MATH 202: VECTOR CALCULUS
FRIDAY WEEK 1 HANDOUT

Problem 1. Consider the sequences

$$\{x_\nu = \left(\frac{\nu^2+\nu-1}{3\nu^2+2}, \frac{\nu-1}{\nu+1}\right)\}$$

in $\mathbb{R}^2$ and the sequence

$$\{y_\nu = (1+(-1)^\nu, 1/\nu, 1+1/\nu)\}$$

in $\mathbb{R}^3$. Do these sequences converge? Prove your assertion.

Problem 2. Suppose $\{x_\nu\}$ is a sequence in $\mathbb{R}^2$ that converges to a point $p \in \mathbb{R}^2$. Let $\theta(x)$ denote the angle between x and the positive horizontal axis. Does the sequence $\{\theta(x_\nu)\}$ converge? Give as complete an answer as you can.

Problem 3. For the following two functions on $\mathbb{R}^2$, either find $b \in \mathbb{R}$ such that the function is continuous at 0 or prove that there is no b for which the function is continuous at 0:

$$f(x,y) = \begin{cases} \frac{x^4-y^4}{(x^2+y^2)^2} & \text{if } (x,y) \neq 0, \\ b & \text{if } (x,y) = 0, \end{cases} \qquad g(x,y) = \begin{cases} \frac{x^2-y^3}{x^2+y^2} & \text{if } (x,y) \neq 0, \\ b & \text{if } (x,y) = 0. \end{cases}$$