MATH 311: COMPLEX ANALYSIS
HOMEWORK DUE FRIDAY WEEK 10

Problem 1. Open sets $U, V \subseteq \mathbb{C}$ are called *homeomorphic* if there exists a continuous function $f : U \to V$ admitting a continuous inverse $f^{-1} : V \to U$. Suppose that U, V are homeomorphic open subsets of $\mathbb{C}$ and that U is simply connected. Prove that V is simply connected.

Problem 2. Which of the following open sets in $\mathbb{C}$ are simply connected? (Prove your assertion, perhaps via Theorem 4.6.16.)

(a) $\mathbb{C} \setminus \{0\}$
(b) $D_2(0) \setminus [-1, 1]$
(c) $\mathbb{C} \setminus (-\infty, 0]$
(d) The bounded open set with boundary the line segments joining 1 to $2 + 2i$ to i to $-2 + 2i$ to -1 to $-2 - 2i$ to $-i$ to $2 - 2i$ to 1, consecutively.

Problem 3. Find $\operatorname{Res}(f, 0)$ and $\operatorname{Res}(f, 2)$ if $f(z) = \frac{\cos z}{2z - z^2}$.

Problem 4. Prove that if f has a simple pole at z_0 and g is analytic in a neighborhood of z_0, then $\operatorname{Res}(gf, z_0) = g(z_0) \operatorname{Res}(f, z_0)$. Show by example that this is not true if f has a pole of degree greater than 1 at z_0.

Problem 5. Use long division of power series to find the residue at 0 of $\tanh(z)/z$. (You may assume the usual Taylor series for $\cosh$ and $\sinh$, but do not assume the power series for $\tanh$.)

Problem 6. Compute the following integrals using the methods of §5.2.

(a) $\int_0^{2\pi} \frac{d\theta}{10 + 6 \sin \theta}$
(b) $\int_0^{\infty} \frac{x^2}{(1 + x^2)^2}\, dx$