

MATH 202: VECTOR CALCULUS
MONDAY WEEK 4 HANDOUT

Problem 1. Consider the function $f(x, y) = \frac{x^2 - y}{y + 1}$ on $\{(x, y) \in \mathbb{R}^2 \mid y \neq -1\}$. At which (a, b) is f differentiable? What is $f'(a, b)$?

Problem 2. Consider the function

$$g(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

At which (a, b) is g differentiable? What is $g'(a, b)$?

Problem 3. Define $h : \mathbb{R} \rightarrow \mathbb{R}$ by $h(x) = x^2 \sin \frac{1}{x}$ if $x \neq 0$, and $h(0) = 0$. Show that $h'(x)$ exists for all x but that h' is discontinuous at 0. [It follows that differentiability does not imply that a function has continuous derivatives!]