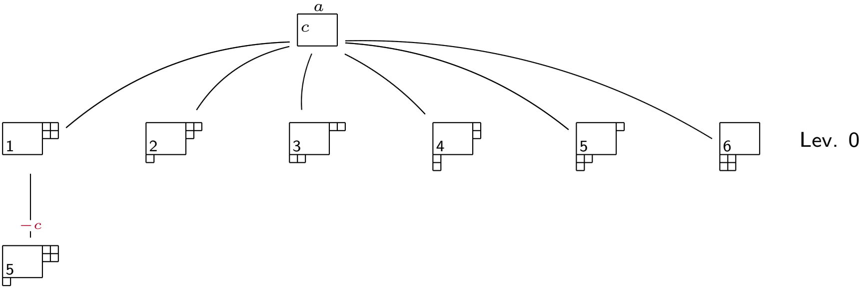

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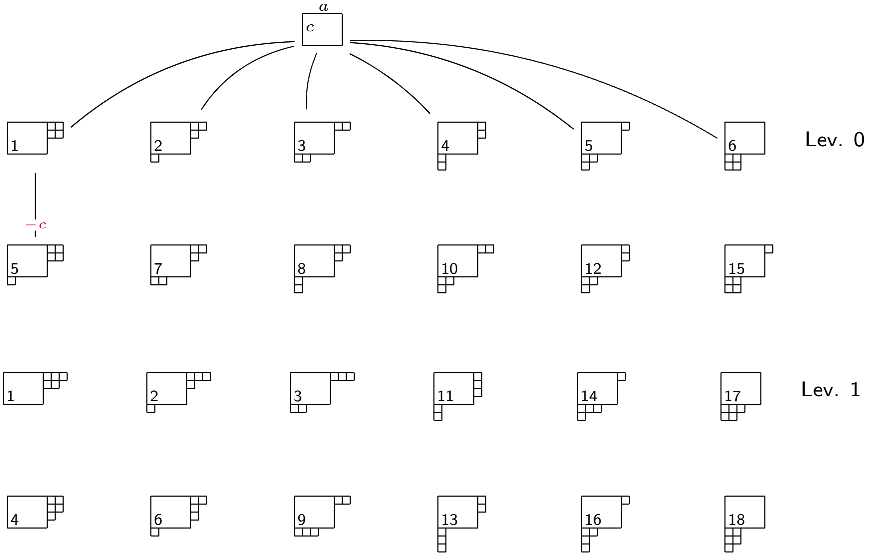

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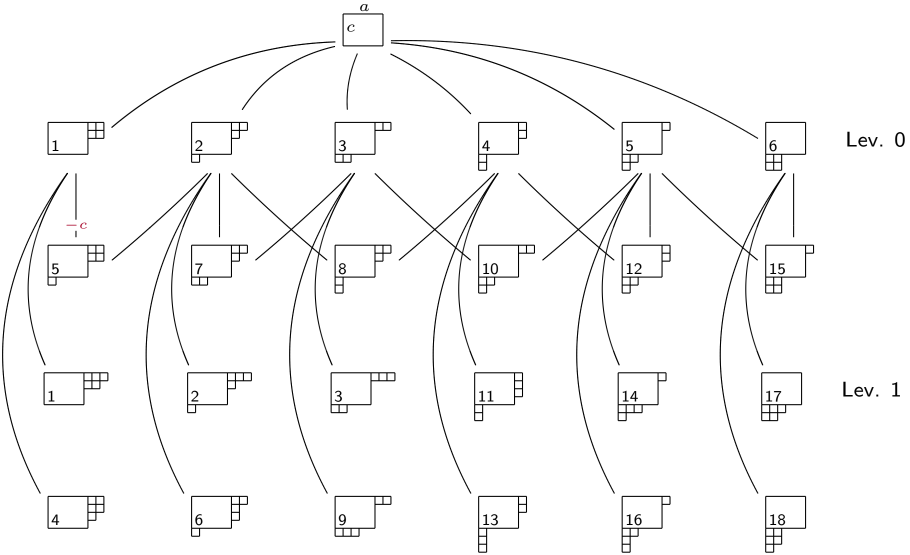

