

**MATH 202: VECTOR CALCULUS**  
**HOMEWORK FOR WEDNESDAY WEEK 2**

*Problem 1.* Show that every multilinear function  $(\mathbb{R}^n)^m \rightarrow \mathbb{R}$  is continuous. (Here multilinear means that the function is linear in each of its  $m$  coordinates of the form  $\mathbb{R}^n$ .) [*Hint:* Look up CAES Exercise 3.6.1 for one potential method.]

*Problem 2 (CAES 3.6.10).* Show that the *Vandermonde matrix*

$$\begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$$

has determinant  $(b-a)(c-a)(c-b)$  *without* resorting to explicitly expanding the determinant. For what values  $a, b, c$  is this matrix invertible? *Bonus:* Generalize the Vandermonde matrix and this determinant identity to  $n \times n$  matrices.

*Problem 3.* Draw a picture of the tetrahedron in  $\mathbb{R}^3$  with vertices at  $(0, 0, 0)$ ,  $(1, -1, 1)$ ,  $(3, 1, -1)$ , and  $(1, 3, 0)$ . Use the determinant function to determine its volume.