MATH 202: VECTOR CALCULUS
FRIDAY WEEK 8 HANDOUT

Problem 1. Let D be the cylindrical shell about the z-axis with inner radius 1, outer radius 2, and $0 \leq z \leq 1$. In this problem, we consider the integral

$$\int_D x^2 + y^2.$$

(a) Make a sketch of D.
(b) Find a region K such that $\Psi(K) = D$ for Ψ the cylindrical change of coordinates.
(c) Compute $|\det \Psi'|$.
(d) Convert the integral $\int_K (f \circ \Psi)|\det \Psi'|$ into a multiple integral where $f(x, y, z) = x^2 + y^2$.
(e) Use the above expression and change of variables theorem to compute $\int_D x^2 + y^2$. Are there any subtleties with the hypotheses of COV?

Problem 2. Let $D = B_3(1) \smallsetminus C$ where $C = \{(x, y, z) \mid z^2 \geq x^2 + y^2\}$. In this problem, we consider the integral

$$\int_D \sqrt{x^2 + y^2 + z^2}.$$

(a) Make a sketch of D.
(b) Find a region K such that $\Xi(K) = D$ for Ξ the spherical change of coordinates.
(c) Compute $|\det \Xi'|$.
(d) Convert the integral $\int_K (g \circ \Xi)|\det \Xi'|$ into a multiple integral where $g(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
(e) Use the above expression and change of variables theorem to compute $\int_D \sqrt{x^2 + y^2 + z^2}$. Are there any subtleties with the hypotheses of COV?

Problem 3 (*CAES* 6.7.8). A closed ball of radius b centered at the origin has density $\delta(x, y, z) = e^{-(x^2+y^2+z^2)^{3/2}}$. Find its mass, $\int_{B_3(b)} \delta$.