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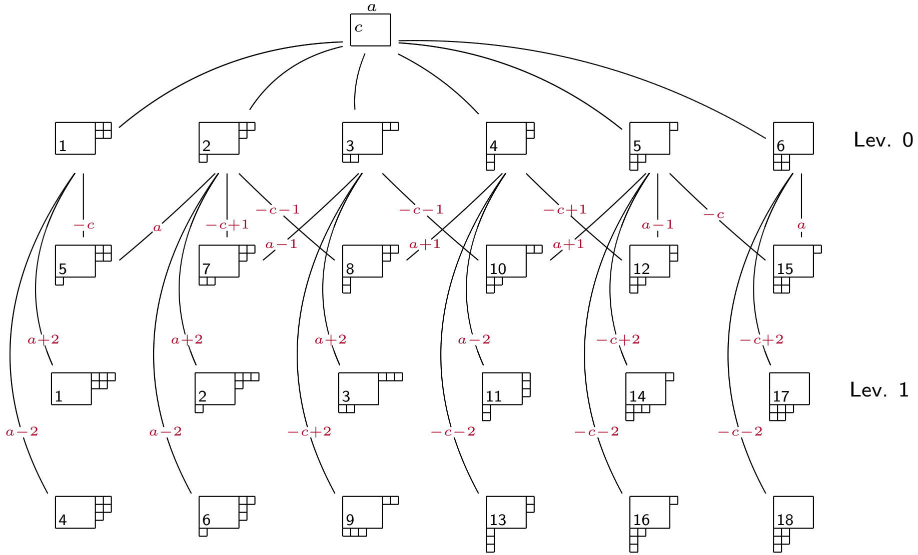

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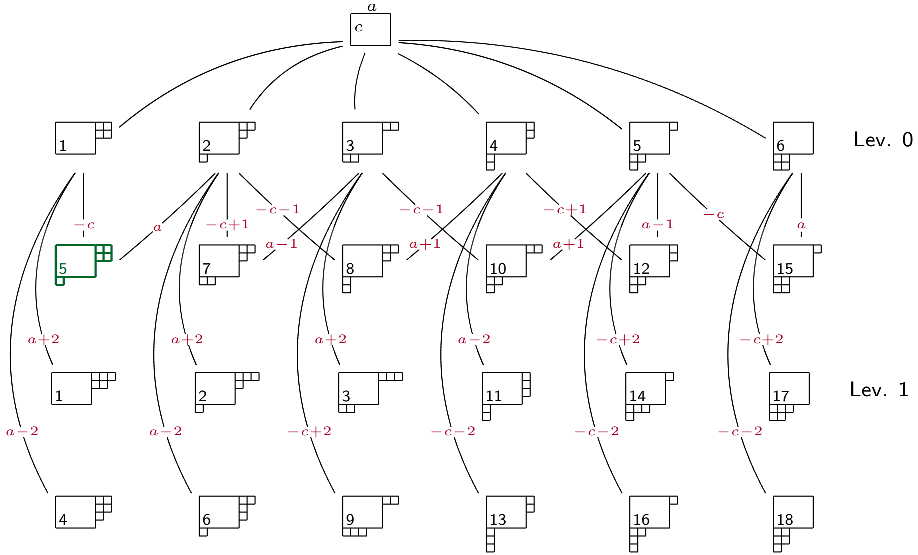

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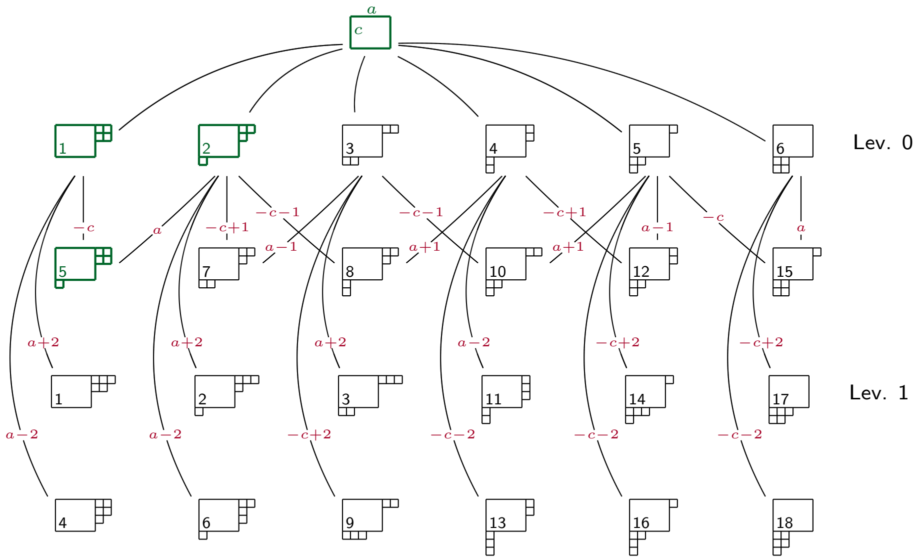
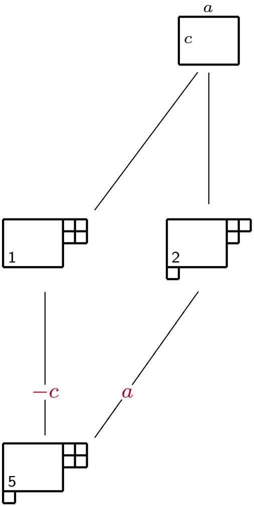


