MATH 202: VECTOR CALCULUS
MONDAY WEEK 2 HANDOUT

Problem 1. Identify the limit points for the following subsets of $\mathbb{R}^n$. Sketch a proof of your assertion, but don't write down all the details.

(1) $(0,1) \subseteq \mathbb{R}$
(2) $\mathbb{Q} \subseteq \mathbb{R}$
(3) $\{x \in \mathbb{R}^3 \mid |x| = 1/n$ for some integer $n > 0\}$

Problem 2. For $A \subseteq \mathbb{R}^n$ let $L(A)$ denote the set of limit points of A. We define the *closure* of A to be $\overline{A} := A \cup L(A)$. Prove that $\overline{A}$ is the smallest closed set containing A, *i.e.*, if $A \subseteq B \subseteq \mathbb{R}^n$ and B is closed, then $A \subseteq B$. Find the closures of the sets in Problem 1.

Problem 3. Let $A \subseteq \mathbb{R}^n$. Prove that if $f \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^m)$ and $C \subseteq \mathbb{R}^m$ is closed, then $f^{-1}(C)$ is closed.

Problem 4. Is the preimage of a compact set under a continuous mapping compact? What does your answer tell you about the set of solutions to the equation $f(x) = 0$ when $f \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^m)$?