MATH 202: VECTOR CALCULUS
FRIDAY WEEK 2 HANDOUT

Problem 1. Determine the area of the triangle in $\mathbb{R}^3$ with vertices $(1,1,1)$, $(-1,0,-1)$, $(0,2,1)$.

Problem 2. Define the *scalar triple product* of three vectors $u, v, w \in \mathbb{R}^3$ to be $u \cdot (v \times w)$. Prove that $|u \cdot (v \times w)|$ is the volume of the parallelipiped spanned by u, v, w.

Problem 3. Let $A \subseteq \mathbb{R}^n$. Prove that if $f \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^m)$ and $C \subseteq \mathbb{R}^m$ is closed, then $f^{-1}(C)$ is closed.

Problem 4. Is the preimage of a compact set under a continuous mapping compact? What does your answer tell you about the set of solutions to the equation $f(x) = 0$ when $f \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^m)$?

Problem 5. Use the fact that 21375, 38798, 34162, 40223, and 79154 are all divisible by 19 to show that

$$\begin{pmatrix} 2 & 1 & 3 & 7 & 5 \\ 3 & 8 & 7 & 9 & 8 \\ 3 & 4 & 1 & 6 & 2 \\ 4 & 0 & 2 & 2 & 3 \\ 7 & 9 & 1 & 5 & 4 \end{pmatrix}$$

is divisible by 19 without directly evaulating the determinant. Generalize this observation to all square matrices with entries single digit natural numbers.