Why the representation theory is type A

A two-dimensional Hecke module ($k = 1$): Generators: Y_1 and T_0



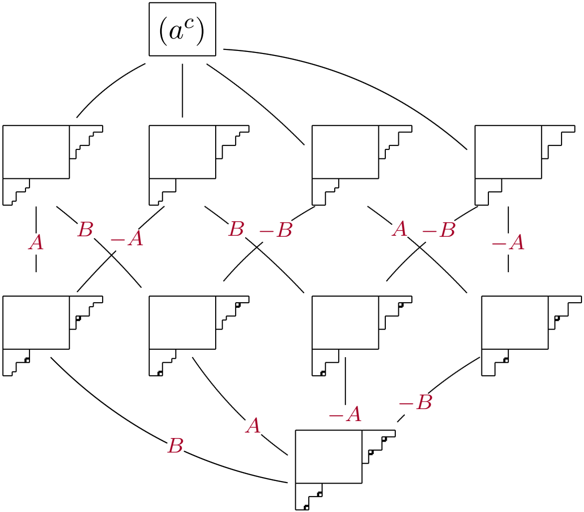
$$Y_1 = \begin{pmatrix} q^{-c} & 0 \\ 0 & q^{a} \end{pmatrix}$$

$$T_0 \sim \begin{pmatrix} q^{-2} & 0 \\ 0 & q^{2} \end{pmatrix}$$

(formulas for T_0 given in terms of contents of added boxes)

Why the representation theory is type A

An eight-dimensional Hecke module ($k = 2$)



Shift: Label edges by action of $q^{-\frac{1}{2}(a-c+b-d)}Y_1$ and $q^{-\frac{1}{2}(a-c+b-d)}Y_2$

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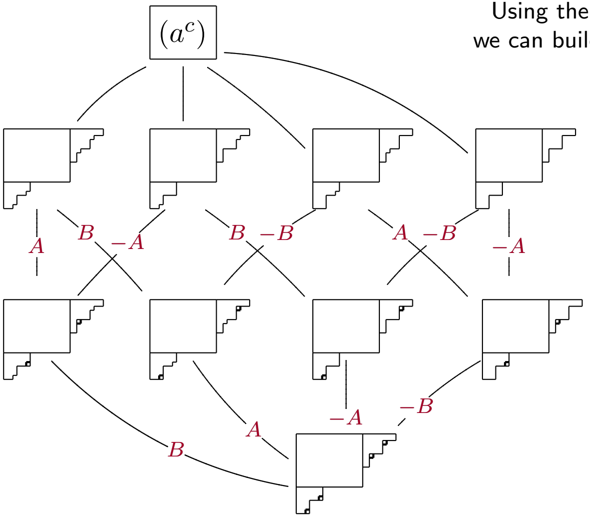

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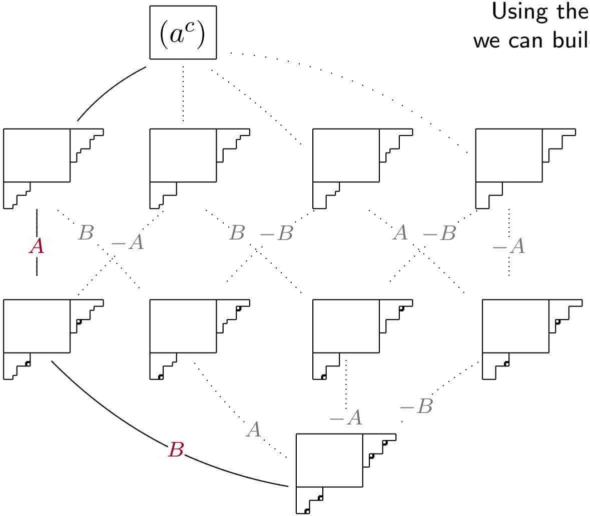

