MATH 202: VECTOR CALCULUS
WEDNESDAY WEEK 4 HANDOUT

Problem 1. For the following functions, determine all second-order partial derivatives:
(a) $f(x, y) = \cos(xy)$,
(b) $g(x, y, z) = e^{ax}\sin(y) + e^{bx}\cos(z)$.

Problem 2. A function $f : \mathbb{R}^n \to \mathbb{R}$ is called *harmonic* :iff $\Delta f = 0$ for $\Delta = D_{11} + D_{22} + \cdots + D_{nn}$ the Laplacian operator.
(a) Is $f(x, y, z) = x^2 + y^2 - 2z^2$ harmonic? What about $f(x, y, z) = x^2 - y^2 + z^2$?
(b) Give an example of a harmonic function of n variables, and verify that your example is correct.

Problem 3. A function $f : \mathbb{R}^n \to \mathbb{R}$ is called *radial* :iff $f = f \circ T$ for all orthognal transformations $T : \mathbb{R}^n \to \mathbb{R}^n$. The value of such a function at $x \in \mathbb{R}^n$ only depends on $|x|$ (but we will not prove this). Show that the Laplacian of a radial function is radial.

Problem 4. Check that $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is an orthogonal matrix. Show that if $f : \mathbb{R}^2 \to \mathbb{R}$ is harmonic, then $f \circ R_\theta$ is harmonic.