MATH 202: VECTOR CALCULUS
WEDNESDAY WEEK 10 HANDOUT

Problem 1. Let $\Phi : B_2(3) \to \mathbb{R}^3$ be the 2-surface taking $(u, v) \mapsto (u, v, 9 - u^2 - v^2)$, and let ω be the syntactic 2-form $2dx \wedge dy - dx \wedge dz - 3dy \wedge dz$.
(a) Make a sketch of the surface which Φ parametrizes.
(b) Compute the Jacobian matrix $\Phi'(u, v)$.
(c) Compute $\int_\Phi \omega$.

Problem 2. Let $\gamma : [0, 2\pi] \to \mathbb{R}^3$ be the 1-surface taking $t \mapsto (3\cos t, 3\sin t, 0)$, and let λ be the syntactic 1-form $(2z - y)dx + (x + z)dy + (3x - 2y)dz$.
(a) Make a sketch of the curve which γ parametrizes.
(b) Compute the Jacobian matrix $\gamma'(t)$.
(c) Compute $\int_\gamma \lambda$.

Problem 3. For a 1-form $\omega = f_1 dx + f_2 dy + f_3 dz$ on $\mathbb{R}^3$ define its *derivative*

$$d\omega := (D_1 f_2 - D_2 f_1)dx \wedge dy + (D_1 f_3 - D_3 f_1)dx \wedge dz + (D_2 f_3 - D_3 f_2)dy \wedge dz.$$

(You will see this deinition in generality in §9.8.) Compute $d\lambda$ for the λ the 1-form in Problem 2. How is it related to ω from Problem 1? Does the equation

$$\int_{\partial \Phi} \lambda = \int_\Phi d\lambda$$

make any sense?