$1 \qquad (A, B)$

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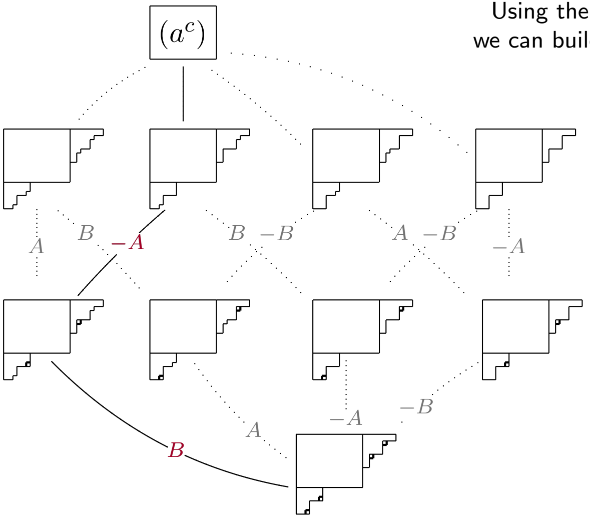

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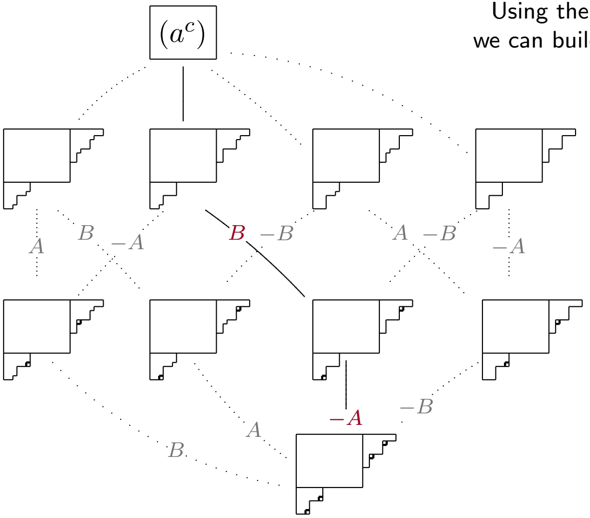

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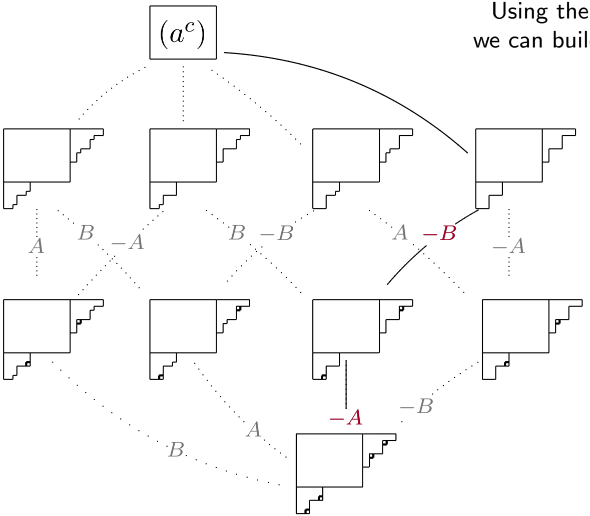

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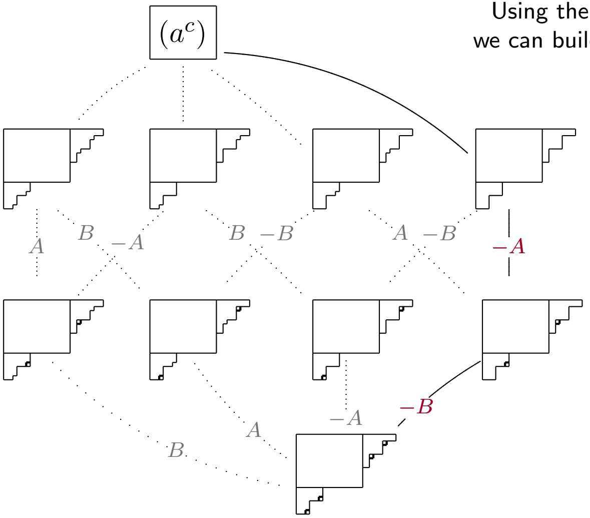

