MATH 111: FINAL REVIEW

We have a final exam on Tuesday, December 16, 6-9P.M. in Physics 123 on the subjects we've covered in the readings, homeworks, and lectures throughout the term. The test will be closed book, closed notes, no calculator. Please note that the test is comprehensive, but will focus on topics covered after the midterm. Also note that while you will have three hours to complete the exam, I am aiming to write a test which will take a well-prepared student only two hours to finish.

Study tips. Please study early and well for the exam. See the midterm review handout for specific pointers.

Review topics. Following is a non-comprehensive but still pretty darn good list of need-to-know items which we've covered since the first midterm. See the midterm review sheet for topics covered earlier. Items marked with a $\star$ will *definitely* appear on the exam.[1]

Theory of integrals.

- Know the definition of upper and lower bounds, suprema and infima for subsets of the real numbers.
- $\star$ You will be asked the definition of the integral of a bounded function. (You are expected to know the definiton from class, not the book's definition. See the INTEGRALS handout on the course website for details.)
- Interpret the intergral as the (signed) area under the graph of a function.
- Know the basic comparison theorems between upper and lower sums, and how they imply that continuous functions are integrable.
- $\star$ You will be asked to state and understand fundamental theorem of calculus (versions 1 and 2) on the exam.
- Know and use basic properties of integrals such as linearity and $\int_a^b + \int_b^c = \int_a^c$.
- Know the mean value theorem for integrals, including its interpretation in terms of averages of functions over intervals.
- Understand the notion of an antiderivative and its relationship to FTC1 and FTC2.
- $\star$ You will be asked to derive the substitution technique of antidifferentiation from the chain rule and FTC1.

Date: 10.XII.14.

[1]Starred items from the midterm review sheet will not necessarily appear on the final.

Exponential, logarithmic, and inverse functions.

- Know the definition of the (natural) logarithm function and how FTC2 then gives the derivative of $\log$.
- Interpret the logarithm as a "multiplication-to-addition machine" and know the properties which follow from this interpretation.
- Know the definition of the exponential function as the inverse to $\log$.
- Know the inverse function theorem and how it implies that $\frac{\mathrm{d}}{\mathrm{d}x}[e^x] = e^x$.
- Interpret the exponential function as an "addition-to-multuplication machine" and know the properties which follow from this interpretation.
- Understand logarithms and exponentials with other bases, including their basic properties and differentiation/integration rules.
- Remember the definition of the various inverse trigonometric functions and their derivatives (at least $\frac{\mathrm{d}}{\mathrm{d}x}[\arcsin x]$ and $\frac{\mathrm{d}}{\mathrm{d}x}[\arctan x]$).

Special integration techniques.

- ⋆ Know the statement of the integration by parts rule and its relation to the product rule for differentiation.
- Be able to use integration by parts as an integration technique.
- Understand basic techniques for integrating functions involving trigonometric functions.
- Be able to implement trigonometric substitutions in simple examples.
- Understand the method of partial fractions as it applies to integrating rational functions, especially for functions which have distinct linear factors in the denominator.
- Know how to define and evaluate improper integrals.

Differential equations.

- Be able to interpret word problems in the language of differential equations.
- Have the ability to solve simple differential equations via separation of variables.

Integration rules. Finally, I've compiled a handy reference sheet of common (indefinite) integrals. All of them are good to know, ⋆'s are necessary to know for the exam. Here f and g are integrable functions, k and c are constants.

- ⋆ $\int kf(x)\,dx = k\int f(x)\,dx$
- ⋆ $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$
- ⋆ $\int x^k\,dx = \frac{1}{k+1}x^{k+1} + c$, $k \neq -1$
- ⋆ $\int e^x\,dx = e^x + c$
- ⋆ $\int \frac{dx}{x} = \log|x| + c$

- $\int a^x \, dx = \frac{1}{\log a} a^x + c$
- $\star$ $\int \sin x \, dx = -\cos x + c$
- $\star$ $\int \cos x \, dx = \sin x + c$
- $\int \tan x \, dx = -\log|\cos x| + c$
- $\int \cot x \, dx = \log|\sin x| + c$
- $\int \sec x \, dx = \log|\sec x + \tan x| + c$
- $\int \csc x \, dx = -\log|\csc x + \cot x| + c$
- $\star$ $\int \sec^2 x \, dx = \tan x + c$
- $\star$ $\int \csc^2 x \, dx = -\cot x + c$
- $\int \sec x \tan x \, dx = \sec x + c$
- $\int \csc x \cot x \, dx = -\csc x + c$
- $\star$ $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$
- $\star$ $\int \frac{dx}{1+x^2} = \arctan x + c$
- $\int \frac{dx}{x\sqrt{x^2-1}} = \operatorname{arcsec}|x| + c$