MATH 202: VECTOR CALCULUS
HOMEWORK FOR MONDAY WEEK 9

Let $W \subseteq \mathbb{R}^3$ be a solid region with density $\delta : W \to \mathbb{R}$ and total mass $M = \int_W \delta$. The gravia-tional potential of W acting on a point at (x_0, y_0, z_0) of mass m is

$$V(x_0, y_0, z_0) = -\int_W \frac{Gm\delta(x, y, z)}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}.$$

Suppose W is the region between two concentric spheres of radii $a < b$, centered at the origin. Assume that W has total mass M and constant density δ. In the following problems, you will compute the gravitational potential $V(x_0, y_0, z_0)$ of W acting on a point mass m concentrated at (x_0, y_0, z_0). Note that by the spherical symmetry of W, there is no loss of generality in taking $(x_0, y_0, z_0) = (0, 0, r)$.

Problem 1. Show that if $r \geq b$, then $V(0, 0, r) = -GMm/r$. (This is the same potential as if W were a point mass M concentrated at the origin, but you should compute the actual integral for V.)

Problem 2. Show that if $r \leq a$, then gravitational force is 0. (The gravitational force is $F = -\nabla V$, so it suffices to show that V is constant for $r \leq a$.)

Problem 3. Find $V(0, 0, r)$ if $a < r < b$. How does this compare to the previous two answers?