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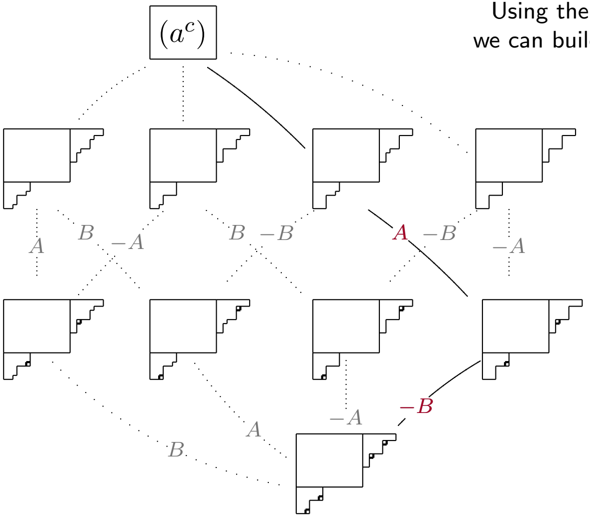

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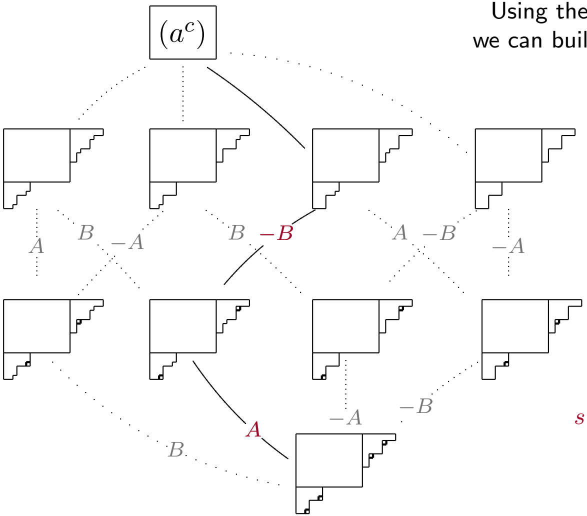

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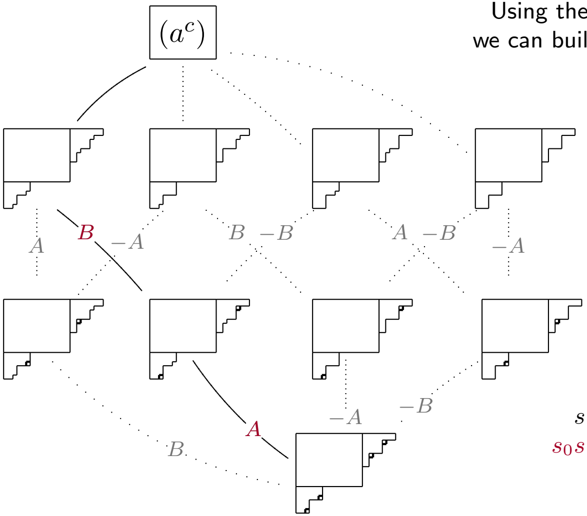

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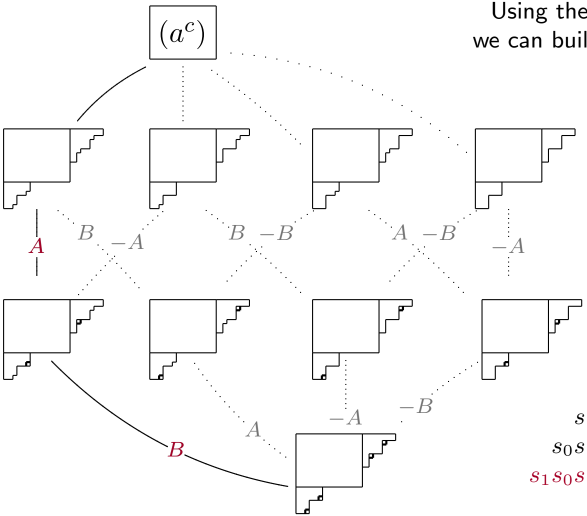

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[GN] J. de Gier and A. Nichols, *The two-boundary Temperley-Lieb algebra*, J. Algebra **321** (2009) 1132–1167.

[OR] R. Orellana and A. Ram, *Affine braids, Markov traces and the category* $\mathcal{O}$, Proceedings of the International Colloquium on Algebraic Groups and Homogeneous Spaces Mumbai 2004, V.B. Mehta ed., Tata Institute of Fundamental Research, Narosa Publishing House, Amer. Math. Soc. (2007) 423–473.

In preparation:

[Da2] Z. Daugherty, *Centralizer properties of the graded Hecke algebra of type C*

[DR] Z. Daugherty, A. Ram, *Two boundary Hecke Algebras and the combinatorics of type* $(C_n^\vee, C_n)$ *Hecke algebras*