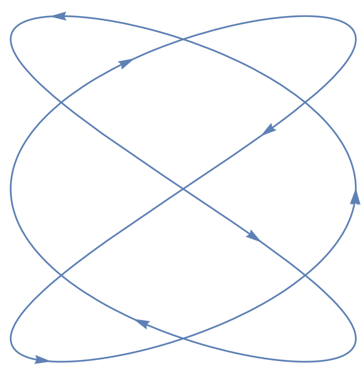
MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 5

Problem 1. Use Cauchy's integral formula to calculate
$$\int_{|z-1|=1} \frac{dz}{z^2-1} \qquad \text{and} \qquad \int_{|z+1|=1} \frac{dz}{z^2-1}.$$
Use these calculations and Cauchy's theorem to compute
$$\int_{|z|=3} \frac{dz}{z^2-1}.$$

Problem 2. Using only the definition of *connected*, prove that the union of a family of connected sets with a point in common is also a connected set.

Problem 3. Determine the value of $\mathrm{Ind}_\gamma(z)$ in each of the components of $\mathbb{C} \smallsetminus \gamma(I)$ for γ the path pictured below with parameter interval I.



Problem 4. Use the Weierstrass M-test to show that the series $\displaystyle\sum_{k=1}^{\infty} \frac{k+z}{k^3+1}$ converges uniformly on $\overline{D}_1(0)$.

Problem 5. Prove that the series $\displaystyle\sum_{k=1}^{\infty} k^{-z}$ converges uniformly on each set of the form $\{z \in \mathbb{C} \mid \mathrm{Re}(z) > s\}$, with $s > 1$. (The function to which it converges is the famous Riemann zeta function, $\zeta(z)$.)

Problem 6. Using the power series expansion $\displaystyle\frac{1}{1+w} = \sum_{k=0}^{\infty} (-1)^k w^k$, find a power series expansion for $\displaystyle\int_0^z \frac{dw}{1+w}$ about 0. What is the radius of convergence of this power series? What function does it converge to?

Problem 7. Find a power series expansion of $\sqrt{1+z}$ about 0, where the square root function is defined in terms of the principal branch of the log function (as in Example 1.4.9 in the textbook). What is the radius of convergence of this series?