

MATH 412: TOPICS IN ALGEBRA
HOMEWORK DUE FRIDAY WEEK 1

Problem 1. Let Σ_n denote the permutation group of $\{1, 2, \dots, n\}$ and let $\text{sgn} : \Sigma_n \rightarrow \{\pm 1\}$ denote the sign permutation. State three or more different ways to compute sgn . Consider the logical dependencies between your methods of computation. Which serves best as a primary definition of sgn ? Write a paragraph or two explaining your reasoning. (You are welcome to use references to look up information on sgn , but frame your discussion in your own words.)

Problem 2. For $n \geq 2$, let x_1, \dots, x_n be variables and define

$$V_n := \prod_{1 \leq i < j \leq n} (x_j - x_i).$$

Is V_n a symmetric polynomial? If so, prove it; if not, determine (with proof) the largest subgroup of Σ_n which fixes V_n .

Problem 3. Let F be a field. Show that the polynomial ring in two variables $F[x, y]$ is not a PID. Conclude that $F[x_1, x_2, \dots, x_n]$ is not a PID for $n \geq 2$.