MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE MONDAY WEEK 9

Problem 1. If γ is a simple closed path with -1 and 1 on its inside and f is an entire function, show that

$$\int_\gamma \frac{f(z)}{z^2-1}\,dz = \pi i(f(1)-f(-1)).$$

Problem 2. Let f be analytic on a region A and let γ be a closed curve in A. For any $z_0 \in A \setminus \gamma(I)$, show that

$$\int_\gamma \frac{f'(\zeta)}{\zeta - z_0}\,d\zeta = \int_\gamma \frac{f(\zeta)}{(\zeta - z_0)^2}\,d\zeta.$$

Bonus: Can you think of a way to generalize this result?

Problem 3. Prove that if f is analytic in a disc $D_r(z_0)$ except at z_0, where it has a pole of order k, then, in the annulus $\{z \mid 0 < |z - z_0| < r\}$, the Laurent series expansion for f has only finitely many terms with negative exponent and is of the form $\sum_{n=-k}^{\infty} c_n(z-z_0)^n$ with $c_{-k} \neq 0$.

Problem 4. Find $\int_{|z|=\pi} \tan z\,dz$.

Problem 5. Find $\int_{|z|=2} e^z/(z^2-1)\,dz$.

Problem 6. Evaluate the contour integral

$$\int_\gamma \frac{e^{-z^2}}{z^2}\,dz,$$

where

(a) γ is the positively oriented square with vertices $-1-i$, $1-i$, $1+i$, and $-1+i$, and
(b) γ is the positively oriented ellipse $\gamma(t) = a\cos t + ib\sin t$, where $a, b > 0$ and $0 \leq t \leq 2\pi$.