MATH 202: VECTOR CALCULUS
MONDAY WEEK 3 HANDOUT

Question 1. How should we define the notations $f(h) = o(g(h))$ and $f(h) = \mathcal{O}(g(h))$ for $f, g : B(0_n, \varepsilon) \to \mathbb{R}^m$?

Problem 2. Continutiy of a linear map $T : \mathbb{R}^n \to \mathbb{R}^m$ and compactness of $S^{n-1} := \{x \in \mathbb{R}^n \mid |x| = 1\}$ imply that there is a positive constant c such that $|T(h_0)| \leq c$ for $h_0 \in S^{n-1}$. The smallest such c is attained (extreme value theorem) and is called the *operator norm* of T, denoted $||T||$. Check that $||\ ||$ is absolute homogeneous, subadditive, and positive definite. [*Fact*: $||T||$ is the square root of the absolute value of the largest eigenvalue of $TT^\top$.]

Problem 3. Check the absorption rules for the compositions $\varphi \circ \psi$ and $\psi \circ \varphi$ of the functions $\varphi(x) = x^3$, $\psi(x) = x \sin(1/x)$.