MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 8

Problem 1. Map out the dependencies of all the major theorems leading up to the general version of Cauchy's theorem for analytic functions and nullhomologous 1-cycles. Draw a directed graph with vertex set consisting of major theorems and edges indicating direct dependence ($A \to B$ when B uses A in its proof). Your vertex set should contain (at least) Cauchy's theorem for triangles, Cauchy's theorem for convex open sets, Cauchy's integral formula for convex open sets, Liouville's theorem, Morera's theorem, and Cauchy's integral formula for nullhomologous 1-cycles.

Problem 2. Let $R \subseteq \mathbb{C}$ be a rectangle and let γ be a path which traverses the boundary of R once in the positive direction. Partition R into four subrectangles R_j, $1 \leq j \leq 4$, by joining each pair of opposite sides of R with a perpendicular line segment. Let γ_j be the path which traverses the boundary of R_j once in the positive direction. Let $\Gamma = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4$ and set $E = \Gamma(I)$. Show that γ and Γ are E-equivalent but not equivalent.

Problem 3. Let $\Gamma = \gamma_1 - \gamma_2 - \gamma_3$, where γ_1, γ_2, and γ_3 are positively oriented circles with radii 5, 1, and 1 and centers 0, -2, and 3, respectively. Find

$$\int_\Gamma \frac{dz}{(z+2)(z-3)}.$$

Problem 4. For the cylce Γ of the previous problem, find

$$\int_\Gamma \frac{dz}{z(z+2)(z-3)}$$

by applying the Cauchy integral formula.

Problem 5. If γ is a simple closed path with 0 on its inside and all other integral multiples of π on its outside, find

$$\int_\gamma \frac{dz}{\sin z}.$$

Problem 6. Find the Laurent series expansions of the following functions in $\mathbb{C} \setminus \{0\}$.

(a) $\sin(1/z)$
(b) $e^z + e^{1/z}$
(c) $e^{z+1/z}$

Problem 7. Let c_n denote the n-th coefficient in the Laurent series expansion of $1/\sin z$ in the annulus $\{z \mid 0 < |z| < \pi\}$.

(a) Show that $c_n = 0$ if n is even or if $n < -1$.
(b) Find c_{-1}, c_1, and c_3.

Problem 8. If γ is a simple closed path with 0 and 3 inside it, use the residue theorem to determine

$$\int_\gamma \frac{dz}{z^2 - 3z}.$$