MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 4

Problem 1. Calculate $\int_\gamma dz/z$ if γ is any path in $\mathbb{C}$ joining $-i$ to i which does not cross the non-positive real axis $\mathbb{R}_{\leq 0}$. (You may use the result of Example 2.2.11 in the book.)

Problem 2. Let Δ be the triangle with vertices $1 - i$, i, and $-1 - i$ and let S be the square with vertices $1 - i, 1 + i, -1 + i$, and $-1 - i$. If f is any function which is analytic on $\mathbb{C} \smallsetminus \{0\}$, prove that

$$\int_{\partial \Delta} f = \int_{\partial S} f$$

where $\partial \Delta$ and ∂S are traversed in the counterclockwise direction.

Problem 3. Calculate $\int_\gamma dz/(1 - e^z)$ where $\gamma : [0, 2\pi] \to \mathbb{C}$ is given by $\gamma(t) = 2i + e^{it}$.

Problem 4. Show that the principal branch of the $\log$ function can be described by the formula $\log(z) = \int_1^z dw/w$ for $z \in \mathbb{C} \smallsetminus \mathbb{R}_{\leq 0}$.

Problem 5. Use Cauchy's integral formula to calculate

$$\int_{|z|=1} \frac{e^z}{z} \, dz.$$