MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 2

Problem 1. For which $z \in \mathbb{C}$ does the series $\sum_{n=0}^{\infty} 1/(n^2 + z^2)$ converge?

Problem 2. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} nz^n$.

Problem 3. Determine the values of z for which e^z is a real number. Also determine all z such that e^z is purely imaginary.[1]

Problem 4. Show that $\cos ix = \cosh x$ and $\sin ix = i \sinh x$, where $\cosh$ and $\sinh$ are the hyperbolic cosine and hyperbolic sine functions.

Problem 5. Recall the geometric identity $1 + z + z^2 + \cdots + z^n = \frac{1-z^{n+1}}{1-z}$. Use this to derive the trigonometric identity

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos n\theta = \frac{1}{2} + \frac{\sin(n + 1/2)\theta}{2\sin\theta/2}.$$

Problem 6. Verify the Cauchy-Riemann equations for the function $z \mapsto z^2 + 5z - 1$.

Problem 7. Prove that $z \mapsto |z|$ is not holomorphic.

[1] We call $w \in \mathbb{C}$ *purely imaginary* when $\operatorname{Re} w = 0$.