# MATH 201: LINEAR ALGEBRA
# HOMEWORK DUE TUESDAY WEEK 7

**Problem 1.** Let $F$ be a field and $T : F^n \to F^n$ a linear transformation. Let $A$ be the matrix of $T$ with respect to the standard ordered basis of $F^n$. Show that the column space of $A$ is equal to the image of $T$.

**Problem 2.** Let $V$ be a two-dimensional vector space, and let $B$ be an ordered basis for $V$. Let $T \in \mathcal{L}(V, V)$, and suppose that the matrix of $T$ with respect to $B$ is $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Show that

$$T^2 - (a + d)T + (ad - bc)I = 0,$$

where $T^2 = T \circ T$ and $I \in \mathcal{L}(V, V)$ is the identity map.

**Problem 3.** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (3x, x - y, 2x + y + z).$$

Show that $T$ is an isomorphism and find an explicit description of its inverse.

**Problem 4** (Bonus)**.** Let $V$ and $W$ be finite-dimensional vector spaces and $T \in \mathcal{L}(V, W)$. Show that there exist bases $B$ and $B'$ for $V$ and $W$, respectively, so that the matrix $A$ of $T$ with respect to these bases satisfies $A_{ii} = 1$ for $1 \leq i \leq \operatorname{rank}(T)$, and all other entries of $A$ are 0.