

MATH 412: TOPICS IN ALGEBRA
HOMEWORK DUE FRIDAY WEEK 11

Let $\mathcal{N}_m = \{f \in \mathbb{F}_p[x] \mid f \text{ is monic irreducible of degree } m\}$ and let $N_m = |\mathcal{N}_m|$. Recall that

$$(1) \quad N_m = \frac{1}{m} \sum_{d|m} \mu(d) p^{m/d}.$$

Problem 1. Use Equation 1 to compute N_6 and N_{36} .

Problem 2. Use Equation 1 to prove that $N_m > 0$ for all $m > 0$. Since $\mathbb{F}_p[x]/(f) \cong \mathbb{F}_{p^m}$ for $f \in \mathcal{N}_m$, this provides an alternate proof that \mathbb{F}_{p^m} exists.