MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 11

Problem 1. Prove that a meromorphic function which is even and has an isolated singularity at 0 has residue 0 at 0.

Problem 2. Find $\zeta(4) = \sum_{n=1}^{\infty} \frac{1}{n^4}$.

Problem 3. Prove that the function $\pi / \sin(\pi z)$ has a pole at each integer n with residue $(-1)^n$ and no other poles. Use this to derive a method for summing series of the form $\sum_{n=-\infty}^{\infty} (-1)^n f(n)$, where f is meromorphic with a finite number of poles.

Problem 4. Find a conformal equivalence from the upper half disc $\{z \in \mathbb{Z} \mid |z| < 1, \operatorname{Im} z > 0\}$ to the unit disc.

Problem 5. If D is the unit disc, find a conformal equivalence from $D \setminus \{0\}$ to $\mathbb{C} \setminus \overline{D}$.

Problem 6. If $f(z) = z^4/(z^2 - 1)$ is considered as an analytic function $S^2 \to S^2$, where are the poles of f and what are their orders?

Problem 7. Show that $\sin z$ is not an analytic function $S^2 \to S^2$. (In fact, the only rational analytic functions $S^2 \to S^2$ are the rational functions. As a challenge problem, you may attempt to prove this deeper fact.)