MATH 412: TOPICS IN ALGEBRA
HOMEWORK DUE FRIDAY WEEK 9

Problem 1. The Möbius function is defined for positive integers n by

$$\mu(n) = \begin{cases} 1 & \text{if } n = 1, \\ (-1)^s & \text{if } n = p_1 \cdots p_s \text{ for distinct primes } p_1, \ldots, p_s, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that

$$\sum_{d|n} \mu(n/d) = 0$$

when $n > 1$.

Problem 2. Let μ be the Möbius function defined above. Prove that

$$\Phi_n(x) = \prod_{d|n} (x^d - 1)^{\mu(n/d)}.$$

Problem 3. Compute $\Phi_{105}(x)$ by hand, either from the definition or using the formula from Problem 2.

Problem 4. Prove that an angle of measure $\pi/3$ cannot be trisected. [*Hint*: Let $x = \cos(\pi/9)$ and use the triple angle formula to express $1/2 = \cos(\pi/3)$ in terms of x.]