MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 12

Problem 1. Find a linear fractional transformation that takes the line $\mathrm{Re}(z) = 1$ to the circle of radius one centered at -1.

Problem 2. What is the image of the unit disc under the linear fractional transformation

$$h(z) = \frac{2iz}{z-1}?$$

Problem 3. Find a conformal automorphism of the open unit disc D which takes $1/2$ to 0 and has derivative $4i/3$ at $1/2$.

Problem 4. Let $\mathfrak{h} = \{z \in \mathbb{C} \mid \mathrm{Im}(z) > 0\}$ denote the upper half plane. Prove that the group of conformal automorphisms of $\mathfrak{h}$ is isomorphic to $\mathrm{PSL}_2(\mathbb{R})$, the group of 2×2 real matrices with determinant 1 modulo $\{\pm I\}$. (You are welcome to expand on the argument sketched in the course notes.)

Problem 5. (a) Determine a matrix representative in $\mathrm{SL}_2(\mathbb{C})$ of the conformal automorphism $z \mapsto e^{i\theta} z$ representing rotation of the Riemann sphere by angle θ.

(b) Which matrix in $\mathrm{SL}_2(\mathbb{C})$ represents rotation about the i-axis by angle θ? (Here the i-axis is considered as the x_2-axis in the sterographic projection model of the Riemann sphere.)

Problem 6. Which of the following sets is conformally equivalent to the open unit disc D? Prove your assertion.

(a) $\mathbb{C}$

(b) an open rectangle in the plane

(c) $\{z \in \mathbb{C} \mid \mathrm{Re}(z) > 0\} \setminus [1, \infty)$

Problem 7. Let U be a proper, open, simply connected subset of $\mathbb{C}$ and fix a conformal equivalence $h : U \to D$ where D is the open unit disc. Describe all other conformal equivalences $U \to D$ in terms of h.