MATH 201: LINEAR ALGEBRA
HOMEWORK DUE TUESDAY WEEK 9

Problem 1. Let A be the 3×3 matrix shown below.
(a) Compute $\det(A)$ by expanding along the second column.
(b) Use the formula $A^{-1} = (\det A)^{-1} \operatorname{adj}(A)$ to compute the inverse of A.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Problem 2.
(a) Use the permutation expansion of the determinant to prove that the determinant of an upper triangular matrix is the product of its diagonal entries.
(b) Use the cofactor expansion of the determinant to prove the same result.

Problem 3. If $\sigma \in \Sigma_n$ is a permutation of the set $\{1, \ldots, n\}$ and F is a field, recall that the permutation matrix $P_\sigma \in M_{n\times n}(F)$ has $e_{\sigma(i)}$ as its i-th column. Prove the following statements.
(a) $P_{\sigma\circ\tau} = P_\sigma P_\tau$ for all $\sigma, \tau \in S_n$.
(b) The set $\{P_\sigma : \sigma \in S_n\}$ is equal to the set of all matrices in $M_{n\times n}(F)$ having exactly one 1 in every row and column, and zeros elsewhere.