MATH 202: VECTOR CALCULUS
HOMEWORK FOR MONDAY WEEK 13

Problem 1. *CAES* 9.16.1.

Problem 2. Throughout this problem, let $\Phi : [0,1] \to \mathbb{R}^2$ denote a singular 1-cube which is also a simple closed curve (*i.e.*, $\Phi(0) = \Phi(1)$ and $\Phi(s) \neq \Phi(t)$ for any other $s \neq t$).

(a) Suppose that the image of Φ does not enclose the origin. Use Green's theorem to determine the value of
$$\int_\Phi \frac{x\,dx + y\,dy}{x^2+y^2}.$$

(b) Now suppose that the image of Φ does enclose the origin. Can you use Green's theorem to determine the above integral? Explain why or why not.

(c) Let $\Phi_1, \Phi_2 : [0,1] \to \mathbb{R}^2$ be simple closed curves which both enclose the origin, are both oriented counterclockwise, and do not intersect. Show that
$$\int_{\Phi_1} \frac{x\,dx + y\,dy}{x^2+y^2} = \int_{\Phi_2} \frac{x\,dx + y\,dy}{x^2+y^2}.$$

(d) Use the result of part (c) to determine the value of
$$\int_\Phi \frac{x\,dx + y\,dy}{x^2+y^2}$$
where Φ is any simple closed curve that encloses the origin.